Synergy trap

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Abstract

This paper sheds light on the realization of synergies through an appropriate allocation of property rights. We utilize the Property Rights theory of the firm developed by Grossman, Hart and Moore (GHM). In this theory, *ownership* of an asset provides 'residual rights of control'. This can improve the ex-post bargaining power of the owner and thus creates ex-ante incentives to invest. Thus ownership provides incentives.

But while in GHM ex-post bargaining is supposed efficient, we study here the optimal allocation of property rights when ex-post bargaining is inefficient. Moreover, de-bundling ownership into the right to *access* an asset and the right to exclude others from it (*veto*), we show the specific roles that access and veto can take to mitigate inefficiencies in ex-ante investment *and* ex-post bargaining.

Starting from a firm with two vertically synergetic activities (1 and 2), we investigate the effect of horizontal synergy between one of them (say 2) and a third activity/firm, when the second synergy imposes a negative externality on the first one (there is a *trade-off* effect characterized by its value k). When managers 2 and 3 decide to exploit their horizontal synergy (i.e. to coordinate), it may drive manager 1 to underinvest, leading to a negative surplus of coordination, and the individual incentives of 2 and 3 may be such that they will implement the horizontal synergy nevertheless (a *synergy trap*).

To solve the problem, we study the allocation of property rights on activity 2 to manager 1. We show that the following allocations are optimal and avoid synergy traps: *partial access* when the trade-off effect is small, *non-integration* when it is 'intermediate' and *partial veto* when it is high. Partial or full ownership is never optimal.

1. Introduction

Conventional wisdom argues that cross-business synergies result from efficiencies that are realized through economies of scope, market power, internal capital markets, and reduced agency and transaction costs. Although the concept is theoretically mature, empirical evidence on the realization of synergies is still ambiguous, and the ways cross-business synergies are achieved need new research directions (Martin and Eisenhardt, 2001). This paper sheds light on the realization of synergies through an appropriate allocation of access and veto rights.

The advantages of synergy are often confounded by the individual incentives of those who implement them. Investment choices usually follow the business unit managers' decision on whether to coordinate on the use of their assets. Depending on the distribution of the surplus from any potential synergy, the ultimate choice of the managers may be inefficient. In an incomplete contracting world, there are inefficiencies generated by the non contractibility of investments (leading to inefficient levels of ex-ante investments). And this effect is compounded by the fact that the decision to coordinate is taken by agents who try to maximize their own share of the ex-post surplus rather than the total surplus (ex-post bargaining is not efficient). This may lead them to coordinate even though the total surplus of coordination is negative (compared to the status-quo situation), leading to a *synergy trap*, or not to coordinate even though the total surplus would be positive (*an efficiency trap*). These inefficiencies will dissipate the surplus generated by a merger[1]. A reallocation of rights can change the distribution of payoffs and thereby promote a more efficient outcome.

This paper uses the insights of Bel (2006) to suggest how an expanded set of contracts (relative to current literature) can help reduce inefficiencies. In particular our contracts will emphasize the dual notion of access and veto.

In Section 2, we consider a firm with two vertically integrated synergetic activities managed by agents 1 and 2. There is another firm with activity 3 that has a (horizontal) synergy with activity 2 but imposes a negative externality on activity 1, parameterized by a variable k (the tradeoff effect). If managers 2 and 3 decide to exploit their horizontal synergy (i.e. to coordinate), it may drive manager 1 to underinvest, leading to a negative surplus of coordination (synergy trap). To solve the problem, we study the allocation of control rights on activity 2 to manager 1. An allocation of rights is a tuple (λ, μ) where λ and μ ($\lambda, \mu \in [0,1]$) denote the extent of her access and veto respectively. That is, λ-access allows her to obtain by herself (without 2) a share λ of the vertical synergy between activities 1 and 2. But with access only, she cannot prevent 2 to manage her activity as she wishes and in particular to implement horizontal synergy with 3. On the other hand, with μ-veto on activity 2, she can prevent 2 and 3 from realizing a portion μ of the horizontal synergy.

[1] In the first case managers will promote and decide an unprofitable diversification/merger. In the second case a profitable diversification/merger will not be implemented by the operational managers.

When the synergies are linear, Proposition 1 shows that the following allocations of rights are optimal and avoid synergy traps: *partial access* when the trade-off effect is small, *non-integration* when it is 'intermediate' and *partial veto* when it is high. Partial or full ownership is never optimal.

