MODELLING SHORT TERM EQUILIBRIUM AND LONG TERM CHANGE IN A NATURAL WAY

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PURPOSE

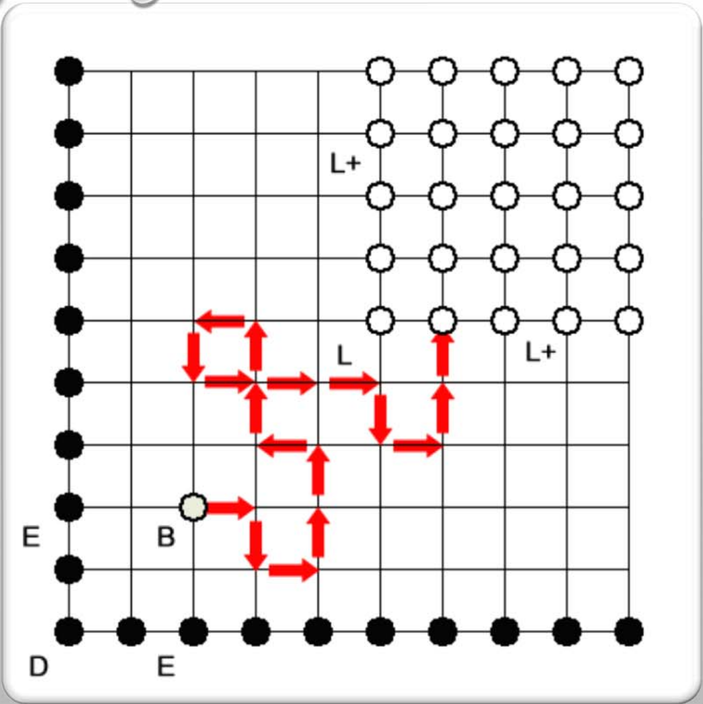
- TO BUILD A MARKOV CHAIN MODEL OF SYSTEMS USING ECONOMIC CONCEPTS
  - SIMPLE
  - EXTENSIBLE
- TO MODEL A BIOLOGY OR A PRE-MARKET ECONOMY AND SHOW THE RELATIONSHIP TO A MARKET ECONOMY
- USE TO MODEL GROWTH AND CHANGE
  - LARGE SCALE TRENDS
  - SPECIALISATION
- NO PREDATION IN THIS ITERATION

LARGE SCALE TRENDS

- "PROGRESS IS A NOXIOUS, CULTURALLY EMBEDDED, UNTESTABLE, NONOPERATIONAL, INTRACTABLE IDEA THAT MUST BE REPLACED IF WE WISH TO UNDERSTAND THE PATTERNS OF HISTORY" S. GOULD (1988)

- **MCSHEA'S LIST FOR BIOLOGY (1998):**
  - ENTROPY
  - ENERGY INTENSIVENESS
  - EVOLUTIONARY VERSATILITY
  - DEVELOPMENTAL DEPTH
  - STRUCTURAL DEPTH
  - ADAPTEDNESS
  - SIZE OF CREATURE
  - COMPLEXITY

- **TRENDS EXPECTED IN ECONOMICS:**
  - EFFICIENCY OF RESOURCE USAGE
  - AMOUNT (OF ECONOMY OR BIOMASS)



DESIGN

- HABITUAL BEHAVIOUR: SURVIVAL AND GROWTH NOT UTILITY AND PROFIT MAXIMISATION
- NON-RECIPROCAL, CO-OPERATIVE TRADING
- LIFE CYCLE
- DIFFERENT AGENT TYPES (EFFICIENCY)

LITERATURE REVIEW

- BIOLOGICAL EQUILIBRIUM MODELS
  - PICCIONE M. AND RUBINSTEIN A. 2007.
    EQUILIBRIUM IN THE JUNGLE, ECONOMIC JOURNAL, 117 (522), PP 883-896.
  - MCLEOD D. 2015.
    "AN ECONOMIC APPROACH TO THE EVOLUTION OF AN ECOLOGY" IN ISCS 2014: INTERDISCIPLINARY SYMPOSIUM ON COMPLEX SYSTEMS EDITED BY A. SANAYEI, O.E. ROSSLER AND I. ZELINKER, SPRINGER, SWITZERLAND, PP 343-58.
  - MARKEY-TOWLER B. 2016.
    LAW OF THE JUNGLE: FIRM SURVIVAL AND PRICE DYNAMICS IN EVOLUTIONARY MARKETS. JOURNAL OF EVOLUTIONARY ECONOMICS, 26(3) (JULY 2016), PP 655-696.
- HABITUAL BEHAVIOUR IN ECONOMICS
  - HODGSON G.M. 2004.
    RECLAIMING HABIT FOR INSTITUTIONAL ECONOMICS. JOURNAL OF ECONOMIC PSYCHOLOGY, 25(5).

