On the Robustness of Multidimensional Counting Poverty Orderings

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Motivation (1)

Aim of the paper

- Multidimensional counting measures: widely used in academic research and policy (eg. World Bank, Eurostat)
- Poverty orderings depend: functions, thresholds and weights
- **Question**: Under which conditions can we claim that poverty rankings are robust to those choices?

Motivation (2)

Aim of the paper

- Income poverty literature: multiple dominance conditions (eg, Lorenz curves). No results for counting poverty measures.
- Typical approach: sensitivity analysis using an restricted and arbitrary set of parameters
- **Our contribution**: dominance conditions (necessary & sufficient) to test the robustness of counting poverty orderings

Outline

Rest of the presentation

- Measurement framework: counting measures
- Some notation
- Theoretical results: dominance conditions
- Empirical application (Australia in 2000s)
- Conclusions

Measurement framework

- Counting approach to multidimensional poverty: count the number of dimensions in which individuals are deprived
- Assume N units and D indicators of wellbeing. If $x_{nd} < z_d$ then n is deprived in d
- For each dimension $\exists\, w_d \in [0,1]$ such that $\sum_{d=1}^{D} w_d = 1$
- The deprivation score: $c_n \equiv \sum_{d=1}^{D} w_d \mathbb{I}(x_{nd} < z_d)$
- A person is poor if: $c_n \geq k$, where $k \in [0,1]$

Measurement Framework

The following class of poverty indices:

$$P(W,k) = \frac{1}{N}\sum_{n=1}^{N} p(c_n)$$

Alkire and Foster (2011): satisfies core axioms of poverty measurement
Includes indices widely used in the literature.

Headcount ratio

$$H(k) \equiv \frac{1}{N}\sum_{n=1}^{N} \mathbb{I}(c_n \geq k)$$

Adjusted headcount ratio

$$M(k) \equiv \frac{1}{N}\sum_{n=1}^{N} \mathbb{I}(c_n \geq k)c_n$$

Some notation

- We denote by $D = \{d_1, d_{2,\ldots,}d_D\}$ the set of dimensions
- $P(D)$ power set with all posible subsets of D excluding the $\varnothing$ set.
- Example if $D = 2$, $P(D) = \{d_1, d_2, d_{12}\}$

Definition *Sub-dimensional intersection headcount* (H_S) *. For any* $S \subset P(D)$, H_S *indicates the proportion deprived only in the dimensions in* S.

For instance, for $S = \{d_1\}$

$$H_S \equiv \frac{1}{N}\sum_{n=1}^{N} \mathbb{I}(x_{n1} \leq z_1 \wedge x_{n2} > z_2)$$

Some notation

Definition *Uncensored deprivation headcount* (U_S). *For any* $S \subset P(D)$, U_S *indicates the proportion deprived at least in the dimensions in* S.

For instance, for $S = \{d_1\}$

$$U_S \equiv \frac{1}{N}\sum_{n=1}^{N} \mathbb{I}(x_{n1} \leq z_1)$$

It is important to note that

$$U_S \geqslant H_S \quad \forall S \subset P(D)$$

Some notation

- $P_\rho(D)$ set of potential poverty sets consistent with the identification rule $\rho(W, k)$
- Example: $D = 2$, $P(D) = \{d_1, d_2, d_{12}\}$. Assume d_1 denotes people deprived in 1, d_2 in 2, and d_{12} in 1 and 2.

$$P_\rho(D) = \{(d_{12}), (d_1, d_{12}), (d_2, d_{12}), (d_1, d_2, d_{12})\}$$

$S = \{d_1\} \notin P_\rho(D) : \forall\, \rho(W, k)$, if those deprived in d_1 then d_{12} should be in S

$$S = \{d_2\} \notin P_\rho(D)$$

Results: dominance conditions

Sufficient condition

Proposition 1 *If* $H_S^A \geq H_S^A \ \forall \, S \subset P(D) \Rightarrow P^A(W,k) \geq P^B(W,k)$ *for any* W and k

Easy to check $(2^D - 1)$ conditions

when $D = 2$, $P(D) = \{d_1, d_2, d_{12}\}$, compute 3 headcounts

when $D = 3$, $P(D) = \{d_1, d_2, d_3, d_{12}, d_{13}, d_{23}, d_{123}\}$, 7 headcounts

Results: dominance conditions

Necessary conditions

Proposition 2

If $P^A(k) \leq P^B(k) \forall k \in [0,1] \wedge \exists k | P^A(k) < P^B(k)$ *for all possible weighting vectors W then* $H^A_{(1,2,...,D)} \leq H^B_{(1,2,...,D)}$

Proposition 3

If $P^A(k) \leq P^B(k)\ \forall k \in [0,1] \wedge \exists k | P^A(k) < P^B(k)$ *for all possible weighting vectors W then* $U^A_d \leq U^B_d \ \forall\ d \in [1,2,...,D]$.