With possibly non-linear tradeoffs, the model is highly intractable. Yet, we obtain certain insights. In particular, access by 1 increases her incentive to invest and decreases 2's, but does not affect 3's incentive to coordinate (for a given level of investment). On the other hand, veto by 1 decreases agents 2 and 3's incentives to invest, as well as their incentive to coordinate (for a given level of investment).

Overall, we show that access and veto prevent synergy traps differently: access by raising the surplus of coordination, veto by preventing inefficient coordination to happen. Access is an incentive device, veto a disciplining device.

The above results derived from methodological advances apart, the model involves a number of subtleties. For instance, ex-post bargaining is inefficient. This is in sharp contrast to the literature that follows GHM[2]. Ex-post inefficiency occurs because the ex-post bargaining process does not involve all agents – only managers 2 and 3 decide whether to coordinate.

We assume that lump-sum transfers between agents at date 0 are not possible. This is an implicit assumption that the agents are cash constrained, as in Aghion and Bolton (1992) and Aghion and Tirole (1994). Aghion and Bolton (1992) show that in some cases it may be optimal for a wealth constrained party to transfer control to another party. But their setting involves only one asset and does not integrate the impact of synergies/complementarities between assets. Aghion and Tirole (1994) show that the existence of a cash constraint on one agent may lead to an inefficient allocation of control rights, if the other agent has substantial bargaining power ex-ante. Aghion *et al.* (2002) follow this with the idea of partial contracting and dynamic *transferable* control to improve efficiency. Here we show that a broader set of control rights can improve efficiency through an ex-ante allocation of *contractible* control.

[2] The seminal papers of Grossman, Hart and Moore are Grossman and Hart (1986) and Hart and Moore (1990).

Rather than partial contracting, here we allow for *partial control*, since (λ, μ) are between zero and one. Aghion and Bolton (1992), Garvey (1995) or Halonen (2002) also consider partial control but they do not split it in terms of access and veto. In other words, in their models $\lambda = \mu$.

Other related papers have studied the problem of efficient investment *and* efficient allocation. For example a recent article by Hori (2006) shows that under a cooperative investment environment (where the seller's investment stochastically determines the buyer's valuation) it is impossible to achieve first-best efficiency (defined as efficient investment and allocation) when the buyer's valuation is distributed continuously. Those models are of bilateral trade and they investigate the conditions that would generate both ex-ante and ex-post efficiency. We assume ex-post inefficiency and try to limit its consequences through the allocation of control rights.

Finally, in a different context, Hart and Holmstrom (2002) have shown that the level of control may impact the level of coordination: there is 'not enough' coordination with a high level of control but 'too much' coordination with a low level of control.

This paper is organized as follows. Section 2 presents the model and the first results. Section 3 studies the optimal allocation of rights in the case of linear synergies and Section 4 concludes.

2. The model

2.1. Technological assumptions

We consider a firm A with two productive assets (or divisions/functions) A_1, A_2 operated by agents 1 and 2. Firm A is contemplating possible synergies between A_2 and another firm (or productive asset) B managed by agent 3[3].

To fix ideas, one can think about a computer manufacturer (firm A) contemplating synergies between its sales division (A_2) and a printer manufacturer (firm B), where A_1 is the manufacturing division of A.

Each agent i makes an investment e_i ($e_i > 0$[4]), whose cost is $c_i(e_i)$, specific to the asset he is working with. Working together and using their respective assets, agents 1 and 2 jointly produce

[3] Firm B is reduced to one asset to keep the model simple while focusing on the effect of a horizontal merger.

revenues equal to $\alpha(e_1, e_2)$. Agents 1 and 2 belong to the same firm and we assume that the corresponding assets are complementary[5]. In the rest of the paper α will be called *vertical synergy*. If the agents do not work together they have an outside option of working for someone else at a wage w. Without loss of generality we normalize w to zero. For Williamson (1985), $\alpha(e_1, e_2)$ represents the aggregate *quasi-rent* of agents 1 and 2, i.e. the difference between what they can realize in the relationship and what they can get if they 'go alone' and for Hart (1995) it represents the *ex-post gain from trade*. In both Williamson and GHM literature, α represents an implicit measure of 'asset specificity'.

