Reconciled Estimates of Monthly GDP in the US

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2021 Australian Conference of Economists

[1]The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Cleveland or the Federal Reserve System.

Introduction

- Main goal of paper
  - To produce estimate of "true" *GDP* at monthly frequency
  - Available data: Two noisy measures of *GDP*: Income and expenditure side *GDP*
  - GDP_I and GDP_E available at the quarterly frequency
- Econometric Model
  - Bayesian Mixed Frequency State Space VAR model with ADNSS measurement error restrictions
- Main contribution/Outcome
  - More high-frequency (monthly) estimates of "True" *GDP* for the US

Introduction (cont.)

- We do this by combining ADNSS and SS
  - ADNSS = Aruoba, Diebold, Nalewaik, Schorfheide and Song. Improving GDP measurement: A measurement-error perspective. Journal of Econometrics, 2016.
  - SS = Schorfheide and Song. Real-time forecasting with a mixed-frequency VAR. JBES, 2015.
  - Our econometric model is a restricted Mixed Frequency Vector Autoregression (MF-VAR) where restrictions reflect measurement error perspective of ADNSS

Review of Economics 101

- What is GDPE?
  - The expenditure approach - the total value of goods and services produced by a country. Y = C+I+G+(X-M)
  - This what most empirical macroeconomist use in the literature
- What is GDPI?
  - The income approach - A measure of economic activity based on incomes.
  - GDPI = compensation of employees + gross operating surplus + gross mixed income + taxes less subsidies on production and imports
  - In theory, GDPI should be equal to GDPE, the different source data yield different results and this is due to measurment error.
  - Nalewaik (2010) shows that GDPI is a better early predictor of a recession than GDPE

The Measurement Error Perspective of ADNSS

- ADNSS work at quarterly frequency
  - GDP_I and GDP_E are observed
  - GDP is not
  - ADNSS start by assuming

$$\begin{bmatrix} GDP_{Et} \\ GDP_{It} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} GDP_t + \begin{bmatrix} \epsilon_{Et} \\ \epsilon_{It} \end{bmatrix} \tag{1}$$

$$GDP_t = \rho \; GDP_{t-1} + \epsilon_{Gt} \tag{2}$$

$$\begin{bmatrix} \epsilon_{Gt} \\ \epsilon_{Et} \\ \epsilon_{It} \end{bmatrix} \sim iidN \left[\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma^2_{GG} & \sigma^2_{GE} & \sigma^2_{GI} \\ \sigma^2_{GE} & \sigma^2_{EE} & \sigma^2_{EI} \\ \sigma^2_{GI} & \sigma^2_{EI} & \sigma^2_{II} \end{pmatrix} \right] \tag{3}$$

The Measurement Error Perspective of ADNSS (cont.)

- The basics intuition: If $\sigma^2_{EE} < \sigma^2_{GG}$ this implies 'News' and $\sigma^2_{EE} > \sigma^2_{GG}$ 'Noise'
- The previous model is not identified
- ADNSS achieve identification by setting this parameter $\xi = 0.8$ (ratio of variance of GDP to variance of GDP_E)
- $\sigma^2_{GI} = \sigma^2_{GE} = 0$ (Noise restriction holds exactly)
- In our paper, we set both restriction of ξ_E and ξ_I to interval $[0.55, 1.15]$

Writing the ADNSS Model as a VAR

- Every model we work with is a VAR (involving latent states):

