

The US Experience with ' $r > g$ ' and Wealth Inequality

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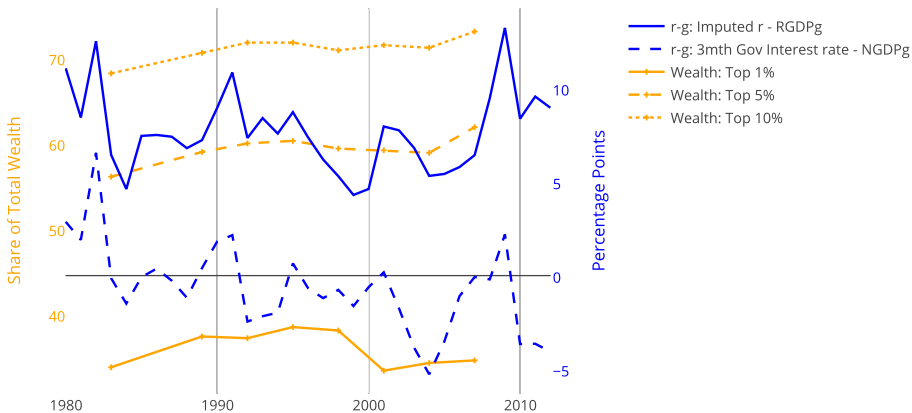
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Piketty posits a positive correlation of Wealth Inequality and 'r>g'. This fundamental inequality [$r > g$] will play a crucial role in this book. In a sense, it sums up the overall logic of my conclusions. When the rate of return on capital significantly exceeds the growth rate of the economy, then it logically follows that inherited wealth grows faster than output and income.

– Thomas Piketty (2014) - Capital in the Twenty-First Century (pg 25-26)

When the rate of return on capital exceeds the rate of growth of output and income, as it did in the nineteenth century and seems quite likely to do again in the twenty-first, capitalism automatically generates arbitrary and unsustainable inequalities that radically undermine the meritocratic values on which democratic societies are based.

– Thomas Piketty (2014) - Capital in the Twenty-First Century



* Data on wealth inequality from the Survey of Consumer Finances (SCF), as summarized Table 2 of Wolff (2010). We show the Share of Total Wealth of the Top 1% (5 & 10) of Wealth. Data on the difference between the rate of return to wealth (r) and the growth rate of income is measured in two ways. One is as the difference between 3-month Treasury bills and the nominal GDP growth rate. The other measures r as the ratio of the Capital Share of Output (rK/Y) divided by the Wealth-Income ratio (K/Y), using data from Karabarbounis and Neiman (2014) and WWID respectively, and then subtracts from this the real GDP growth rate.

Figure: Negative or Zero Correlation between Wealth Inequality and $r-g$ in US since mid-1980s.

Table: Negative or Zero Correlation between Wealth Inequality and $r-g$ in US

Correlation Coefficients between Wealth Inequality and r (for US)

	Share of Top 10% (SCF)	Wealth Inequality Measure	
		Gini Coeff. (SCF)	Share of Top 10% (SZ2016)
		[OLS coefficient (p-value)]	
Rate of Return to Wealth (KN,WWID)	0.2182 (0.6539)	0.0012 (0.7478)	0.2693 (0.3158)
Rate of Return to Wealth (OECD,WWID)	0.3238 (0.5113)	0.0019 (0.6167)	0.2625 (0.2988)
Rate of Return to Capital (GommeEtAl)	0.1376 (0.0906)	0.0014 (0.0169)	0.1546 (0.0060)
Interest rate (IMF 3mth, OECD CPI)	-0.0426 (0.8878)	-0.0004 (0.8572)	-0.3268 (0.0833)
Interest rate (IMF 3mth, OECD GDPDEF)	-0.1820 (0.5448)	-0.0017 (0.4666)	-0.3189 (0.1111)
Interest rate (FedRes 10yr, BLS CPI)	-0.2971 (0.4508)	-0.0034 (0.2468)	-0.4099 (0.0054)
Interest rate (OECD 24hrIB, OECD CPI)	-0.4581 (0.1834)	-0.0048 (0.0522)	-0.3471 (0.0261)

- US displays negative or zero correlation between Wealth Inequality and $r-g$ in US since mid-1980s.
- In conflict with (appealing) logic given by Piketty predicting a positive correlation.
- Can we make sense of this?
- Yes.

- Two key reasons not to expect positive relationship between ' $r > g$ ' and wealth inequality.
 - Negative relationship between wealth inequality and r .
Increased wealth inequality is associated with increased wealth (relative-to-income), leads to fall in r .
 - ' $r > g$ ' does not really capture role of g .
Increase in g leads to fall in aggregate wealth-to-income, leads to fall in r . So ' $r > g$ ' does not decrease one-for-one with increase in g .