Similarly agents 2 and 3 working on assets A_2 and B could produce a return $\beta(e_2, e_3)$ (β measures the complementarity between A_2 and B). It is a function of investments by 2 and 3 and will be called *horizontal synergy*. The outside option of agent 3 is also w, normalized to zero.

Finally, for simplicity, we assume that A_1 and B are independent (the technologies used by computers and printers are different) so that 2 and 3 working together cannot generate a revenue, besides their outside options (there is no synergy between A_1 and B).

The model can be drawn as in Figure 1.

Firm A Firm B

(agent 1) A_1

χ Vertical synergy (α)

(agent 2) A_2 ← Horizontal synergy (β) → B *(agent 3)*

Figure 1

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[4] If $e_i = 0$, the agents do not produce any synergy.

[5] In Hart (1995)'s terminology, α represents the 'gain from trade' between 1 and 2, and e_1, e_2 are 'relationship-specific': they pay more if 'trade' occurs (i.e. if coordination takes place) between the two agents than if it does not.

In our framework, there is a trade-off between investing for vertical synergy and investing for horizontal synergy[6]. The realization of horizontal synergy generates an externality on the vertical synergy, reducing its value by a factor $k(\geq 0)$[7] so that the three agents working together generate a revenue:

$$R(e_1, e_2, e_3) = (1-k)\alpha(e_1, e_2) + \beta(e_2, e_3)$$

Given that agents 1 and 2 are already part of the same firm and realize the vertical synergy α, we will be interested in the conditions for agents 2 and 3 to realize the horizontal synergy, i.e. to 'coordinate'.

2.2. First-Best

The timing of the model is described in Figure 2.

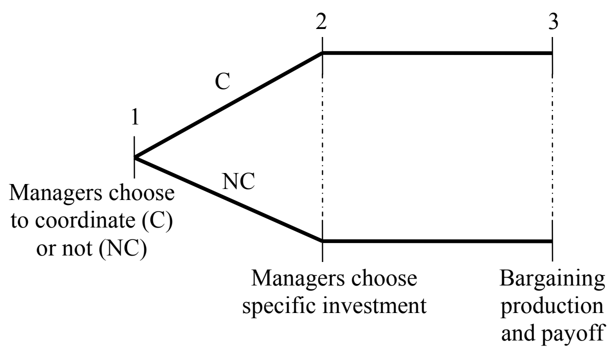

Figure 2

We begin by discussing the incentives to coordinate at date 1 assuming that the efficient choices are made at date 2. This offers a benchmark for the latter case where individual incentives cause inefficiencies.

The net total surplus S is given by the surplus generated by the agents minus their cost of investment.

In *absence of horizontal coordination*, the net total surplus S^{NC} is given by $S^{NC}(e_1, e_2, e_3) = \alpha(e_1, e_2) - \sum_{i=1,2,3} c_i(e_i)$. In a first best world where agents would cooperate, the

[6] With this trade-off, R is not *supermodular* in e_i (i.e. the cross-partial derivatives are negative) and the game is not convex. Formally $\exists S,T$ with $S \cap T \neq \varnothing$ such that $v(S) + v(T) \geq v(S \cup T) + v(S \cap T)$ (see Gul, 1999, or Hendrickx & al., 2001). In our case, $R(A_1, A_2) + R(A_2, B) \geq R(A_1, A_2, B) + R(A_2)$ ($(\alpha) + (\beta) \geq (1-k)\alpha + \beta$).

[7] In other words, the 'presence' of B decreases the complementarity between A_1 and A_2.

efficient levels of investments would maximize the net total surplus and solve: $Max_{e_1,e_2,e_3} S^{NC}(e_1,e_2,e_3) = Max_{e_1,e_2,e_3} \alpha(e_1,e_2) - \sum_{i=1,2,3} c_i(e_i)$

The equilibrium investments (e_1^*, e_2^*, e_3^*) are given by the first order condition $\alpha_i^{'}(e_1^*, e_2^*) = c_i^{'}(e_i^*), i = 1,2$ (where the subscript i to function α indicates partial with respect to i) and $e_3^* = 0$ and the resulting first best surplus is $S^{NC}(e_1^*, e_2^*, 0)$.

