

**Validity of NPV Rule and IRR Criterion in Cost-benefit Analysis
New Approach to Perfect the Estimates and to Resolve the Common Criticisms
of Reinvestment, Multiple IRR and Spurious NPV**

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HISTORICAL PERSPECTIVES

Lorie and Savage (1956) considered:

- a. the DCF method for CBA as “ambiguous or anomalous” as early as 1956
- b. *With such an ambiguity “the rate-of-return solution does not result in maximizing the present worth of the firm's net worth.”*

Merlo (2017) argued that:

- a. the DCF cannot be a correct approach when NPV function is nonmonotonic
- b. it is wrong to ‘treat symptoms of disease with out finding out the cause’.

What Next?

What causes the problems of **Reinvestment, multiple IRR and spurious NPV (hither to unreported)** and resolving them by applying a new approach.

The Current Situation in Cost-Benefit Analysis (CBA)

1. The Discounted Cash Flow (DCF) method used in CBA, is subject to three serious criticisms (among other minor problems):

1.1. the assumption of **reinvestment** of interim income.

1.2 The problem of **multiple IRR** (more than one IRR).

1.3 The IRR is **not the return on total capital** but on balance of capital.

2. These criticisms are evaluated applying a **new approach**:

2.1. to evaluate the cause and effects of these problems.

2.2 to resolve the problems of **reinvestment, multiple IRR and Spurious NPV.**

The Net Present Value (NPV) Rule

THE NPV RULE: Accept investments with positive NPV.

Major criticisms against Cost-benefit Analysis (CBA):

- the assumption of reinvestment of interim income.
- The problem of multiple IRR (more than one IRR).
- The IRR is not the return on total capital (on balance of capital)

Three questions:

Will the NPV rule be valid in all cases (**Spurious NPV**)?

Whether the underlying causes affect **ONLY** the IRR not NPV?

What causes coexistence of spurious NPV and multiple IRR?

COEXISTENCE OF MULTIPLE IRR AND SPURIOUS NPV (NPV RULE ?)

Investments with no net profit or benefit end up with spurious +ve NPV and Multiple IRR

Years	1. Solomon (1956:128)	2. Excel user Microsoft	3. Damodaran (2011)	4. Arnold & Hope (1990)
0	-1600	-1000	-1000	-2000
1	10000	6000	800	5100
2	-10000	-11000	1000	-3150
3	0	6000	1300	0
4	0	0	-2200	0
Sum of NCF (NPV at 0%)	-1600	0	-100	-50
NPV AND IRR ESTIMATED BY DCF METHOD				
NPV at 10%	-\$774	-\$128	\$28	\$33
NPV at IRR1	\$0	\$0	\$0	\$0
NPV at IRR2	\$0	\$0	\$0	\$0
IRR 1	25%	100.0%	6.6%	5.0%
IRR 2	400.0%	200.0%	36.5%	50.0%
IRR 3	NIL	0.0%	NIL	NIL
NPV AND IRR ESTIMATED BY A MODIFIED CAS METHOD				
NPV at 10%	-3636	-667	-157	-207
IRR	-100%	0%	-9%	-2%

REINVESTMENT OF INTERMEDIATE INCOME

- **Reinvestment Assumption:** *Interim* income would get *reinvested* at the same rate as IRR or at cost of capital (hurdle rate) in the case of NPV.
- “The prospective return on another independent investment should never be allowed to influence the investment decision” (Brealey et. al, (2011).
- Is it correct to assume that both NPV and IRR have implicit reinvestment rate? With DCF method, reinvestment is prevalent in some cases.
- A new approach is introduced to identify and demonstrate that the reinvestment causes the multiple IRR and spurious NPV.

Normal and Non-normal cash flow

Non-normal cash flow stream leads to non-monotonic NPV functions with resulting reinvestment problem, multiple-IRR and spurious NPV.

1. Normal cash flow (NCF) - No interim capital out flow

Investment Year	Year 1	Year 2	Year 3	Year 4	Year 5
-1000	600	500	400	300	300
Capital cost (Negative flow)	Income or Benefit flow (Positives)				

2. Non-normal cash flow (NNCF) - One or more interim capital outflow

Investment Year	Year 1	Year 2	Year 3	Year 4	Year 5
-1000	600	500	-400	300	-300
Capital cost (Negative flow)	Income flow (Positives)		Cost flow (Negatives)		

The Approach

- Introducing a Capital Amortization Schedule (**CAS**) based method.
- **Estimating IRR and NPV and identifying any reinvestment of income.**
- **Modifying the CAS method to eliminate the reinvestment.**
- Comparing the estimated IRR and NPV by the CAS and DCF methods.
- Resolving the decades old **reinvestment and multiple IRR problems.**
- **Illustrating the appropriateness of IRR vs NPV**
- **Estimating IRR on total capital (DCF estimates IRR on balance of capital).**

WHAT IS CAPITAL AMORTIZATION SCHEDULE ? AN EXAMPLE

Capital amortization schedule is a table showing how each periodic **future income** from an investment is allocated towards **return on invested capital (ROIC)** and **return of capital (ROC)**.