NINE AGENT PROCESSES

- RESTING — NOTHING HAPPENS
- PRODUCTION OF ENDOWED RESOURCES (+) — $N \cdot p^r = L^r$
- PRODUCTION OF MANUFACTURED RESOURCES — $\mathbf{a}^{PRO\,s}(R \times 1) = \begin{matrix} R \\ S \end{matrix} \begin{bmatrix} -1 \\ 2 \end{bmatrix}$
- CONSUMPTION (-) — $\mathbf{a}^{CON}(R \times 1) = \begin{matrix} R \\ S \end{matrix} \begin{bmatrix} -1 \\ -1 \end{bmatrix}$
- SHEDDING EXCESS RESOURCES (-) — UPPER LIMIT U
- TRADE — $b_j^{TRA\,r} = k^r \left(\bar{\rho} - \rho_j \right)$
- REPRODUCTION — NET RESOURCE USE IS ZERO
- EXTINCTION — IF ANY RESOURCE RUNS OUT
- RECYCLING OF RESOURCES — FROM EXTINCT AGENTS

THE MARKOV CHAIN MATRIX

$$\mathbf{M} = \begin{array}{c} \\ E \\ R1S1 \\ R1S2 \\ R2S1 \\ L \end{array} \begin{array}{c} \begin{array}{ccccc} E & R1S1 & R1S2 & R2S1 & L \end{array} \\ \begin{bmatrix} 1 & p & p & p & . \\ . & p & p & p & . \\ . & p & p & p & . \\ . & p & p & p & . \\ . & p & p & p & 1 \end{bmatrix} \end{array}$$

- RHS EIGENVECTOR REPRESENTS THE DISTRIBUTION

$$\mathbf{M}\boldsymbol{\mu} = \lambda\boldsymbol{\mu}$$

RESOURCES

- RESOURCES HELD ARE GIVEN BY:

$$\mathbf{R} = \mathbf{XM}\boldsymbol{\mu} = \begin{array}{c} \\ R \\ S \end{array} \begin{array}{c} \begin{array}{ccccc} E & R1S1 & R1S2 & R2S1 & L \end{array} \\ \left[\begin{array}{ccccc} . & 1 & 1 & 2 & 2 \\ . & 1 & 2 & 1 & 2 \end{array} \right] \end{array} \begin{array}{c} \\ E \\ R1S1 \\ R1S2 \\ R2S1 \\ L \end{array} \begin{array}{c} \begin{array}{ccccc} E & R1S1 & R1S2 & R2S1 & L \end{array} \\ \left[\begin{array}{ccccc} 1 & p & p & p & . \\ . & p & p & p & . \\ . & p & p & p & . \\ . & p & p & p & . \\ . & p & p & p & 1 \end{array} \right] \end{array} \cdot \boldsymbol{\mu}$$

- IN EQUILIBRIUM:

$$\begin{aligned} \Delta\mathbf{R} &= \mathbf{X}\boldsymbol{\mu}_1 - \mathbf{X}\boldsymbol{\mu}_0 = \mathbf{XM}\boldsymbol{\mu}_0 - \mathbf{X}\boldsymbol{\mu}_0 \\ &= (\lambda - 1)\mathbf{X}\boldsymbol{\mu}_0 \\ &= 0 \end{aligned}$$

