Income Distribution in a Dynamic Assignment Model

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Introduction

- The top 1% of the richest households hold more than one-third of wealth in the United States, while the bottom 40% hold less than 1% wealth (Wolff 2004);

- The share of assets held by the rich has increased rapidly (Piketty and Saez 2003)

- Factors helps explain the skewness of wealth distribution:

  persistent income shocks, discount rate heterogeneity, savings and bequests, entrepreneurship and idiosyncratic returns to investment, etc.

A new mechnism

In a static assignment model, sorting affects wage rate, not vice versa. Eeckhout and Kircher (2018) introduced firm size effect to resolve this issue.

In our dynamic assignment model, sorting and wage rate are determined endogenously and simultaneously, irrespetive of firm size.

Our model

- Positive assortative matching between bosses and workers;
- Heterogeneity exists on both sides:
  - The human capital of the boss accumulates over time: determined by a random innate ability shock, parents' human capital and bequest;
  - The human capital of the employee is exogenously given;
- Employees are hand to mouth;
- The matching rule is determined by the human capital distributions of bosses and workers;
- The human capital of bosses and workers are observable.

The experiments conducted to examine the effects of **the weight of boss** in the output production, the **technology**, and the **mean** and **variance** of the employee's human capital in the static and dynamic model environment.

- In the static model, the distribution of boss' human capital is exogenously given.
- In the dynamic model, its distribution is dynamically determined.
- In the dynamic model, the capital accumulation plays important roles in matching, labor and firm size while the static model takes capital as given and lowers the interactions among the variables.

Literature review

Our paper links **three streams** of literature.

The **first** follows the early work of Becker (1973) in which a standard frictionless matching model is used to explore the sorting between firms and workers.

Eeckhout and Kircher (2018): positive assortative matching (PAM) and negative assortative matching (NAM); find that PAM could help explain why more productive firms tend to become larger, e.g. Google, Apple etc,

Using assignment models, investigate the increase in the top income earners: the high level CEO payment; the taxation of superstars, e.g., Gabaix and Landier (2008) and Tervio (2008), Scheuer and Werning (2017), Bao et al (2022).

Literature review

The **second stream** of literature is related to the inheritance effect on income inequality in the dynamic model: Becker and Tomes (1979) and Benhabib et al. (2011).

The **third stream** of literature is on the human capital accumulation through learning from coworkers.

Anderson and Smith (2010): consider the matched coworker's contribution to the accumulation of human capital. No sorting is considered.

Anderson (2015): peer's effect on one's productivity is important; perfect sorting is considered.

Herkenhoff et al (2018): nearly 60% of learning on the job could be from coworkers.

Jarosch et al (2021): between 4 and 9% of worker's compensation is benefited from learning.

The model

Bosses and employees match and produce outputs in an assignment framework.

All of bosses and employees live for one period. At the end of the period, each of them give birth to one child so that the population keeps constant. Bosses leave bequests to their children when they die.

The boss in period t with human capital x_t chooses his consumption $c_{B,t}$ and bequest $b_{B,t+1}$ to maximize utility,

$$\max_{c_{B,t}, b_{B,t+1}} \frac{c_{B,t}^{1-\gamma}}{1-\gamma} + \chi_B \frac{b_{B,t+1}^{1-\gamma}}{1-\gamma}, \tag{1}$$

$$s.t. \; c_{B,t} + b_{B,t+1} = \pi(x_t). \tag{2}$$

where γ is the reciprocal of the intertemporal elasticity of substitution and χ_B represents the intensity of the bequest motive. The boss' income $\pi(x_t)$ is from the firm's profit.

The optimal solutions to boss' problem are,

$$c_{B,t} = \frac{1}{1 + \chi_B^{\frac{1}{\gamma}}} \pi(x_t), \tag{3}$$

and

$$b_{B,t+1} = \frac{\chi_B^{\frac{1}{\gamma}}}{1 + \chi_B^{\frac{1}{\gamma}}} \pi(x_t). \tag{4}$$

The human-capital accumulation follows

$$x_{t+1} = \kappa \theta_{t+1} x_t^{\epsilon} b_{B,t+1}^{\eta}, \tag{5}$$

where $\kappa > 0$ is a constant and θ_{t+1} denotes the innate ability and follows a lognormal distribution, $\ln \theta_{t+1} \sim N(\mu_\theta, \sigma_\theta)$.

The employee problem

The employee consumes all his wage income,

$$c_{e,t} = W_t(y). \tag{6}$$

where $W_t(y)$ is the worker's wage income in period t, y is the human capital of the employee, drawn from an exogenous distribution function $F_Y(y)$.

Since employees are hand-to-mouth, they do not have human-capital accumulation.

Matching

The labor market has no friction.

The firm decides the quality and quantity of labor hiring.

A firm with a boss of human capital x hires L employees of human capital y to produce output according to the function,

$$f(x, y, L) = A\left[\beta x^{\frac{\alpha-1}{\alpha}} + (1-\beta) y^{\frac{\alpha-1}{\alpha}}\right]^{\frac{\alpha}{\alpha-1}} L^{\phi}, \tag{7}$$

where L represents the firm size, $A > 0$ is the firm's productivity, and $0 < \beta < 1$ denotes the weight of bosses in production. $\alpha > 0$ measures the elasticity of substitution between x and y. We assume that $\alpha < 1$, i.e., positive assortative matching as that in Eeckhout and Kircher (2018).