On the other hand, *if agents 2 and 3 decide to coordinate*, the net total surplus S^C is given by $S^C(e_1,e_2,e_3) = (1-k)\alpha(e_1,e_2) + \beta(e_2,e_3) - \sum_{i=1,2,3} c_i(e_i)$ and the efficient levels of investments solve:

$Max_{e_1,e_2,e_3} (1-k)\alpha(e_1,e_2) + \beta(e_2,e_3) - \sum_{i=1,2,3} c_i(e_i)$

The equilibrium investments $(e_1^{**}, e_2^{**}, e_3^{**})$ are given by $(1-k)\alpha_1^{'}(e_1^{**}, e_2^{**}) = c_1^{'}(e_1^{**})$, $(1-k)\alpha_2^{'}(e_1^{**}, e_2^{**}) + \beta_2^{'}(e_2^{**}, e_3^{**}) = c_2^{'}(e_2^{**})$ and $\beta_3^{'}(e_2^{**}, e_3^{**}) = c_3^{'}(e_3^{**})$, and the resulting first best surplus is $S^C(e_1^{**}, e_2^{**}, e_3^{**})$.

It is easy to see (since α, β are concave) that $e_1^{**} \leq e_1^*$, $e_3^{**} > e_3^*$ and that the relationship between e_2^{**} and e_2^* depends on k. Agent 1 will invest less with horizontal coordination because her marginal return on vertical synergy is reduced by the trade-off effect. Agent 3 will invest in horizontal coordination because she contributes to it. For agent 2, the effect of coordination is not clear. She will invest more with coordination only if her marginal return in horizontal synergy more than compensates the negative impact of the trade-off effect on her marginal return in vertical synergy.

In the first-best world, agents 2 and 3 will decide to coordinate only if the total net surplus of coordination is higher than the total net surplus in non-coordination ($S^C(e_1^{**}, e_2^{**}, e_3^{**}) > S^{NC}(e_1^*, e_2^*, e_3^*)$). If this is the case, coordination will be implemented and will be efficient. Otherwise, non-coordination will prevail and will be efficient.

2.3. Access and veto contracts

Following the incomplete contracts literature[8], we assume that trade-off level and agents' investments are not contractible (they are not *verifiable*)[9]. Therefore agents choose their investments non-cooperatively. But ex ante contracts can be written on the allocation of control rights. As in Bel (2006), we adopt a broad definition of control rights encompassing both veto and access rights. Control rights can be viewed as 'residual control rights' or rights of ownership (as in GH and HM), as exclusive dealing rights or veto rights (as in Segal and Whinston, 2000), as access rights (as in RZ) or a mix of those. But in any case, control rights give both decision rights (i.e. the rights to decide on implementing horizontal synergy or not) and income rights (determine the share of ex-post surplus). Therefore the allocation of control rights ex-ante provides bargaining power ex-post.

Following a traditional view of ownership we assume that an agent who 'owns' an asset can use it as she wishes and in particular can use it in her relation with another agent (she has *access* to the asset) and that no other agent can use the asset without her (she has *veto* right on the asset). In our setting, if agent 1 owns asset A_2 (in addition to asset A_1) she can realize the vertical synergy α (between assets A_1 and A_2) by herself and agent 2 cannot realize the horizontal synergy β (between assets A_2 and A_3) without the presence of 1.

We think of a contract as a tuple (λ, μ) where λ is the level of access and μ is the extent of veto. Giving λ-access on A_2 ($\lambda \in [0,1]$) to agent 1 means that 1 can access asset A_2 as she wishes and enjoy by herself (without 2) a portion $\lambda\alpha$ of the complementarity between A_1 and A_2 (and she benefits from agent 2's investment on A_2 [10]). But with access only, she cannot prevent 2 to use A_2 as she wishes and in particular to implement horizontal synergy with 3 (and in that case, 1 does not get a share of β). On the other hand, if 1 gets μ-veto only on A_2 ($\mu \in [0,1]$), she cannot use A_2 (and thus does not get a share of α) but she can prevent 2 to implement a portion μ of the horizontal

[8] See in particular Hart (1995) for a justification of these arguments.
[9] In the GHM framework, revenue sharing contracts are also ruled out, but this assumption is not critical to their models, as long as the agents have an option to trade under the existing contract but are not forced to trade under this contract (Hart, 1995).
[10] Unlike Hart & Moore (1990) and our initial framework (Bel, 2006), we do not restrict the model to 'human capital' investments, i.e. an agent's investment may enhance the productivity of the asset she is working with. Here, access by 1 will have the effect of reducing 2's incentive to invest, even if it does not affect her outside option.

coordination with 3 (so 2 and 3 can only get a share $(1-\mu)\beta$ of the synergy). The timing of the model is described in Figure 3.