$$Ay_t = By_{t-1} + \epsilon_t, \epsilon_t \sim N(0, \Sigma), \tag{4}$$

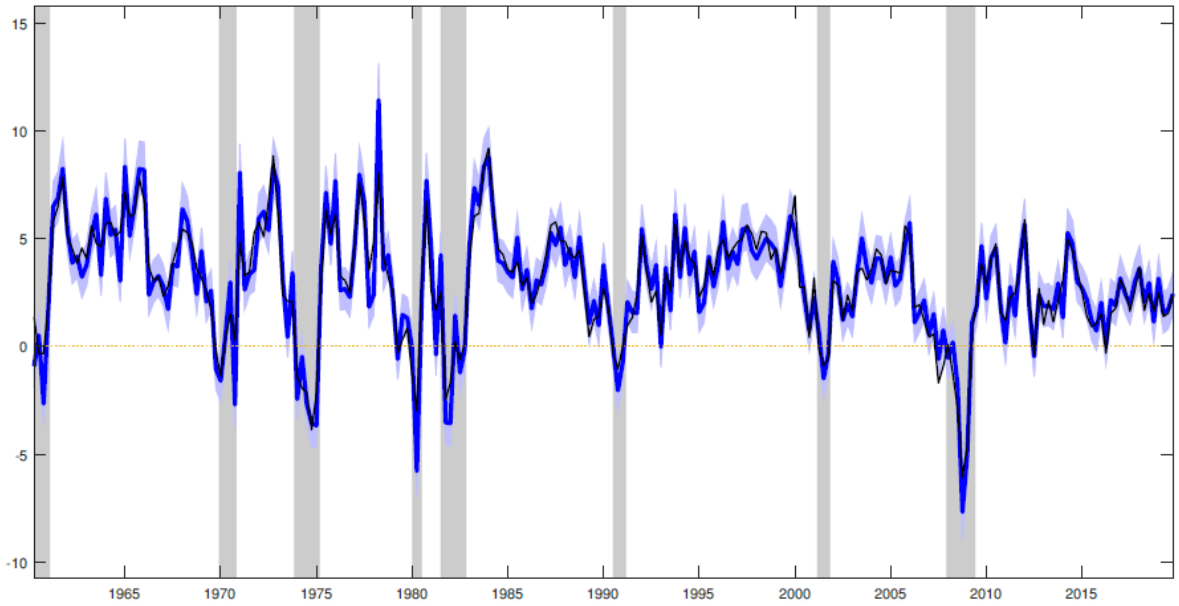
- A is lower triangular and Σ is diagonal
- Σ contains error variances and A determines covariances (contemporaneous relationships)
- Computational advantage: writing VAR in this way allows for equation-by-equation estimation
- ADNSS model has $y_t = (GDP_t, GDP_{Et}, GDP_{It})'$
- All elements of B zero except for b_{11}
- Identification is achieved via the A

ADNSS' Identification Scheme

- Identification can be achieved by three ways:
  1. If $\sigma^2_{GI} = \sigma^2_{GE} = 0$ then $a_{21} = -1$, $a_{31} = -1 + \sigma_{EI}$ and $a_{32} = -\sigma_{EI}$ (Noise restriction hold exactly)
     - This specification ensures that the volatility of true *GDP* is less than the volatility of GDP_E or GDP_I.
     - This implies the measurement error is purely noise, as opposed to the idiosyncratic variation in GDP_E and GDP_I containing news or information about the true state of the economy.
     - If the measurement error is pure news, true *GDP* is more volatile than either GDP_E or GDP_I.
  2. Include the unemployment rate, U_t, into the VAR. This treats unemployment is an instrumental variable.
     - U_t is uncorrelated with measurement errors in GDP_E and GDP_I but correlated with *GDP*
  3. Set Identification
     - We ensure both ξ_E and ξ_I (ratio of variance of *GDP* to variance of GDP_E or GDP_I) are bounded to the interval $[0.55, 1.15]$.

Quarterly results

Figure 4: Quarterly posterior median estimates of true US real GDP growth (blue line) versus the Philadelphia Fed's GDPplus (black line)



Notes: GDP growth in quarterly annualized percent changes from 1960q1-2019q4 (blue line) from the VAR model in GDP_E, GDP_I and unemployment, as seen in Table 2. Blue shaded region is the 16th and 84th percentile interval of the posterior density of true GDP. Vertical shaded areas represent NBER-defined recessions

Mixed Frequency VAR

- So far every time series variable at quarterly frequency
- But unemployment (and many other macroeconomic variables) are available monthly
- We still use the VAR

$$Ay_t = By_{t-1} + \epsilon_t, \epsilon_t \sim N(0, \Sigma), \tag{5}$$

- Now $t = 1, \ldots, T$ is time at the monthly frequency
- $y_t = (X_t', U_t, GDP_t, GDP_{Et}, GDP_{It})'$
- X_t contains other monthly variables, consider three versions:
- X_t contains 8 variables of SS
- X_t contains 50 variables (Big Data)
- X_t contains no variables (ADNSS)

Mixed Frequency VAR

- X_t and U_t are observed, but the other elements of y_t are not
- We never observe monthly values of GDP, GDP_E, GDP_I
- For GDP_E and GDP_I we observe quarterly values
- This is an MF-VAR
- If true GDP was omitted from it, it would be a conventional MF-VAR as in SS
- Our model adds the model of ADNSS to SS
- Main benefit: monthly estimates of true GDP
- Side benefit: monthly estimates of GDP_E and GDP_I