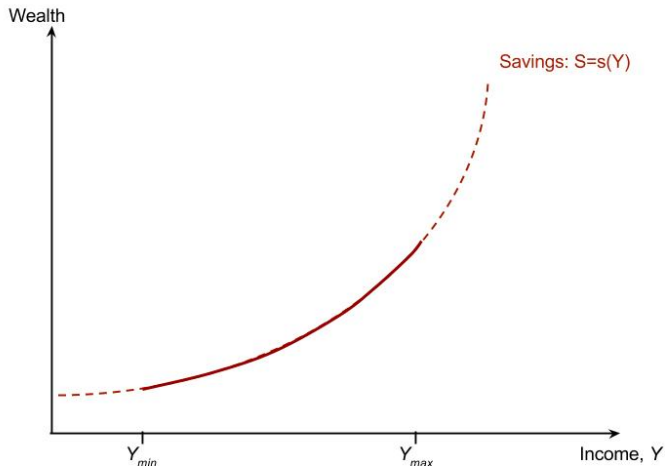
' $r > g$ ' and Wealth Inequality

- This paper:
- Show that a general equilibrium heterogeneous agent model with incomplete markets, exogenous growth, and borrowing constraints can be used to understand why we might expect negative relationship between ' $r-g$ ' and wealth inequality.
- Simple model that captures logic of larger model about why we might expect a negative correlation between Wealth Inequality and $r-g$.
- Key mechanisms of simple model—which 'summarize' the larger model—are in line with the data.

- Two key reasons not to expect positive relationship between ' $r > g$ ' and wealth inequality.
 - **Negative relationship between r and wealth inequality.**
Increased wealth inequality is associated with increased wealth (relative-to-income), leads to fall in r .
 - ' $r > g$ ' does not really capture role of g .
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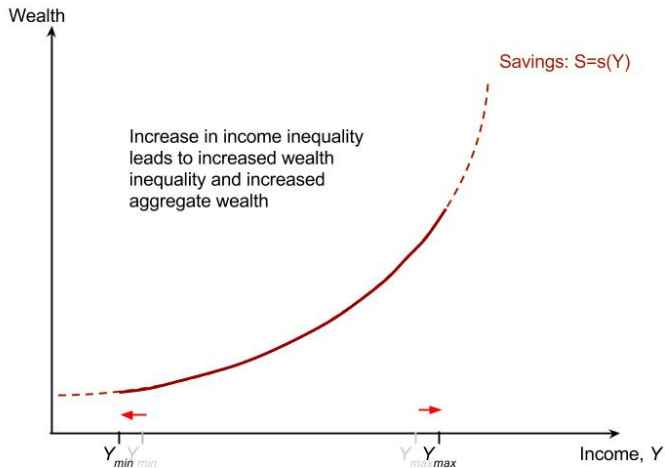
' $r > g$ ' and Wealth Inequality: Simple model

- Household income distribution together with savings decisions determine wealth distribution.



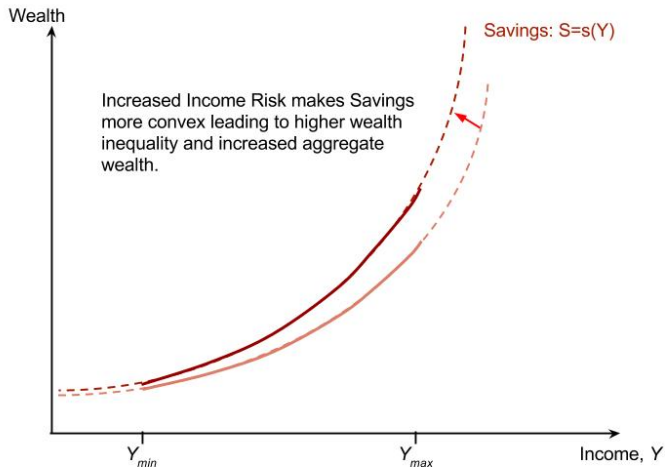
' $r > g$ ' and Wealth Inequality: Simple model

- Increased income inequality increases wealth inequality and total wealth



' $r > g$ ' and Wealth Inequality: Simple model

- Increased income risk increases wealth inequality and total wealth.



' $r > g$ ' and Wealth Inequality

- Two key reasons not to expect positive relationship between ' $r > g$ ' and wealth inequality.
 - Negative relationship between r and wealth inequality.
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 - **' $r > g$ ' does not really capture role of g .**
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' $r > g$ ' and Wealth Inequality

- Decrease in economic growth g increases aggregate wealth-to-income ratio.
- Logic is same as in Solow growth model.
- Increase in wealth-to-income ratio decreases return to wealth.
- Return to wealth is marginal product of wealth, which is decreasing in wealth-to-income ratio.

Larger model includes an additional channel, vs Solow model, through effect of growth on patience. Also issue of distribution of agents requires change in approach of proof.

' $r > g$ ' and Wealth Inequality: Key mechanisms of small model

- So it makes sense that we might expect negative relationship between ' $r-g$ ' and wealth inequality.
- Key aspects of simple model:
 - 1 Negative relation between the rate of income growth and the wealth-income ratio.
 - 2 Negative relation between wealth inequality and the rate of return to wealth (r).
 - 3 Positive relation between income inequality and wealth inequality.
 - 4 Positive relation between income risk and income inequality.
 - 5 Convexity of savings function.

Model Mechanism: Convexity of Savings Function

- Key to the model predictions about wealth inequality is the savings behaviour of households.
- Specifically, the analytical results depend on convexity of savings function in wealth.
- “The increase in [US] wealth inequality in recent decades is due to the upsurge of top incomes combined with an increase in saving rate inequality.” (Saez and Zucman, 2016)
- So model mechanism fits with main micro-level facts on increasing wealth inequality.