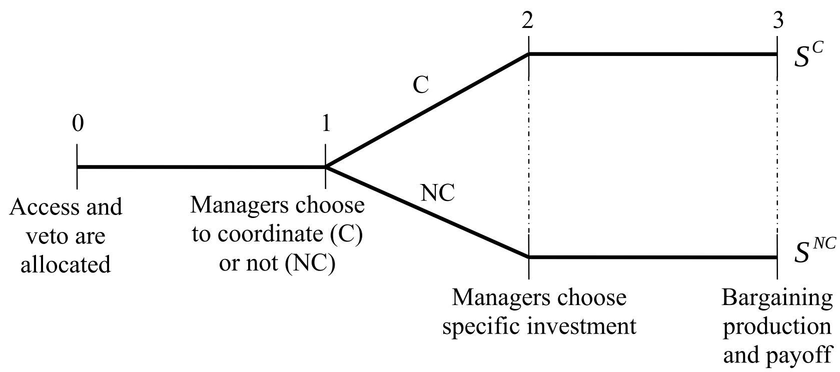

Figure 3

Access and veto are allocated at date 0 and are contractible. There are two decisions in this framework. At date 2, managers choose (non-cooperatively) their level of investment by maximizing their *ex-ante* share of surplus. Understanding what will be their optimal level of investment in the 'non-coordination' vs. 'coordination' scenarii, the relevant agents reason backward and decide or not to coordinate at date 1, based on their ex-ante share of surplus. At date 3, all the aspects of the relationship become contractible and agents bargain over the share of the surplus. We follow a large literature[11] in taking the Shapley value[12] as the solution concept of the bargaining game. Finally production takes place and payoff is allocated according to date 3 bargaining.

In the following, the payoff of agent i will be denoted π_i, her net payoff will be $S_i = \pi_i(e_1, e_2, e_3) - c_i(e_i)$ and the net total surplus will be $S = \sum_{i=1,2,3} \pi_i(e_1, e_2, e_3) - c_i(e_i)$. We will

[11] For example Hart and Moore (1990). For a non-cooperative justification of the Shapley value, see Gul (1989) and Stole and Zwiebel (1996).

[12] The Shapley value gives each agent i her expected contribution to a coalition, where the expectation is taken over all coalitions that i might belong to. For example in a game with three players i, j, k where v_{ij} represents the revenue that the coalition of agents i and j can produce, the expected contribution of agent i is given by: $v(i) = \frac{1}{3}v_i + \frac{1}{6}(v_{ij} - v_j) + \frac{1}{6}(v_{ik} - v_k) + \frac{1}{3}(v_{ijk} - v_{jk})$

use the superscript C (respectively NC) if the agents coordinate (respectively do not coordinate).

- Investment decision at date 2

Horizontal coordination between 2 and 3 creates a negative externality on the vertical synergy between 1 and 2 through the trade-off effect. This tends to reduce agent 1's ex-ante investment and in turn may affect the overall surplus. To alleviate this effect we will investigate the allocation of control rights on A_2 to agent 1.

When 1 is given λ-access and μ-veto, the value function takes the following form:

$$\begin{cases} v_1 = \lambda\alpha(e_1,e_2); v_2 = 0; v_3 = 0 \\ v_{12} = \alpha(e_1,e_2); v_{13} = \lambda\alpha(e_1,e_2); v_{23} = (1-\mu)\beta(e_2,e_3) \\ v_{123} = (1-k)\alpha(e_1,e_2) + \beta(e_2,e_3) \end{cases}$$

Where v_{ij} represent the revenue that agents i and j can generate by working together.

If agent 1 has λ-access on A_2, she gets a portion λ of the complementarity between both assets. Hence $v_1 = \lambda\alpha(e_1,e_2)$ (and $v_{13} = \lambda\alpha(e_1,e_2)$)[13].

Moreover, agent 1 exercises her veto power on A_2 by preventing 2 and 3 to get the full benefit of their complementarity. Since 1 has μ-veto on A_2, agents 2 and 3 can only enjoy a portion $(1-\mu)$ of their assets' complementarity. Hence $v_{23} = (1-\mu)\beta(e_2,e_3)$.