The Intertemporal Restriction

- Let Y_t^Q be a quarterly variable in levels
  - We work with log quarterly differences: $y_t^Q = \Delta_3 \ln Y_t^Q$
  - And produce estimates of log monthly differences: $y_t^M = \Delta \ln Y_t$ where Y_t is the monthly variable
  - Easy to show link between two is (approx.)

$$y_t^Q = \frac{1}{3} y_t^M + \frac{2}{3} y_{t-1}^M + y_{t-2}^M + \frac{2}{3} y_{t-3}^M + \frac{1}{3} y_{t-4}^M \tag{6}$$

  - This is the intertemporal restriction

Model Estimation - Priors

- Two groups of coefficients
- "Economic" parameters:
  - Parameters of ADNSS model (reflect measurement error perspective)
  - We implement a subjective prior, centered over the noise restriction
- "VAR" parameters:
  - The remaining parameters for VAR block of the model
  - No subjective prior info about these (and potentially high dimensional)
  - We implement the Dirichlet-Laplace global-local shrinkage prior
  - Minimal prior hyperparameter choice and can automatically decide which coefficients to shrink to zero.

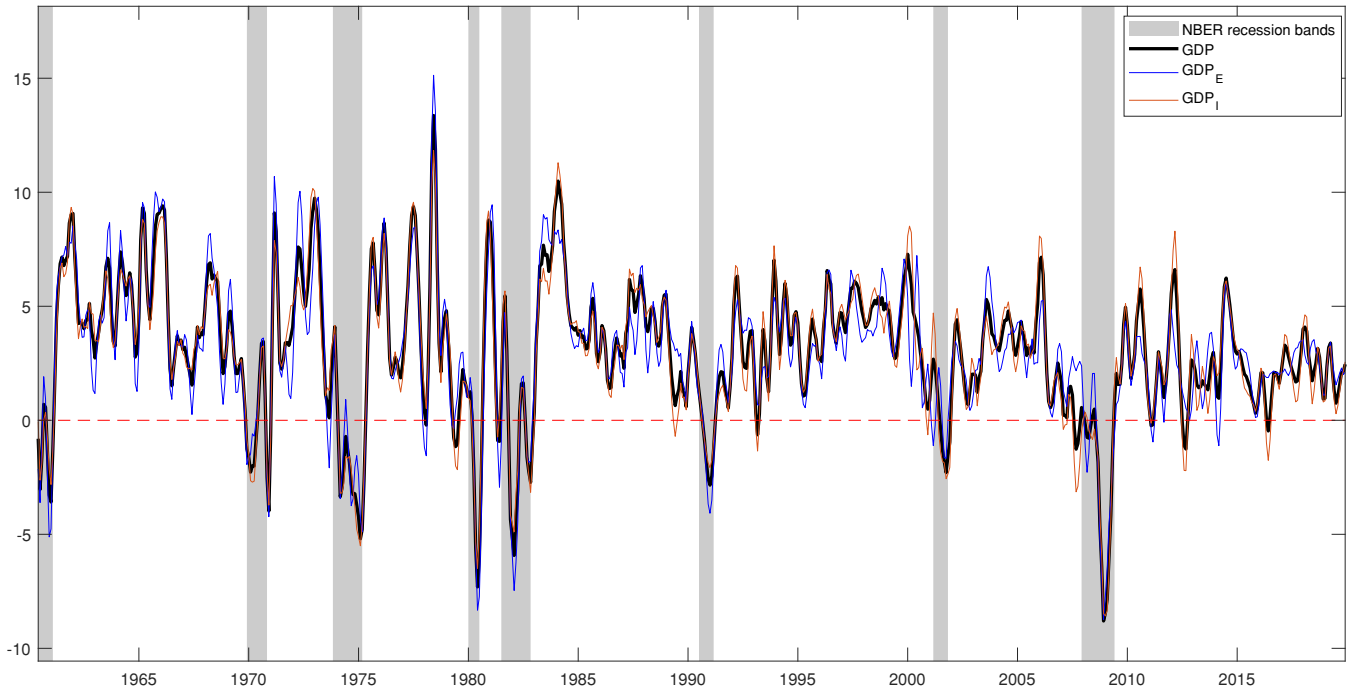
Model Estimation - ADNSS Restriction

- Unemployment is an instrument
- Noise restriction holds
- All of X_t are instruments (model breaks into SS block and ADNSS block with true GDP being only variable present in both)
- ξ_E and ξ_I are bounded to the interval $[0.55, 1.15]$.
- We consider models with various combinations of these restrictions imposed