- Four correlations for US since mid-1980s.
 - 1 Negative correlation between the rate of income growth and the wealth-income ratio.
 - 2 Negative correlation between wealth inequality and the rate of return to wealth (r).
 - 3 Positive correlation between income inequality and wealth inequality.
 - 4 Positive correlation between income risk and income inequality.

1st

2nd

3rd

4th

' $r > g$ ' and Wealth Inequality: Comparative statics results in large model

- Four comparative distributional balanced growth path results in large model.

- 1 Increase in rate of economic (TFP) growth leads to decrease in capital-output ratio.

Negative correlation between the rate of income growth and the wealth-income ratio.

- 2 Increase in income inequality (caused by mean-preserving spread to income shock process) leads to decrease in marginal product of capital (r).

Negative correlation between income inequality and the rate of return to wealth (r).

- 3 Increase in income inequality (caused by mean-preserving spread to income shock process) leads to increase in wealth inequality.

Positive correlation between income inequality and wealth inequality.

(I have demonstrated this quantitatively, but so far no analytical proof.)

- 4 Increase in income risk (caused by mean-preserving spread to income shock process) leads to increase in income inequality.

Positive correlation between income risk and income inequality.

- Neoclassical growth model with heterogeneous agents and incomplete markets.
- Constant rate of growth, g , and no aggregate shocks.
- Households are borrowing constrained, labour income is subject to idiosyncratic shocks.
- Huggett (1997) gives definition of competitive equilibrium. Krueger and Lustig (2010) show how to relate the competitive equilibria to competitive equilibria of the 'growth-adjusted' model.
- The growth-adjusted model is similar to that of Aiyagari (1994).

- Infinitely-lived households face income fluctuations and make savings decisions.
- The household problem is maximize utility subject to a budget constraint and a borrowing constraint,

$$\max_{\{c_t(z^t), k_{t+1}(z^t)\}} U(\{c_t(z^t)\})$$

$$s.t. c_t(z^t) + k_{t+1}(z^t) = w_t z_t E_t + (1 + r_t) k_t(z^{t-1}), \quad \forall z^t, \forall t$$

$$k_{t+1}(z^t) \geq \underline{k}(E_t), \quad \forall z^t, \forall t$$

k_1 given

- Labour-augmenting aggregate technology E_t grows deterministically as $E_{t+1} = (1 + g)E_t$.
- Idiosyncratic z_t follows a stationary 1st order Markov process.
- Per-period utility function $u(c) = \frac{c^{1-\mu}}{1-\mu}$. U is discounted expected utility.
- So a Household is identified by a pair (k_t, z_t) . Optimal policy will be a choice of k_{t+1}

- Wealth inequality has been increasing.

At least it has in the few countries we have good data for.

- What is driving increases in Wealth Inequality?
- Not $r > g$!
- Appears that slowing economic growth and increased income risk played a role.

Raw correlations in data, together with model's suggestion.

- Other possibilities: Globalization. Changes in taxation. Political economy factors (oligopolies). Increased negotiating power of managers. Skill-biased technology change (increased education premium). Different r 's: larger fortunes earn higher returns.

More

- Four correlations capture major aspects of US relationship between Macroeconomics, r & g , and inequality.
 - ① Negative correlation between the rate of income growth and the wealth-income ratio.
 - ② Negative correlation between wealth inequality and the rate of return to wealth (r).
 - ③ Positive correlation between income inequality and wealth inequality.
 - ④ Positive correlation between income risk and income inequality.
- All four can be understood as arising analytically in a neoclassical growth model with heterogeneous agents and incomplete markets.

- $r > g$ is not the key to Wealth Inequality.
- Seems likely that lower economic growth and income risk are playing a role.

- What are empirical facts in Piketty's book?
Summary at: robertdkirkby.com/blog/2015/summary-of-piketty
- **Model** from Jones (2015), of kind Piketty describes as capturing his idea that $r-g$ is related to Wealth Inequality.

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- S. Rao Aiyagari. Uninsured idiosyncratic risk and aggregate saving. *Quarterly Journal of Economics*, 109(3):659–684, 1994.
- Manuel Arellano, Richard Blundell, and Stephane Bonhomme. Earnings and consumption dynamics: A nonlinear panel data framework. *IZA Discussion Papers*, No. 9344:1–66, 2015.
- Orazio Attanasio and Luigi Pistaferri. Consumption inequality over the last half century: Some evidence using the new PSID consumption measure. *American Economic Review*, 104(5):122–126, 2014.
- Jess Benhabib, Alberto Bisin, and Shenghao Zhu. The distribution of wealth and fiscal policy in economies with finitely lived agents. *Econometrica*, 79(1):123–157, 2011.
- Paul Gomme, B. Ravikumar, and Peter Rupert. The return to capital and the business cycle. *Review of Economic Dynamics*, 14(2):262–278, 2011.
- Fatih Guvenen and Anthony Smith. Inferring labor income risk and partial insurance from economic choices. *Econometrica*, 82:2085–2129, 2014.