This representation captures the independent effects of access and veto in a simple form as well as the characteristics of more usual forms of control: ownership ($\lambda = \mu = 1$), non-integration ($\lambda = \mu = 0$), joint ownership ($\lambda = \mu = 1/2$), partial ownership ($0 < \lambda = \mu < 1$).

As noted above, we suppose that the bargaining process between the agents will give to each agent i an ex-post share of surplus π_i given by her Shapley value. It takes the following form:

$$\pi_1(e_1,e_2,e_3) = \left(\frac{(1+\lambda)}{2} - \frac{k}{3}\right)\alpha(e_1,e_2) - \frac{\mu}{3}\beta(e_2,e_3) \qquad (1.1)$$

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[13] Because we do not restrict the model to human capital investments, access here is not 'inclusive' (see Bel, 2006), i.e. access may *reduce* the marginal productivity of the agent who is giving access.

$$\pi_2(e_1,e_2,e_3) = (\frac{(1-\lambda)}{2} - \frac{k}{3})\alpha(e_1,e_2) - \frac{(3-\mu)}{6}\beta(e_2,e_3) \quad (1.2)$$

$$\pi_3(e_1,e_2,e_3) = \frac{(3-\mu)}{6}\beta(e_2,e_3) \quad (1.3)$$

In the second-best world, an agent i anticipates her ex-post share of surplus and sets her investment ex-ante in order to maximize her net payoff, $S_i(e_1,e_2,e_3) = \pi_i(e_1,e_2,e_3) - c_i(e_i)$. Her investment level e_i^c is determined by the first order condition: $\left.\frac{\partial \pi_i(e_1,e_2,e_3)}{\partial e_i}\right|_{e_i=e_i^c} = c_i^{'}(e_i^c)$ (and the total surplus is given by $S = \sum_{i=1,2,3} S_i(e_1^c,e_2^c,e_3^c)$).

In non-coordination ($\beta = 0, k = 0$), agent i's equilibrium investment, e_i^{nc}, is determined with the same process.

But setting their level of investment is not the only decision that agents take in our framework. They also decide to coordinate or not.

- *Coordination decision at date 1*

Taking their optimal investment choices at date 2 into account, at date 1, agents 2 and 3 will decide to coordinate if their respective net payoffs are higher with coordination than without, i.e. when:

$$S_2^C(e_1^c,e_2^c,e_3^c) > S_2^{NC}(e_1^{nc},e_2^{nc}) \quad \text{(C2)}$$

And $$S_3^C(e_1^c,e_2^c,e_3^c) > 0 \quad \text{(C3)}$$

And we assume that coordination will not take place if at least one of the agents does not want to coordinate (i.e. if C2 *or* C3 does not hold).

- *Allocation of rights at date 0*

To calculate the optimal allocation of rights ($\hat{\lambda},\hat{\mu}$), one must first choose (λ^C,μ^C) that maximizes the *total* surplus of coordination subject to these incentive coordination constraints (i.e. that solves $Max_{\lambda,\mu} S^C(e_1^c,e_2^c,e_3^c)$, *s.t.* C2 and C3) and (λ^{NC},μ^{NC}) that maximizes the total surplus in non-coordination, given that either 2 or 3 does not want to coordinate (i.e. that solves $Max_{\lambda,\mu} S^{NC}(e_1^{nc},e_2^{nc})$, *s.t.* $\neg$C2 or $\neg$C3).

Overall, the *optimal* allocation of rights ($\hat{\lambda}, \hat{\mu}$) will be the one that will generate a *second-best* level of social surplus $\hat{S}$, given by comparing the optimal surplus of coordination and non-coordination: $\hat{S} = Max(\hat{S}^C, \hat{S}^{NC})$.

From now on, we will simplify notation and drop the investment levels e_i in the arguments of functions α, β, c, π, S whenever it is convenient and unambiguous.

2.4. Synergy and efficiency traps

- *The status-quo*

In the rest of the article, we will refer to the *status-quo* as the situation prevailing *before* allocating control rights to agent 1 and before an opportunity of horizontal coordination arises. In the status-quo ($\lambda = 0, \mu = 0$, $\beta = 0, k = 0$), each agent owns her own asset (*non-integration* (NI) in Hart's terminology), and there is no coordination.