Summary of Models

Model	Monthly Variables	Noise imposed	Instruments
SS(IV)	$X^8, U, GDP, GDP_E, GDP_I$	No	X^8, U
SS(IV+N)	$X^8, U, GDP, GDP_E, GDP_I$	Yes	X^8, U
SS	$X^8, U, GDP, GDP_E, GDP_I$	No	U
SS(N)	$X^8, U, GDP, GDP_E, GDP_I$	Yes	U
SS+	$X^{50}, U, GDP, GDP_E, GDP_I$	No	U
ADNSS	U, GDP, GDP_E, GDP_I	No	U
ADNSS(N)	U, GDP, GDP_E, GDP_I	Yes	U

Empirical Results - Monthly 'True' GDP estimates (SS model)



Does GDP_I or GDP_E play bigger role?

- We estimate this λ^* that explains the proportion of GDP_E in explaining "true" GDP_t

$$\lambda^* = \underset{\lambda}{\text{argmin}} \sum_{t=1}^{T} [(\lambda GDP_{E,t} + (1-\lambda) GDP_{I,t}) - GDP_t]^2,$$

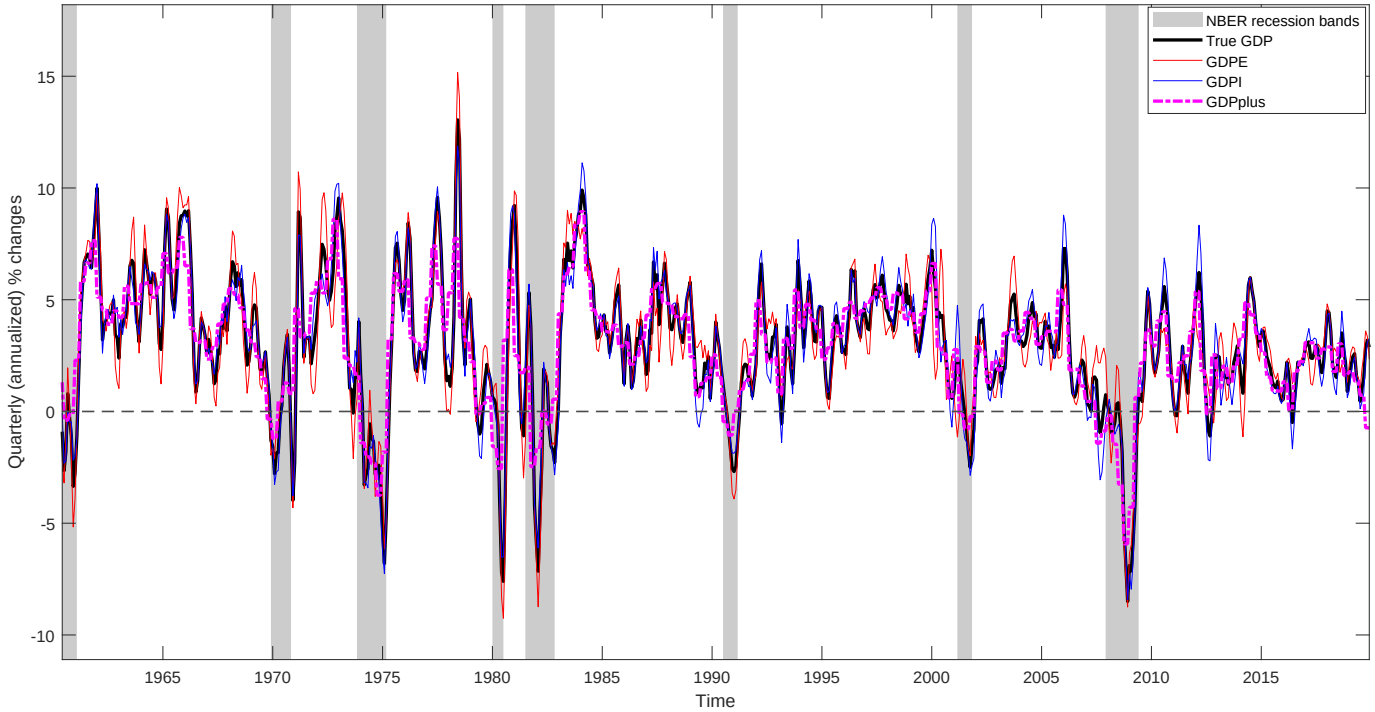
	λ^*
SS(IV)	0.37
SS(IV+N)	0.44
SS	0.35
SS(N)	0.40
ADNSS	0.34
ADNSS(N)	0.44
SS+	0.27