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- Jonathan Heathcote, Fabrizio Perri, and Giovanni Violante. Unequal we stand: An empirical analysis of economic inequality in the United States, 1967-2006. *Review of Economic Dynamics*, 13:15–51, 2010.
- Mark Huggett. The one-sector growth model with idiosyncratic shocks: Steady states and dynamics. *Journal of Monetary Economics*, 39(3): 385–403, 1997.
- Charles Jones. Pareto and piketty: The macroeconomics of top income and wealth inequality. *Journal of Economic Perspectives*, 29(1):29–46, 2015.
- Loukas Karabarbounis and Brent Neiman. The global decline of the labor share. *Quarterly Journal of Economics*, 129(1):61–103, 2014.
- Dirk Krueger and Hanno Lustig. When is market incompleteness irrelevant for the price of aggregate risk (and when is it not)? *Journal of Economic Theory*, 145(1):1–41, 2010.
- Dirk Krueger, Fabrizio Perri, Luigi Pistaferri, and Giovanni Violante. Cross sectional facts for macroeconomists. *Review of Economic Dynamics*, 13(1):1–14, 2010.
- Robert Moffitt and Peter Gottschalk. The rising instability of U.S. earnings. *Journal of Economic Perspective*, 23(4):3–24, 2009.

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- Robert Moffitt and Peter Gottschalk. Trends in the transitory variance of male earnings: Methods and evidence. *Journal of Human Resources*, 47(1):204–236, 2012.
- Thomas Piketty. *Capital in the Twenty-First Century*. Harvard University Press, first (english) edition, 2014.
- Thomas Piketty. Putting distribution back at the center of economics: Reflections on capital in the twenty-first century. *Journal of Economic Perspectives*, 29(1):67–88, 2015.
- Thomas Piketty and Gabriel Zucman. Capital is back: Wealth-income ratios in rich countries, 1700-2010. *Quarterly Journal of Economics*, 129(3):1255–1310, 2014.
- Emmanuel Saez and Gabriel Zucman. Wealth inequality in the united states since 1913: Evidence from capitalized income tax data. *Quarterly Journal of Economics*, 131(2):519–578, 2016.
- Edward Wolff. Recent trends in household wealth in the United States—rising debt and the middle-class squeeze—an update to 2007. *Levy Economics Institute Economics Working Paper Archive*, No. 589:1–59, 2010.

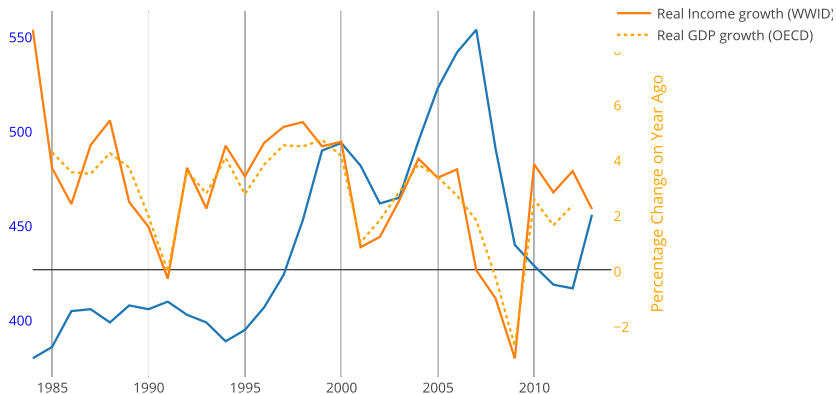
- Now turning to the data.
- Four correlations for US since mid-1980s.
 - 1 Negative correlation between the rate of income growth and the wealth-income ratio.
 - 2 Negative correlation between wealth inequality and the rate of return to wealth (r).
 - 3 Positive correlation between income inequality and wealth inequality.
 - 4 Positive correlation between income risk and income inequality.

Correlations in the US since mid-1980s

- Focus on US since mid-1980s due to limitations of data.
- Main limitation is data on wealth inequality.
- Use both Survey of Consumer Finances data on wealth, and Saez and Zucman (2016) data on Top Capital Shares for US.
- Main measures of r and g : 90-day nominal interest rates (annual averages, from IMF) and nominal GDP growth rates (OECD)
- Paper finds robustness to other sources (eg. Luxembourg Income Study income inequality measures, UK and Swedish data on Top Wealth Shares, World Wealth and Income Database data on income inequality, other measures of interest rates (eg. longer-term bonds), and conversions to real interest rates and real GDP growth rates (using CPI, core CPI, and GDP deflator).
- Main weakness in robustness is using Gomme, Ravikumar, and Rupert (2011) measure of marginal product of capital for r . Robust to using r as implied by capital income shares from Karabarbounis and Neiman (2014) and OECD. Even here in the few cases where the correlation is positive and statistically significant it is economically irrelevant.

Correlations in the US since mid-1980s: 1/4

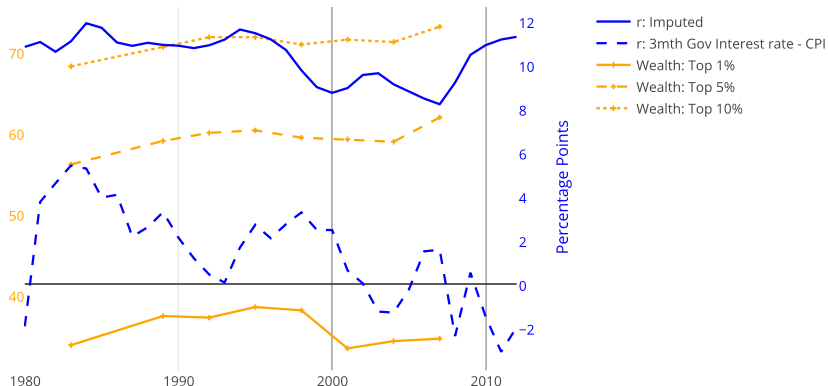
- 1 Negative Correlation between the rate of income growth and the wealth-income ratio in the US since the mid-1980s.