Years	0	1	2	3	4	
Net Cash flow (NCF) \$	-1000	300	400	500	600	
Capital Amortization Schedule (CAS) Method:						
					Allocation	
	1	2	3	4	Items	Amount
Opening Capital (OC)	-1000	-800	-480	-28	ROC	-1000
Interest/ Return (ROIC) 10.0%	-100	-80	-48	-3	ROIC	-231
Income (CIF)	300	400	500	600	CIF	1800
Closing Balance (CB)	-800	-480	-28	569	CB	569
NOTE: CB = OC + ROIC + CIF		(ROC + ROIC + CIF = CB)				

INTRODUCING CAS METHOD AND THE ESTIMATED IRR & NPV

A	B	C	D	E	F	G	H
Capital Amortization Schedule (CAS) Method: Normal NCF							
IRR by CAS							
Net Cash flow (NCF)	-1000	300	400	500	600	IRR	
Opening Capital		-1000	-949	-785	-480	IRR is the rate that makes the CB = 0 by using the "Goal Seek" function.	
Interest/ Return	24.9%	-249	-236	-195	-120		
Income		300	400	500	600		
Closing Balance		-949	-785	-480	0	24.9%	
NPV at 10% by CAS							
NCF	-1000	300	400	500	600	NPV at 10%	
Opening Capital		-1000	-800	-480	-28	The PV of the CB is the NPV of the NCF at say 10% - $(F13/(1+10\%)^4)$	
Interest/ Return	10.0%	-100	-80	-48	-2.8		
Income		300	400	500	600		
Closing Balance (CB)		-800	-480	-28	569	(F13/(1.1)^4)	→ 388.8
		IRR	NPV at 10%		IRR ON TOTAL CAPITAL		
DCF METHOD ESTIMATE		24.9%	\$388.8		20.0%		

B C D E F G H
Non-normal NCF (NNCF), Reinvestment of Intermediate Income and Multiple IRR
Data Source: Damodaran (<http://people.stern.nyu.edu/adamodar/pdfiles/acf3E/book/ch1thru4.pdf>)

Multiple IRR by DCF Method						
Details / Years	0	1	2	3	4	Multiple IRR by DCF
NNCF	-1000	800	1000	1300	-2200	6.6%
(non-normal cash flow)						36.5%
Multiple IRR by CAS Method						
NNCF	-1000	800	1000	1300	-2200	Multiple IRR
Opening Capital	-1000	-266	716	2064		IRR 1
Interest/ Return	6.6%	-66	-18	47	136	6.6%
Income		800	1000	1300	-2200	
Closing Balance		-266	716	2064	0	
NNCF	-1000	800	1000	1300	-2200	income: Interest
Opening Capital	-1000	-565	228	1611		with positive signs
Interest/ Return	36.5%	-365	-207	83	589	are reinvestment.
Income		800	1000	1300	-2200	IRR 2
Closing Balance		-565	228	1611	0	36.5%

Modified CAS (MCAS) method that Eliminates Reinvestment and Multiple IRR

Data Source: Damodaran (<http://people.stern.nyu.edu/adamodar/pdfiles/acf3E/book/ch1thru4.pdf>)