EXISTENCE OF SOLUTION

- Trading $\overset{TRA\,r}{b_j} = k^r\left(\bar{\rho} - \rho_j\right) \;\Rightarrow\; \mathbf{M} \;\Rightarrow\; \boldsymbol{\mu} \;\Rightarrow\; \rho \;\Rightarrow\; \overset{TRA\,r}{b_j}$???
- WE CAN REPRESENT THE WHOLE SYSTEM OF MANY AGENTS BY KRONECKER PRODUCT:

$$\mathbf{M}^{SYS} = \mathbf{M}_1 \otimes \mathbf{M}_2 \otimes \mathbf{M}_N$$

- EVERY POSSIBLE STATE OF THE SYSTEM IS REPRESENTED BY A STATE IN $\mathbf{M}^{SYS}$
- WE CAN EVALUATE ALL THE FUNCTIONS
- APPLY THE PERRON-FROBENIUS THEOREM: THERE EXISTS $\boldsymbol{\mu}^{SYS} > \mathbf{0}$ SUCH THAT:

$$\mathbf{M}^{SYS} \cdot \boldsymbol{\mu}^{SYS} = \boldsymbol{\mu}^{SYS}$$

- THE SOLUTION IS UNIQUE AND STABLE
- **BUT** SOLUTION IS DISTRIBUTIONAL, NOT SITUATIONAL

THE LINEAR PROGRAMMING PROBLEM

$$\mathbf{R}_1 = \mathbf{XM\mu}$$

$$= \mathbf{X}\left(\overset{PRO\,r}{\mathbf{M}} + \overset{PRO\,r}{\mathbf{M}} + \overset{CON}{\mathbf{M}} + \overset{SHED}{\mathbf{M}} + \overset{TRA\,r}{\mathbf{M}} + \mathbf{I}\right)\boldsymbol{\mu}$$

$$= \overset{PRO\,r}{\mathbf{b}} + \overset{PRO\,r}{\mathbf{a}} + \overset{CON}{\mathbf{a}} + \overset{SHED}{\mathbf{b}} + \overset{TRA\,r}{\mathbf{b}} + \mathbf{R}_0$$

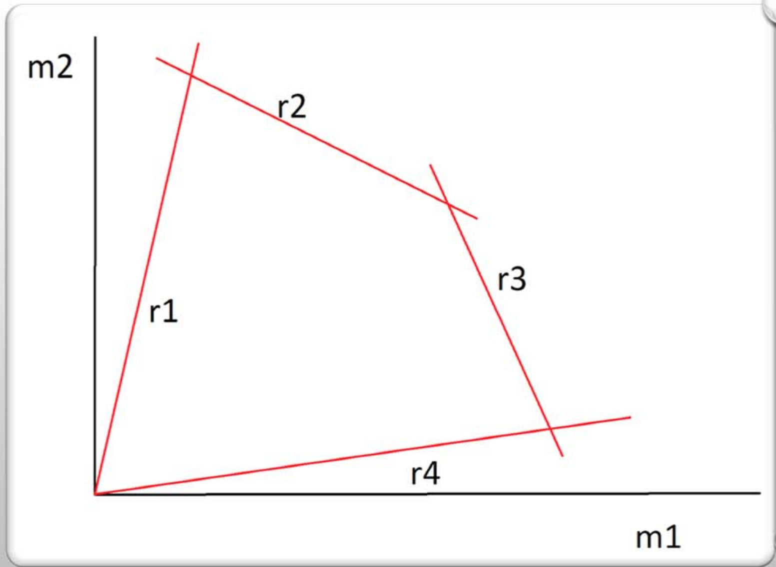
$$\Delta\mathbf{R} = \overset{PRO\,r}{\mathbf{b}} + \overset{PRO\,r}{\mathbf{a}} + \overset{CON}{\mathbf{a}} + \overset{SHED}{\mathbf{b}} + \overset{TRA\,r}{\mathbf{b}}$$

Summing over agents (trade nets out):

$$\Delta\mathbf{R} = \overset{PRO\,r}{\mathbf{B}}\ \mathbf{m} + \overset{PRO\,r}{\mathbf{A}}\ \mathbf{m} + \overset{CON}{\mathbf{A}}\ \mathbf{m} + \overset{SHED}{\mathbf{B}}\ \mathbf{m}$$

Sustainable points where $\Delta\mathbf{R} \geq \mathbf{0}$

$$\mathbf{L} + \overset{PRO\,r}{\mathbf{A}}\ \mathbf{m} + \overset{CON}{\mathbf{A}}\ \mathbf{m} + \overset{SHED}{\mathbf{B}}\ \mathbf{m} = \mathbf{0}$$