* Data on (Net National) Wealth-Income ratio and Income growth rate (calculated as percentage change in income) from World Wealth and Income Database (WWID). For comparison OECD data on growth rate of real GDP is also included.

Correlations in the US since mid-1980s: 2/4

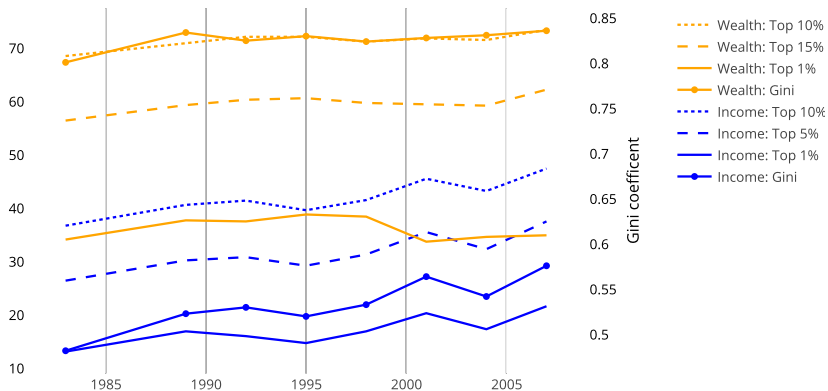
- 2 Negative correlation between wealth inequality and the rate of return to wealth (r) in the US since the mid-1980s.



* Data on wealth inequality from the Survey of Consumer Finances (SCF), as summarized Table 2 of Wolff (2010). We show the Share of Total Wealth of the Top 1% (5 & 10) of Wealth. Data on the rate of return to wealth (r) is measured in two ways. One, following the literature standard is as the difference between 3-month Treasury bills and the CPI inflation rate. The other is as the ratio of the Capital Share of Output (rK/Y) divided by the Wealth-Income ratio (K/Y), using data from Karabarbounis and Neiman (2014) and WWID respectively.

Correlations in the US since mid-1980s: 3/4

3 Positive Correlation between Income Inequality and Wealth Inequality in the US since the mid-1980s.



* Data all taken from the Survey of Consumer Finances (SCF), as summarized Table 2 of Wolff (2010). We show the Gini coefficients for income and wealth. We show the Total Income (Wealth) shares of the Top 1% of Incomes (Wealth); note that while there is overlap these are not the same households.

[Back](#)

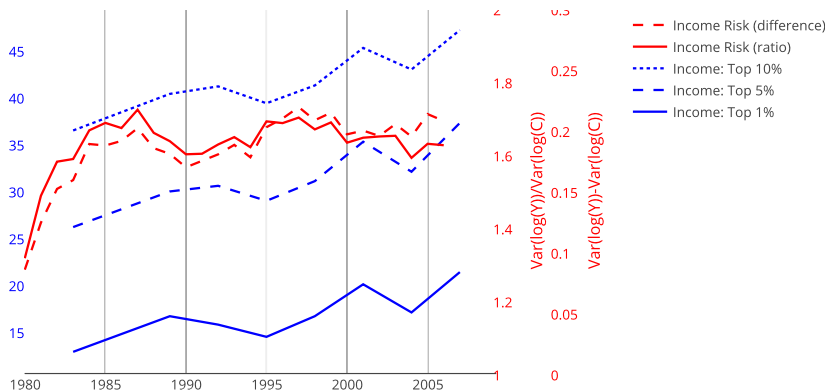
[Longer Time Period](#)

Peering beyond data limitations on 'r-g' and wealth inequality?

- Given strength of correlation between income inequality and wealth inequality in data (also for UK & Sweden), and our finding of negative or zero correlation between wealth inequality and 'r-g', we might expect negative or zero correlation between income inequality and 'r-g'.
- We can do this for more countries, as more income inequality data.
- Find that majority of correlations are negative, but some are positive (eg. Top 0.1% in Australia and Canada, 1978 to present). Substantial majority are anyway statistically insignificant.
- Suggests that if we had more wealth inequality data we would not find any substantial differences from findings on 'r-g' reported here.

Correlations in the US since mid-1980s: 4/4

- 4 Positive correlation between income risk and income inequality in the US since the mid-1980s.



* Data on income inequality from the Survey of Consumer Finances (SCF), as summarized Table 2 of Wolff (2010). We show the Total Income shares of the Top percentiles of Income. Data on Income Risk is from the Consumer Expenditure Survey (CEX), as summarized in Heathcote, Perri, and Violante (2010).

[Back](#)

[More on Income Risk](#)

Model capturing Piketty's view that ' $r-g$ ' increases wealth inequality

- Piketty argues that *Inequality* is increasing in ' $r-g$ '
- Logic: when $r-g$ is high, people who are already wealthy will get higher incomes, and so become even more wealthy (even relative to others).
- g reflects amount of capital needed to maintain level of wealth relative to incomes.
- Formal model: occurs because $r-g$ determines the rate at which assets will accumulate over time. A higher $r-g$ allows individuals with higher assets to have higher incomes, and since savings are a fixed fraction of income, or at least not declining in income, leads to ever higher assets relative to other individuals.
- Note absence of any income risk.