The boss chooses the quality of the worker, y, and the number of workers, L, to maximize his profit,

$$\pi(x) = \max_{y,L} A\left[\beta x^{\frac{\alpha-1}{\alpha}} + (1-\beta) y^{\frac{\alpha-1}{\alpha}}\right]^{\frac{\alpha}{\alpha-1}} L^{\phi} - W(y)L. \tag{8}$$

FOC with respect to L and y,

$$W(y) = A\phi\left[\beta x^{\frac{\alpha-1}{\alpha}} + (1-\beta) y^{\frac{\alpha-1}{\alpha}}\right]^{\frac{\alpha}{\alpha-1}} L^{\phi-1}, \tag{9}$$

and

$$W'(y) = A(1-\beta)\left[\beta x^{\frac{\alpha-1}{\alpha}} + (1-\beta) y^{\frac{\alpha-1}{\alpha}}\right]^{\frac{1}{\alpha-1}} y^{-\frac{1}{\alpha}} L^{\phi-1}. \tag{10}$$

Each boss would choose workers with ability y for the profit maximization such that the matching rule follows,

$$x = m(y), \tag{11}$$

or

$$y = m^{-1}(x) \equiv \nu(x). \tag{12}$$

The stationary distribution

In our paper, the human capital accumulation is the new mechanism, which connects the matching rule and the equilibrium wage in the labor market, such that

$$x_{t+1} = \theta_{t+1} g(x_t), \tag{13}$$

where

$$g(x_t) = \rho x_t^{\epsilon} \left[\beta x_t^{\frac{\alpha-1}{\alpha}} + (1-\beta)\nu(x_t)^{\frac{\alpha-1}{\alpha}} \right]^{\frac{\alpha\eta}{\alpha-1}} L(x_t)^{\phi\eta}, \tag{14}$$

with $\rho = \kappa \left[\frac{(1-\phi)A\chi_B^{\frac{1}{\gamma}}}{1+\chi_B^{\frac{1}{\gamma}}} \right]^{\eta}$.

The function $g(x)$ represents the deterministic part of the human capital accumulation.

Theorem

Suppose that $g(x)$ is an increasing function of $x > 0$, $\lim_{x \to 0} \frac{g(x)}{x} = \infty$, and $\lim_{x \to \infty} \frac{g(x)}{x} = 0$. The human-capital accumulation process $\{x_t\}_{t=0}^{\infty}$ is ergodic and hence has a unique stationary distribution.

The matching rule: $m(x)$ is determined by the labor-market clearing condition,

$$\int_{m(\underline{y})}^{m(y)} L(z) f_X(z) dz = \int_{\underline{y}}^{y} f_Y(z) dz, \tag{15}$$

where $f_X(z)$ and $f_Y(z)$ are the probability density function of X and Y, and $f_Y(z)$ is exogenously given.

Figure 1: Matching and employee wage comparison with different channels

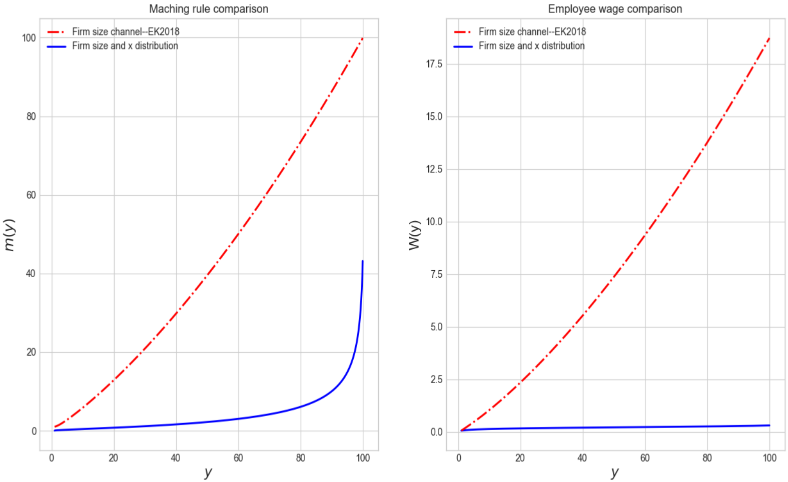
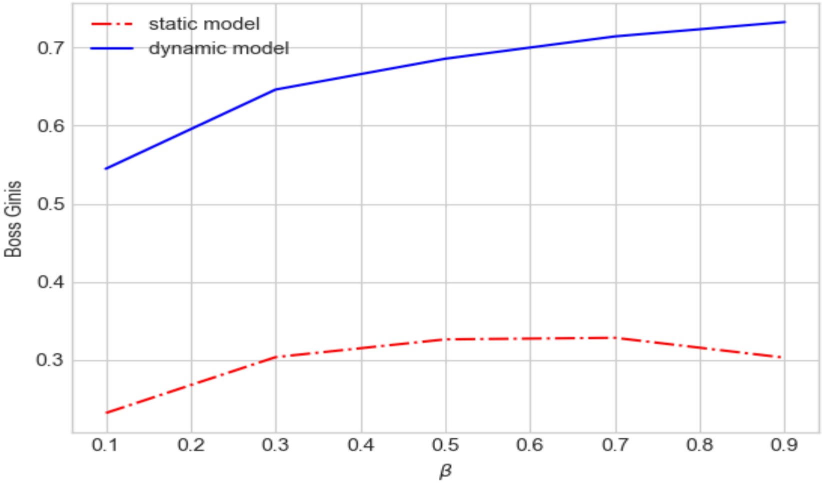


Figure 2: Bosses' Gini trend in the dynamic and static models when firm size is endogenous



Conclusion

Income distribution in assortative matching in the labor-market equilibrium.

Extend Eeckhout and Kircher (2018) to a dynamic assignment model.

We find:

- Higher levels of employee's human capital is needed to match the same level of the boss' human capital when the boss' human capital distribution is endogenous.
- Bosses' Gini increases with β and the variance of the employee's human capital, but decreases with A and the mean of the employee's human capital.