DYNAMICS I: THE LESLEY MATRIX

$$\underline{\mathbf{M}} = \begin{array}{c} \\ E \\ R1S1 \\ R1S2 \\ R2S1 \\ L \end{array} \begin{array}{c} \begin{array}{ccccc} E & R1S1 & R1S2 & R2S1 & L \end{array} \\ \left[\begin{array}{ccccc} 1 & p & p & p & . \\ . & p & p & p & S \\ . & p & p & p & . \\ . & p & p & p & . \\ . & p & p & p & . \end{array}\right] \end{array}$$

- LHS EIGENVECTOR REPRESENTS THE EXPECTED NUMBER OF DESCENDANTS
- EIGENVALUE λ REPRESENTS THE POPULATION GROWTH FACTOR $\mathbf{vM} = \lambda \mathbf{v}$

DYNAMICS II: GROWTH RATE EQUATION

- WE CAN SHOW $d\lambda = \mathbf{v} \cdot \mathbf{d}\underline{\mathbf{M}} \cdot \boldsymbol{\mu}$

$$= \sum_{RESOURCES} \mathbf{v} \cdot \begin{bmatrix} . & . & . & . & . \\ . & -\Delta p & . & . & . \\ . & \Delta p & -\Delta p & . & . \\ . & . & \Delta p & -\Delta p & . \\ . & . & . & \Delta p & . \end{bmatrix} \cdot \boldsymbol{\mu}$$

$$= \sum_{RESOURCES} \Delta p \cdot \mathbf{v} \begin{bmatrix} . & . & . & . & . \\ . & -1 & . & . & . \\ . & 1 & -1 & . & . \\ . & . & 1 & -1 & . \\ . & . & . & 1 & . \end{bmatrix} \boldsymbol{\mu}$$

$$= \mathbf{U} \cdot \boldsymbol{\Delta}\mathbf{p}$$

- WE ASSUME INTERIOR POINTS MUST INCREASE

DYNAMICS III: GROWTH ALONG A RAY

$$\overset{NET\ r1}{a_j} M_j + \overset{NET\ r1}{a_k} M_k + \overset{NET\ r1}{a_l} M_l = 0$$

$$\overset{NET\ r2}{a_j} M_j + \overset{NET\ r2}{a_k} M_k + \overset{NET\ r2}{a_l} M_l = 0$$

Differentiate:

$$\Delta\lambda_j \cdot \overset{NET\ r1}{a_j} M_j + \Delta\lambda_k \cdot \overset{NET\ r1}{a_k} M_k + \Delta\lambda_l \cdot \overset{NET\ r1}{a_l} M_l = 0$$

$$\Delta\lambda_j \cdot \overset{NET\ r2}{a_j} M_j + \Delta\lambda_k \cdot \overset{NET\ r2}{a_k} M_k + \Delta\lambda_l \cdot \overset{NET\ r2}{a_l} M_l = 0$$

Solution:

$$\mathbf{\Delta\lambda} = \Gamma \cdot \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

- GROWTH RATES MUST BE THE SAME
- SIMILARLY AS MORE RESOURCE CONSTRAINTS ARE ENCOUNTERED

DYNAMICS IV: THE COMMON GROWTH RATE

$$\mathbf{\Delta p} = \boldsymbol{\alpha} \cdot \boldsymbol{\varepsilon} = \begin{bmatrix} + & - & - \\ - & + & - \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{bmatrix}$$

$$\boldsymbol{\Delta\lambda} = \mathbf{U} \cdot \mathbf{\Delta p} = \begin{bmatrix} 0 & + & 0 \\ + & 0 & 0 \\ + & + & 0 \end{bmatrix} \begin{bmatrix} \Delta p_1 \\ \Delta p_2 \\ \Delta p_3 \end{bmatrix}$$

$$\boldsymbol{\Delta\lambda} = \mathbf{U}\boldsymbol{\alpha} \cdot \boldsymbol{\varepsilon} = \mathbf{K} \cdot \boldsymbol{\varepsilon} = \begin{bmatrix} - & + & - \\ + & - & - \\ + & + & - \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{bmatrix}$$

$$\boldsymbol{\lambda}_1 = \boldsymbol{\lambda}_0 + \mathbf{K} \cdot \boldsymbol{\varepsilon}$$