Model capturing Piketty's view that ' $r-g$ ' increases wealth inequality

- 'Pareto inequality' model.
- Piketty specifically refers to this as kind of model he has in mind when arguing that $r-g$ increases inequality.
- Partial equilibrium model, on balanced-growth-path it has a Pareto distribution of wealth.
- Useful: Pareto distribution has fat tails.
- Useful: Inverted Pareto coefficient is an empirical measure of inequality.
- Pareto distribution arises from exponential growth over an variable which has exponential distribution.
- Here: Wealth accumulates exponentially in age, and age has an exponential distribution.

Pareto Dist.

Exponential Dist.

Model capturing Piketty's view that 'r-g' increases wealth inequality

- Individual's assets a accumulate over time according to,
 $\dot{a}(t) = ra(t) - \tau a(t) - c(t)$
- Assume consumption is a constant fraction α of wealth, so
 $\dot{a}(t) = (r - \tau - \alpha)a(t)$
- So the wealth of an individual of age x at date t is
 $a_t(x) = a_{t-x}(0)e^{(r-\tau-\alpha)x}$
- So we have that assets grow exponentially in age.

a =assets; r \equiv interest rate; τ \equiv wealth tax rate; c \equiv consumption; $a_{t-x}(0)$ is the initial wealth of a newborn at date $t - x$.

Model capturing Piketty's view that 'r-g' increases wealth inequality

- Assume that the number of people born at time t is
 $B(t) = B(0)e^{nt}$
- Assume that death is a Poisson process with arrival rate d .
- Stationary dist. for this birth-death process is an exponential distribution in age,
 $Pr[age > x] = e^{-(n+d)x}$

$n \equiv$ growth rate of births; $B \equiv$ number of births; $d \equiv$ prob. of death.

Model capturing Piketty's view that 'r-g' increases wealth inequality

- Before combining wealth growing exponentially in age with the exponential dist of age to get a Pareto dist of wealth, need two more things.
- Initial wealth of newborns: equally inherit wealth of people who die.
$$a_t(0) = \frac{dK(t)}{(n+d)N(t)} = \frac{d}{n+d} k(t)$$
- Assume the economy is on a balanced growth path, so capital per capita grows with exogenous technology at a constant rate g , so
$$k(t) = k(0)e^{gt}.$$

$a_t(0)$ \equiv assets of newborn born at time t ; n \equiv growth rate of births; d \equiv prob. of death; g \equiv growth rate of exogenous technology; K \equiv aggregate capital (wealth); k \equiv capital per capita (ie. the average amount of capital per person)

Model capturing Piketty's view that 'r-g' increases wealth inequality

- Combining, we get that the wealth is Pareto distributed

$$Pr[\textit{Wealth} > a] = \left(\frac{a}{\left(\frac{d}{n+d}\right)k(t)} \right)^{-\frac{n+d}{r-g-\tau-\alpha}}$$

- The Inverted Pareto coefficient (aka. Pareto inequality index) is just the inverse of the exponent in this equation, namely

$$\eta_{\textit{wealth}} = \frac{r-g-\tau-\alpha}{n+d}$$

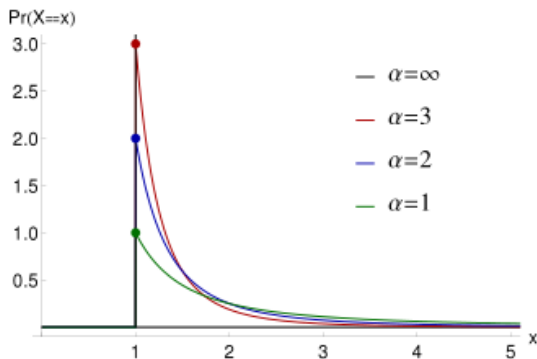
- This is the key result that inequality is increasing in $r-g$!

[Back to 'Model Intuition'](#)

[Back to 'Further Material'](#)

Pareto Distribution

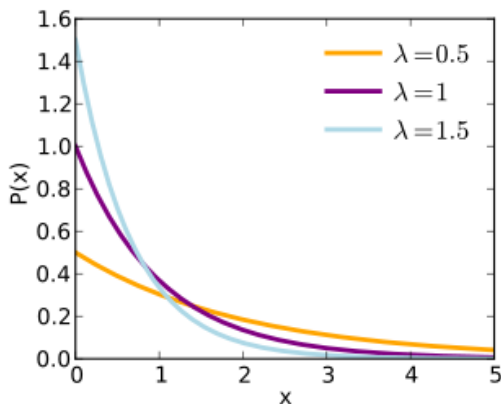
- Prob. Dist. Fn of the Pareto Distribution (with $x_m = 1$)



- PDF: $\frac{\alpha x_m^\alpha}{x^{\alpha+1}}$ for $x \geq x_m$ (zero below x_m).
- Has fat-tail.

Exponential Distribution

- Prob. Dist. Fn of the Exponential Distribution



- PDF: $\lambda e^{-\lambda x}$ for $x \geq 0$.