Probabilities that Noise Restriction Does Not Hold

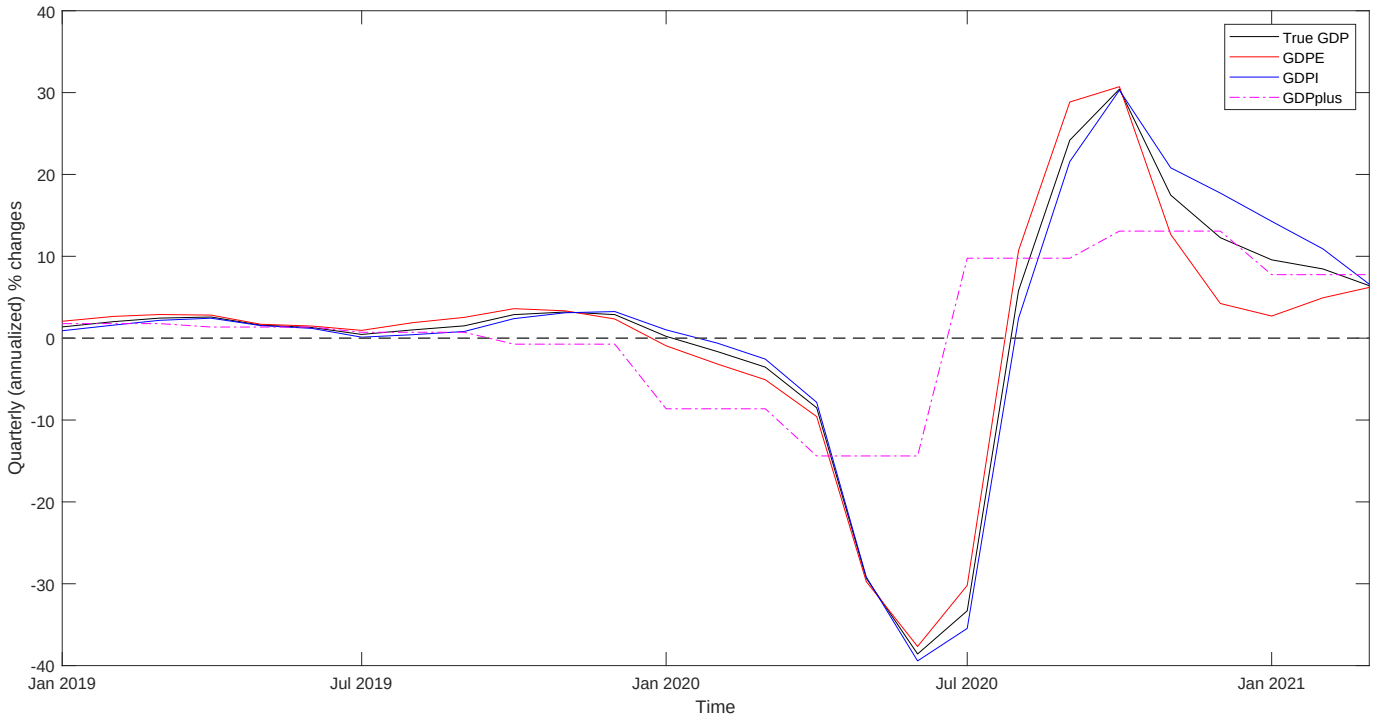
News versus noise by model: posterior probabilities that ξ_E and ξ_I are greater than one implying news

	$p(\xi_E > 1)$	$p(\xi_I > 1)$	$p(\xi_E > 1 \text{ and } \xi_I > 1)$
SS(IV)	0.00	0.01	0.00
SS(IV+N)	0.00	0.00	0.00
SS	0.01	0.37	0.01
SS(N)	0.00	0.00	0.00
ADNSS	0.00	0.01	0.00
ADNSS(N)	0.00	0.00	0.00
SS+	0.00	0.51	0.00

Pre-Pandemic results - latest vintage data



Pandemic results - latest vintage data



Conclusion

- Develop class of restricted MF-VAR model to produce monthly estimates of US *GDP*
- Combines conventional MF-VAR (SS) with measurement error perspective (ADNSS)
- Works well in practice as evidenced by econometric evidence shown here

Thank You

- For more information about this paper and my research, please go to my personal webpage https://sites.google.com/view/aubreybcpoon/home