If any of these conditions is not satisfied then we can conclude that A does not dominate B

Results: dominance conditions

Necessary and sufficient

Proposition 4

$P^A(k) \leq P^B(k)\ \forall k \in [0, v2, ..., 1] \wedge \exists\, k | P^A(k) < P^B(k)$ for all possible weighting vectors W if and only if $H^A_S(s) \leq H^B_S(s)\ \forall S \subset P_\rho(D)$

Testing this condition requires comparing multiple pairs of $H_S(s)$ statistics. For $D = 2$, four statistics:

$$P_\rho(D) = \{(d_{12}), (d_1, d_{12}), (d_2, d_{12}), (d_1, d_2, d_{12})\}$$

The number grows exponentially with D: 18 for $D = 3$, 166 for $D = 4$, 7,579 for $D = 5 \Rightarrow$ Necessary conditions!!

Results: dominance conditions

General testing framework

Evaluating new dominance conditions requires comparing $R \geq 1$ statistics (eg. H or H_S). Let $z(r) \equiv \frac{X^A(r) - X^B(r)}{SE[X^A(r) - X^B(r)]}$. We propose the following test:

$$Ho : z(r) = 0 \; \forall r = 1, 2, ..., R$$

$$Ha : z(r) < 0 \; \forall r = 1, 2, ..., R$$

We reject the null if $\max\{z(1), z(2), ..., z(R)\} < z_\alpha < 0$, where z_α is a left-tail critical value.

Empirical application

Poverty in Australia in the 2000s: strong growth but hit by the GFC.

Data from the Household Income and Labour Dynamics in Australia (HILDA) survey for the period 2000-10.

Three indicators of financial disadvantage:

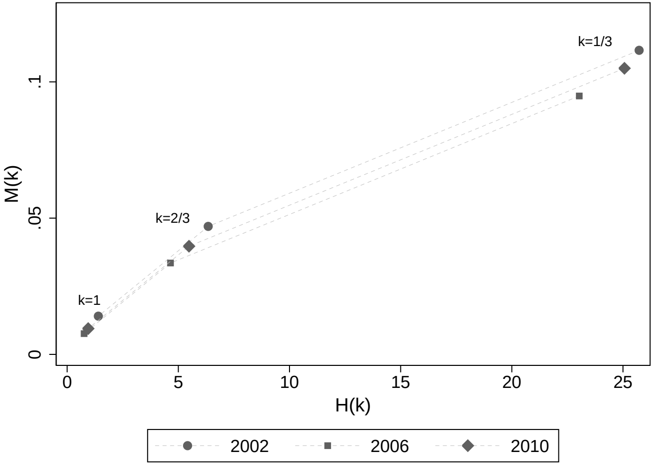
- Income poverty: household income below 60% of the median
- Lack of assess: no wealth to keep family members above the poverty line for three months
- Financial stress: could not pay bills on time; mortgage on time; pawned something; went without meals; unable to heat home; asked for financial help

Table: Poverty indicators in Australia (%)

Year	Income	Wealth	Financial hardship
2002	18.47	8.36	6.65
2006	16.55	6.97	4.93
2010	18.21	7.39	5.89

Empirical application

Figure: M(k) and H(k) indices (**equal weights**)



Empirical application

Test of sufficient condition:

$$Ho : H_s^{t_A} = H_s^{t_B} \; \forall s \subset S(D) \text{ versus } Ha : H_s^{t_A} < H_s^{t_B} \; \forall s \subset S(D)$$

Table: Test of sufficient condition 1 (maximum statistics)

$t_B \backslash t_A$	2002	2006	2010
2002	0.00	0.07	1.05
2006	4.70	0.00	3.33
2010	3.19	0.29	0.00

Empirical application

Test of necessary condition:

$$Ho : U_d^{t_A} = U_d^{t_B} \ \forall d \in [1, 2, ..., D] \text{ versus } Ha : U_d^{t_A} < U_d^{t_B} \forall d$$

Table: Test of necessary condition 3 (maximum statistics)

$t_B \backslash t_A$	2002	2006	2010
2002	0.00	**-3.82**	-0.50
2006	5.60	0.00	3.26
2010	2.69	-1.22	0.00

Empirical application

Test of necessary and sufficient condition:

$$Ho : H^{t_A}(k) = H^{t_B}(k) \forall k \text{ versus } Ha : H^{t_A}(k) < H^{t_B}(k) \forall k$$

Table: Test of condition 4 (maximum statistics)

$t_B \backslash t_A$	2002	2006	2010
2002	0.00	**-4.70**	-1.14
2006	5.64	0.00	3.54
2010	3.19	-1.49	0.00

Conclusions

- Literature on dominance for multidimensional poverty orderings still scarce
- New dominance results and testing framework. Superior to current approaches (arbitrary)
- Robustness to thresholds and weights. Easy to implement