- Growth-Adjusted Model is given by

$$\hat{c}_t \equiv c_t/E_t$$

$$\hat{\beta} \equiv \beta(1+g)^{1-\gamma}$$

$$\hat{k}_t(z^{t-1}) \equiv k_t(z^{t-1})/E_t$$

$$\hat{k}_t(z) \equiv \hat{k}_t(z)/E_{t+1}$$

$$\hat{w}_t \equiv w_t, \hat{r}_t \equiv r_t$$

$$\hat{K}_t \equiv K_t/E_t, \hat{L}_t \equiv L_t/E_t$$

$$\hat{\mu}_t(\hat{k}, z) \equiv \mu_t(k/E_t, z)$$

for $t = 0, 1, 2, \dots$

- The Growth-Adjusted Model is a standard Bewley-Huggett-Aiyagari model.

- Not $r-g$! But interest rates may still be important.
- ① Maybe large fortunes get access to higher interest rates?
 - Benhabib et al. (2011) model shows (qualitatively) that interest rate risk can generate wealth inequality; similarly to how we saw income risk generate wealth inequality.
 - Saez and Zucman (2016) data on US Wealth inequality suggest that it is purely a portfolio effect.
Find that people with higher wealth on average receive higher asset returns, but that this is due to differences across, not within, asset classes. (loosely, high wealth is disproportionately invested in equities, and so disproportionately benefit from equity premium)

- Not $r-g$! But interest rates may still be important.
- ② Factor shares: capital income as share of total income.
 - Capital income as share of total income has increased in recent decades Karabarbounis and Neiman (2014), while capital-output ratio has increased Piketty and Zucman (2014).
 - So Macro evidence suggests elasticity of substitution between capital and labour greater than one. [Formal Estimate](#)
 - But Micro (firm and sector level) evidence finds elasticity of substitutions lower than one.
 - In any case, given that capital income accounts for a smaller fraction of top incomes than in the past (Piketty, 2014) the importance of this seems limited.

[back](#)

- The Constant Elasticity of Substitution production function is,

$$Y = (\theta_1 K^{\theta_2} + (1 - \theta_1)L^{\theta_2})^{1/\theta_2}$$

- The Cobb-Douglas is nested as the limit when θ_2 goes to zero (in which case θ_1 plays the role of θ in the formula given earlier for the Cobb-Douglas prodn fn; hence the notation).
- The elasticity of substitution between the two inputs — capital and labour — is given by $1/(1 - \theta_2)$, a constant and thus the name.
- Can show:

$$\text{capital share of income} \equiv MP_K K = \theta_1 (K/Y)^{\theta_2}$$

CES Production Function

- Estimate previous equation from US data on Capital Share of Income and on Capital-Output Ratio (annual, 1975-2010; non-linear least squares).
- The estimated values for the parameters are $\hat{\theta}_1 = 0.2651$, $\hat{\theta}_2 = 0.2208$.
- So estimated elasticity of substitution between the two inputs — capital and labour — is given by $1/(1 - \theta_2) = 1.28$

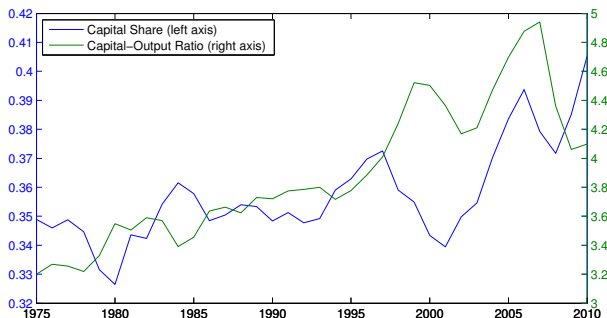


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Rate of Return to Wealth (OECD,WWID)	0.3238 (0.5113)	0.0019 (0.6167)	0.2625 (0.2988)
Rate of Return to Capital (GommeEtAl)	0.1376 (0.0906)	0.0014 (0.0169)	0.1546 (0.0060)
Interest rate (IMF 3mth, OECD CPI)	-0.0426 (0.8878)	-0.0004 (0.8572)	-0.3268 (0.0833)
Interest rate (IMF 3mth, OECD GDPDEF)	-0.1820 (0.5448)	-0.0017 (0.4666)	-0.3189 (0.1111)
Interest rate (FedRes 10yr, BLS CPI)	-0.2971 (0.4508)	-0.0034 (0.2468)	-0.4099 (0.0054)
Interest rate (OECD 24hrIB, OECD CPI)	-0.4581 (0.1834)	-0.0048 (0.0522)	-0.3471 (0.0261)

All regressions are run on the available annual data; note that SCF data is only periodic so contains few observations. The number of observations varies across and reflects all available data for each time series post-1954. Correlation coefficients are calculated by OLS regression (with a constant term) and p-values are for the standard null hypothesis of H_0 : coefficient=0. For purpose of regression the shares are measured as 0 to 100. In all cases NGDP or RGDP growth is subtracted as appropriate (ie. NGDP for rows 4 and 6). First data observations are 1975 for KN data, 1970 for OECD data, 1954 otherwise.