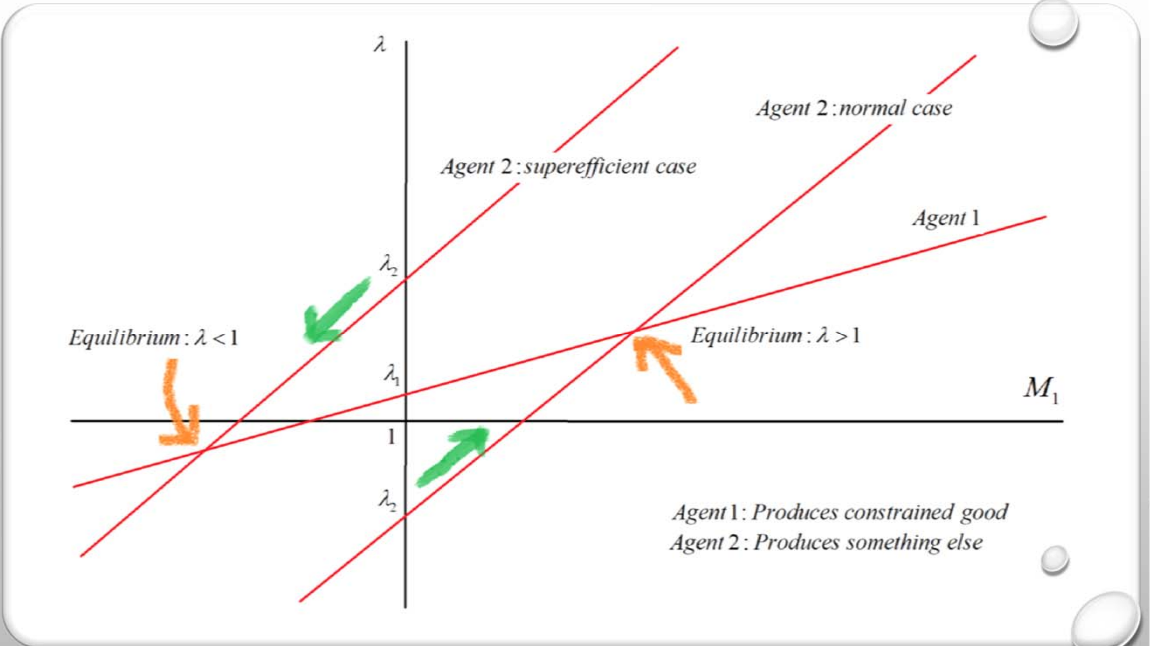
$$\boldsymbol{\iota}' \cdot \mathbf{K}^{-1} \cdot \boldsymbol{\iota} \cdot \Gamma = \boldsymbol{\iota}' \cdot \mathbf{K}^{-1} \cdot \boldsymbol{\lambda}_0 + \boldsymbol{\iota}' \cdot \boldsymbol{\varepsilon}$$

$$\Gamma = \left(\boldsymbol{\iota}' \cdot \mathbf{K}^{-1} \cdot \boldsymbol{\iota}\right)^{-1} \left(\boldsymbol{\iota}' \cdot \mathbf{K}^{-1} \cdot \boldsymbol{\lambda}_0\right)$$

- MODIFY THE FIXED MINIMUM STOCK LEVELS ASSUMPTION

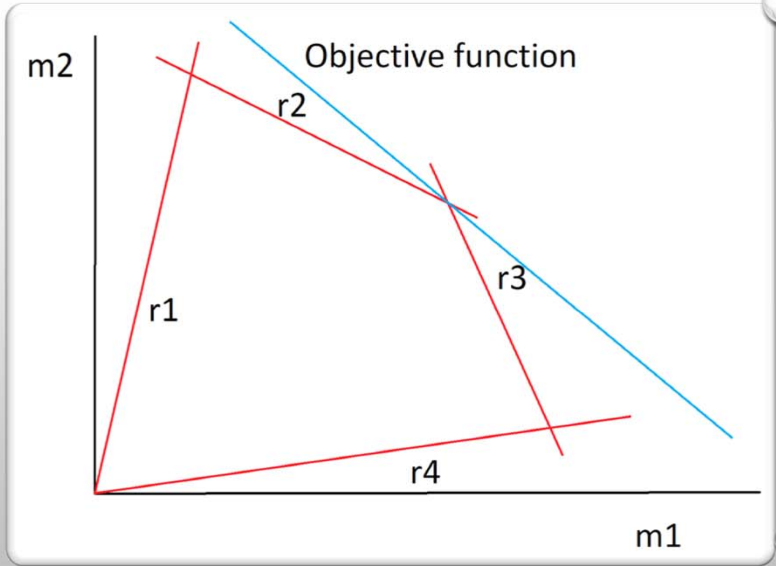
DYNAMICS V: EQUILIBRIUM CONDITIONS

- Can an agent be **too** efficient?