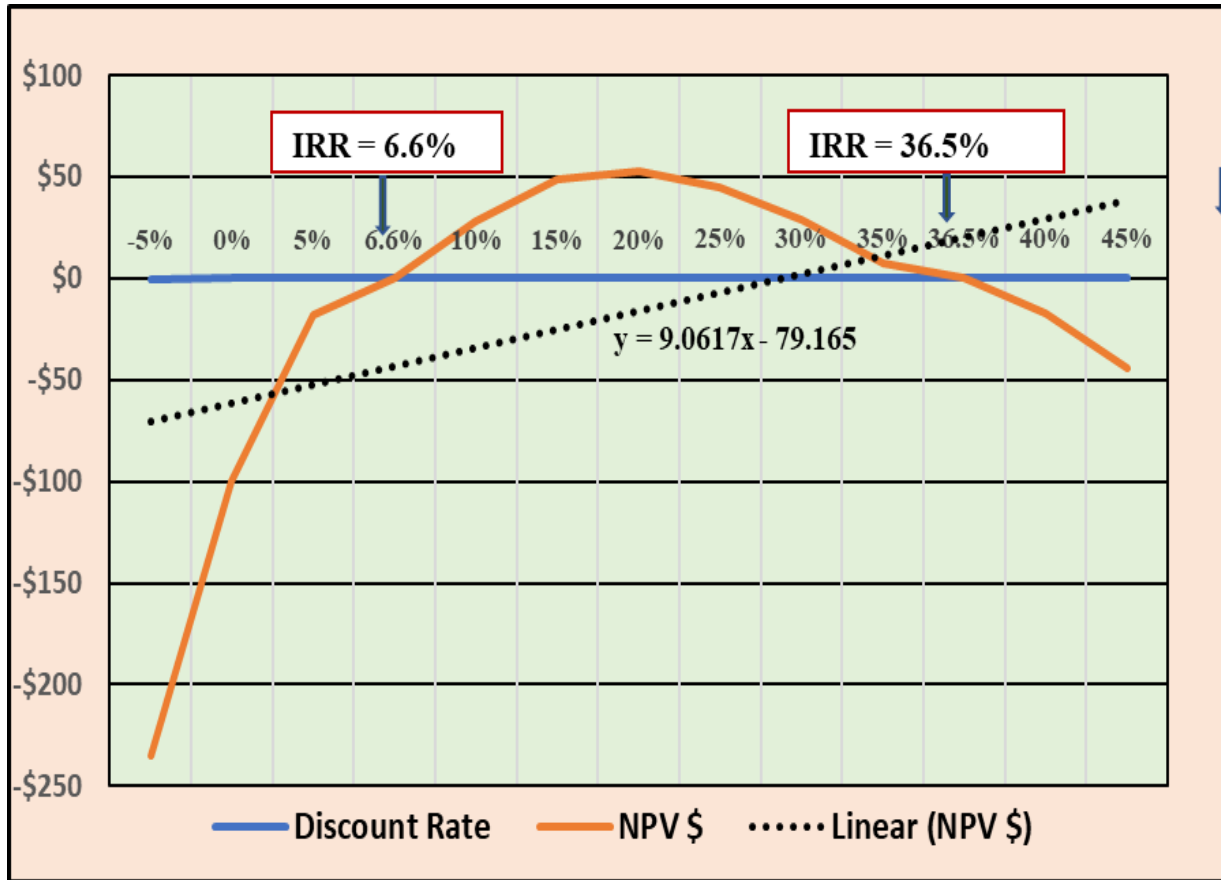
Details / Years	0	1	2	3	4	Multiple IRR with DCF
CAS and DCF - IRR	-1000	800	1000	1300	-2200	6.6%
Sum of NCF = -100	NPV at 10% by DCF				27.8	36.5%
Modified CAS (MCAS) Method						
MCAS - IRR	-1000	800	1000	1300	-2200	Unique IRR by MCAS
Opening Capital		-1000	-110	900	2200	-9.0%
Interest/ Return	-9.0%	90	10	0	0	
Income		800	1000	1300	-2200	NPV at -9.0%
Closing Balance		-110	900	2200	0	0.0
MCAS - NPV	-1000	800	1000	1300	-2200	Spurious NPV by DCF
Opening Capital		-1000	-300	670	1970	27.8
Interest/ Return	10%	-100	-30	0	0	
Income		800	1000	1300	-2200	NPV at 10% by MCAS
Closing Balance		-300	670	1970	-230	-157.1

Note: For similar NCF data with multiple IRR problems please read reference item 2 in the last page 12