Table: Negative Correlation between Wealth-Income ratio and rate of income growth (g)

	Country Correlation Coefficients between Wealth-Income and g			
	Wealth-Income Ratio		Capital-Income Ratio	
	OLS coefficient and (p-value)			
Australia	-6.0061	(0.4313)	-2.8042	(0.0415)
Canada	-3.6113	(0.1568)	-6.2509	(0.0155)
France	-5.3643	(0.0066)	-7.3981	(0.0000)
Germany	-1.5521	(0.3170)	-2.0071	(0.2200)
Spain	0.1395	(0.9693)	-9.8835	(0.0000)
Sweden	-0.9348	(0.1719)	-1.9495	(0.0456)
UK	-2.7143	(0.5521)	-1.9374	(0.0001)
US	-2.4310	(0.0366)	-1.0525	(0.2871)

All data are observed annually. The number of observations varies across countries, and for each country reflects all available data. Correlation coefficients are calculated by OLS regression (with a constant term) and p-values are for the standard null hypothesis of $H_0: \text{coefficient}=0$.

Depending on country data begin anywhere from late-19th century to 1970s, and for all countries are through till the present. Wealth-Income data from WWID. Capital-Output data from PWT8.

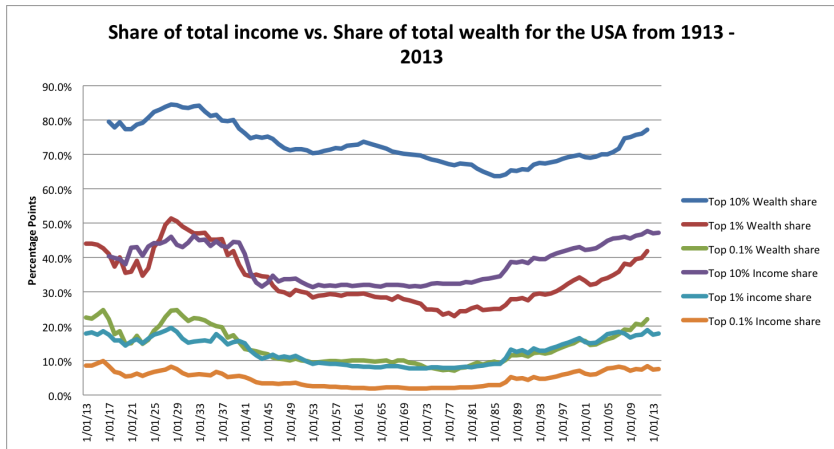
2 Negative Correlation between Wealth Inequality and rate of return to wealth (r)

Correlation Coefficients between Wealth Inequality and r (for US)

	Share of Top 10% (SCF)	Wealth Inequality Measure	
		Gini Coeff. (SCF)	Share of Top 10% (SZ2016)
		[OLS coefficient (p-value)]	
Rate of Return to Wealth (KN,WWID)	-0.5958 (0.1985)	-0.0038 (0.2988)	-1.0631 (0.0276)
Rate of Return to Wealth (OECD,WWID)	-0.5996 (0.2423)	-0.0038 (0.3489)	-1.2546 (0.0100)
Rate of Return to Capital (GommeEtAl)	0.1259 (0.1649)	0.0014 (0.0329)	0.1346 (0.0048)
Interest rate (IMF 3mth, OECD CPI)	-0.4262 (0.0899)	-0.0031 (0.1146)	-0.8261 (0.0000)
Interest rate (IMF 3mth, OECD GDPDEF)	-0.3481 (0.2023)	-0.0023 (0.2807)	-0.9320 (0.0000)
Interest rate (FedRes 10yr, BLS CPI)	-0.5908 (0.0090)	-0.0049 (0.0024)	-0.6606 (0.0002)
Interest rate (OECD 24hrIB, OECD CPI)	0.0048 (0.9835)	-0.0002 (0.8940)	-0.9413 (0.0035)

All regressions are run on the available annual data; note that SCF data is only periodic so contains small number of observations. The number of observations varies across and reflects all available data for each time series post-1954. Correlation coefficients are calculated by OLS regression (with a constant term) and p-values are for the standard null hypothesis of H_0 : coefficient=0. For purpose of regression the shares are measured as 0 to 100. First data observations are 1975 for KN data, 1970 for OECD data, 1954 otherwise.

3 Positive Correlation between Income Inequality and Wealth Inequality in the US



This longer time period is based on WWID income data and Saez and Zucman (2016) wealth data, both of which use tax returns. Rather than SCF.

- ④ Positive Correlation between Income Inequality and Income Risk in the US
 - Differentiating between two possibilities: increase in income inequality reflects a divergence between different 'fixed types' of people, or that the increase in income inequality reflects an increased variance of 'shocks' to individuals incomes.
 - 'fixed types' would see variances of income and consumption increase; ratio unchanged.
 - Consumption-smoothing means 'shocks' would see variance of income variance increase, but not variance of consumption; ratio increases.
 - 'shocks' is the idea of income risk.

- This measure is not without issues. The main one is that the effects of income risk are deeply intertwined with the availability of insurance and separating the two requires some assumptions.

Another issue is income risk vs social mobility.

- Guvenen and Smith (2014) provide one approach to this, but do not look at changes in income risk over time.
- Measures presented here do not control for any changes in the availability of (self- or market) insurance but we are confident in concluding that while they do not provide an accurate picture of 'how much' income risk has increased they do capture the direction of the change.
- This confidence mostly follows from an extensive microeconomic literature that finds similar trends of increasing income risk using a variety of different methods (Arellano, Blundell, and Bonhomme, 2015; Attanasio and Pistaferri, 2014; Krueger, Perri, Pistaferri, and Violante, 2010; Moffitt and Gottschalk, 2009, 2012).