MARKET CLEARING PRICES

- DEFINE AN OBJECTIVE FUNCTION WHICH TOUCHES THE EQUILIBRIUM POINT
- THE SHADOW PRICES CLEAR THE MARKET FOR EACH AGENT



MUTATION / LEARNING

- INCREMENTAL
  - MUTATION
  - MISTAKE
  - PRODUCT OF RATIONAL THOUGHT
- DELETION OF A PROCESS
  - THIS CAN HAPPEN SURPRISINGLY QUICKLY IN BIOLOGICAL SYSTEMS
- ADDITION OF A PROCESS
  - KNOWN AS THE “HOPEFUL MONSTER” HYPOTHESIS (GOLDSCHMIDT 1940)
  - CONTROVERSIAL IN BIOLOGY
  - NOT CONSIDERED HERE

INCREMENTAL MUTATION

- IF RESOURCE IS SCARCE FOR A1
  - A2 IMPROVES EFFICIENCY
  - A2 DISPLACES A1
- FINAL POPULATION IS LARGER

$$m_j a_j + m_k a_k = L$$

$$\Delta\lambda_j \cdot m_j a_j + \Delta\lambda_k \cdot m_k a_k = 0$$

Now $d\lambda_j = u_j \cdot \Delta p_j > 0$

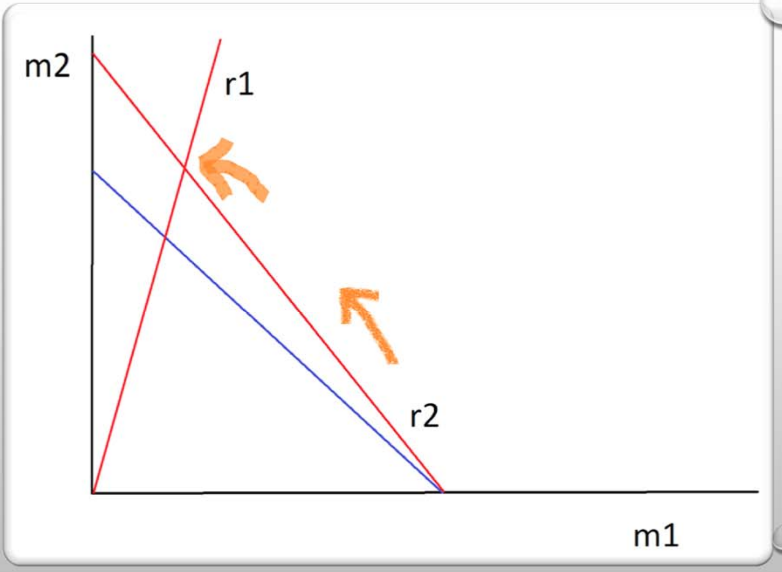
So $\Delta\lambda_j > 0$

$\Delta\lambda_k < 0$

DELETION MUTATION

- R1 IS PRODUCED USING R2
  - MORE THAN NEEDED
- AGENT J STOPS PRODUCING IT
  - SAVING R2
- FINAL POPULATION IS LARGER

$$\Delta\lambda_j = u_j^{r1} \cdot \Delta p_j^{r1} + u_j^{r2} \cdot \Delta p_j^{r2}$$
$$= 0 \cdot -ve + u_j^{r2} \cdot +ve$$
$$> 0$$



CONCLUSIONS

- AN ECOLOGY / ECONOMY IS LIKELY TO ARISE FROM
  - RESOURCE CONSIDERATIONS NOT FITNESS
  - SPECIALIZATION DEVELOPS LATER
  - DELETION MUTATIONS NOT INCREMENTAL MUTATIONS

- A NON-COMMERCIAL ECONOMY WITH NON-RECIPROCAL TRADING CAN ALSO DISPLAY EFFICIENCY IN RESOURCE ALLOCATION BUT

- AN AGENT CAN BE TOO EFFICIENT

- AS A SYSTEM DEVELOPS, EFFICIENCY OF RESOURCE UTILIZATION AND NUMBER OF AGENTS WILL INCREASE.

  LARGE SCALE TRENDS (PROGRESS)