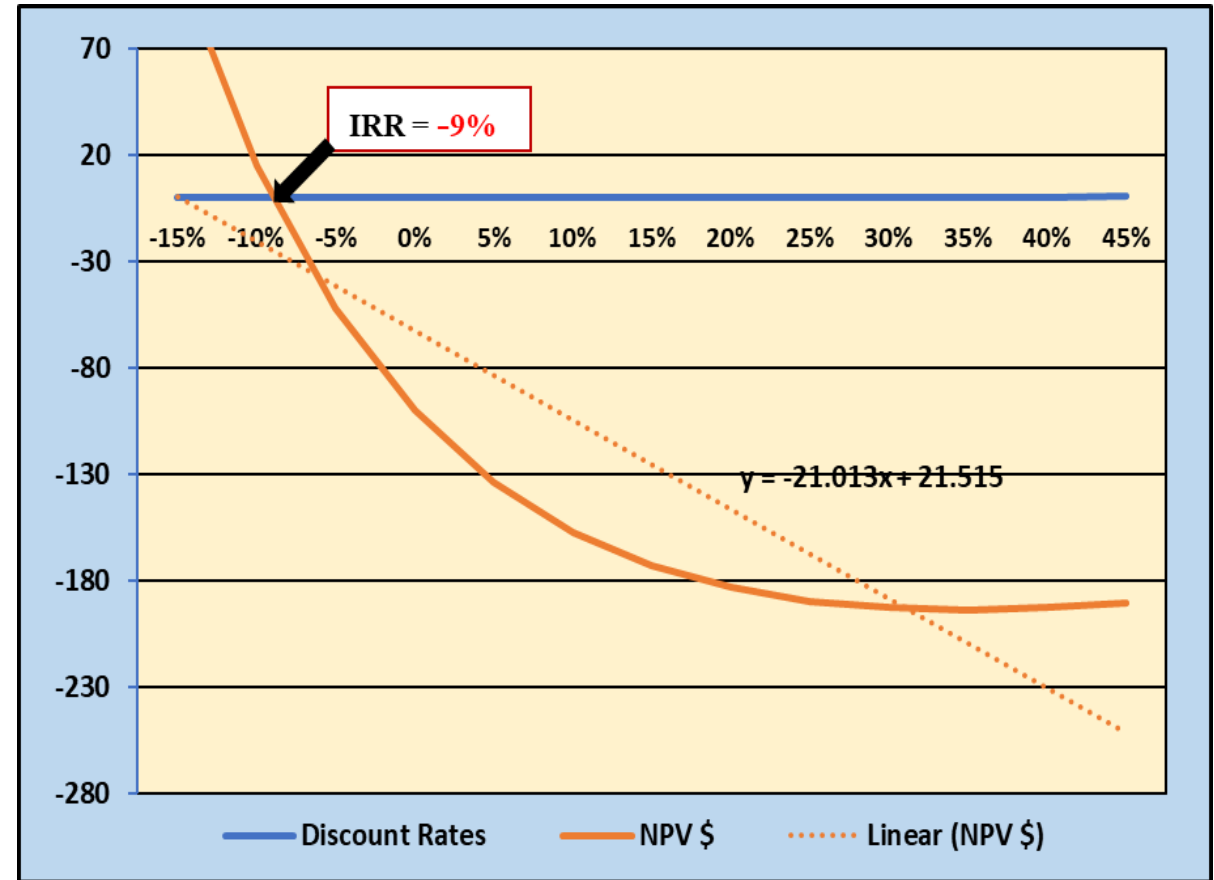
Monotonic vs Non-monotonic NPV functions

Non-normal cash flow stream leads to non-monotonic NPV functions with resulting reinvestment problem, multiple-IRR and spurious NPV (Refer page 9).

Non-monotonic NPV function



Monotonic NPV Function



B

C

D

E

F

G

H

Comparison of Estimated IRR and NPV by DCF, CAS and MCAS methods

NORMAL NCF INVESTMENT WITH CONSISTENT IRR AND NPV

Details / Years	0	1	2	3	4	IRR	NPV at 10%
BY DCF METHOD	-1000	300	400	500	600	24.9%	\$388.8
BY CAS METHOD	-1000	300	400	500	600	24.9%	\$388.8

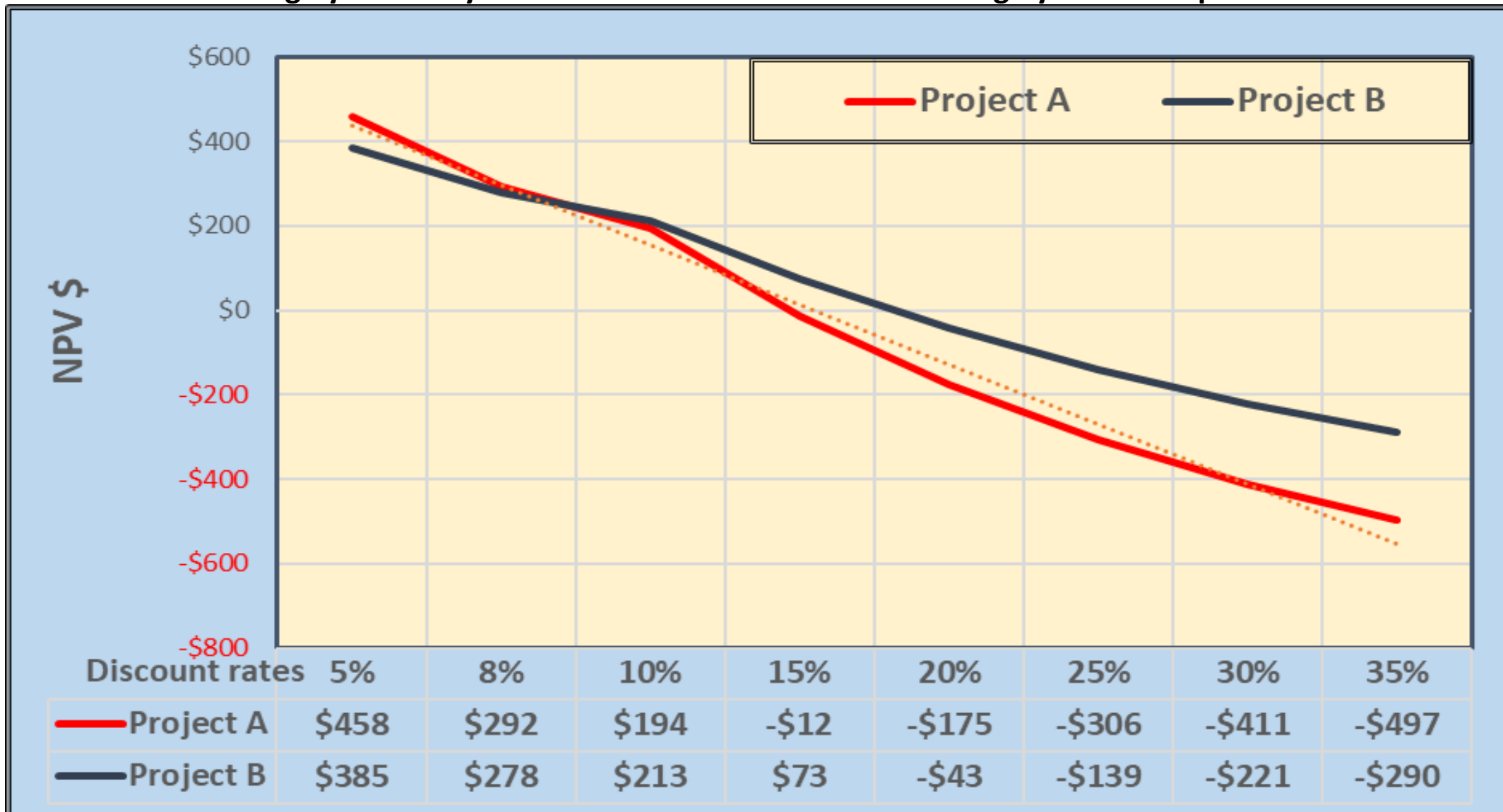
NON-NORMAL NCF (NNCF) INVESTMENT WITH INCONSISTENT NPV AND IRR

Details / Years	0	1	2	3	4	IRR -1	IRR - 2	NPV at 10%
BY DCF METHOD	-1000	800	1000	1300	-2200	6.6%	36.5%	27.8
BY CAS METHOD	-1000	800	1000	1300	-2200	6.6%	36.5%	27.8
BY MCAS METHO	-1000	800	1000	1300	-2200	-9.0%		-151.7

MUTUALLY EXCLUSIVE PROJECTS RANKING (e.g. Silber 2021)

Project A - IRR =15% and Project B IRR =18%

Ranking by NPV vary with discount rates where as Ranking by IRR is unique



SUMMARY AND CONCLUSIONS

1. **The NPV rule is not valid always and the criticism against IRR criterion is also not valid as revealed by an analysis using a capital amortization schedule (CAS) method .**
2. **The DCF method is not appropriate for all investments (e.g. non-normal NCF)**
3. **The CAS method: a. eliminates reinvestment, spurious NPV and multiple IRR problems; b. leads to a unique IRR; c. enables estimation of IRR on total capital.**
4. **The assumption of reinvestment at IRR or at hurdle rate in NPV in ALL the cases of normal NCF and NNCFs is FALSE .**
5. **Reinvestment is evident with some NNCFs that affects both NPV and IRR. With some normal NCF, it affects ONLY the NPV not the IRR if the discount rate is low;**
6. **Ranking of mutually exclusive projects by NPV is not unique but unique by IRR.**
7. **As MIRR is built on the assumption of reinvestment, it becomes spurious indicator.**

REFERENCES

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