The agents' investment levels take the form $e_1^{SQ} = e_2^{SQ} = e^{SQ}$ (determined by $\frac{1}{2}\alpha_i^{'} = c_i^{'}, i = 1,2$) and $e_3^{SQ} = 0$, and the total surplus is $S^{SQ}(e^{SQ}, e^{SQ}, 0)$.

If agents 2 and 3 have the opportunity to exploit an (horizontal) synergy, there will be two consequences.

(i) Agent 1's underinvestment

With coordination, investment levels (e_1^c, e_2^c, e_3^c) are given by $(\frac{1}{2} - \frac{k}{3})\alpha_1^{'} = c_1^{'}$, $(\frac{1}{2} - \frac{k}{3})\alpha_2^{'} + \frac{1}{2}\beta_2^{'} = c_2^{'}$, and $\frac{1}{2}\beta_3^{'} = c_3^{'}$. Given that α, β are concave, it is easy to see that $e_1^c \le e_1^{SQ}$, and $e_3^c > e_3^{SQ} = 0$ (the relationship between e_2^c and e_2^{SQ} is unclear). Agent 1 will underinvest because she suffers from the tradeoff effect. If this underinvestment is too strong, it may overcome the increased investment of 3 (and may be 2) and lead to a surplus of coordination lower than the surplus in status-quo.

(ii) Synergy traps

The decision to coordinate will be taken by agents 2 and 3, based on their anticipated share of surplus: they will decide to coordinate if their *ex-ante* share of surplus in coordination is higher than their *ex-ante* share in status-quo (i.e. if $S_i^C > S_i^{SQ}, \forall i = 2,3$):

$$2:\ (\frac{1}{2}-\frac{k}{3})\alpha^c + \frac{1}{2}\beta^c - c_2(e_2^c) > \frac{1}{2}\alpha^{SQ} - c_2(e_2^{SQ})$$

$$3:\ \frac{1}{2}\beta^c - c_3(e_3^c) > 0$$

where $\alpha^c = \alpha(e_1^c, e_2^c)$, $\beta^c = \beta(e_2^c, e_2^c)$, and $\alpha^{SQ} = \alpha(e_1^{SQ}, e_2^{SQ})$

Assuming that the net payoff for agent 3 is positive she will always want to coordinate. Hence agent 2 is 'binding' in the decision to coordinate, which is taken when $k < \dfrac{3(\beta^c + (\alpha^c - \alpha^{SQ}) - (c_2(e_2^c) - c_2(e_2^{SQ})))}{2\alpha^c}$. But because agent 2 takes the decision to coordinate based on her own share of surplus this does not guarantee that the *total* surplus of coordination ($S^C = \sum_{i=1,2,3} S_i^C$) will be higher than the surplus in status-quo ($S^{SQ} = \sum_{i=1,2} S_i^{SQ}$). In fact, it is easy to see that there will always be values of k for which the decision to coordinate will be taken even though the total surplus of coordination is lower than the surplus in status-quo: we will refer to this situation as a *synergy trap* and will use the following definition.

DEFINITION. There is a *synergy trap* if the agents decide to coordinate even though the total surplus of coordination is lower than the surplus in status-quo ($S^C < S^{SQ}$). There is an *efficiency trap* when the agents *do not* decide to coordinate but the surplus of coordination would have been higher than in status-quo ($S^C > S^{SQ}$).

A synergy trap arises because the attraction of horizontal synergy is too high for the agents who implement it but this leads to investment levels that reduce the total surplus. Loosely speaking, the agents' incentives to coordinate are higher than their incentives to invest. On the other hand, an efficiency trap arises when the agents' incentives to coordinate are low and prevent them to implement an 'efficient' coordination. In what follows, a synergy (or an efficiency) trap will be characterized by the range of values of k for which such a trap may occur.

In the present case, it is easy to show that a synergy trap will occur when:

$$P-(c_1(e_1^c)-c_1(e_1^{SQ})+c_3(e_3^c))<k<\frac{3}{2}P$$

where $P=\dfrac{\beta^c+(\alpha^c-\alpha^{SQ})-(c_2(e_2^c)-c_2(e_2^{SQ}))}{\alpha^c}$

Overall, with coordination, 1's *incentive to invest* may be too low, while 2's (and 3's) *incentive to coordinate* may be too high and it may generate synergy traps. One way to solve the problem may be to give control rights on A_2 to agent 1. This we investigate now.

- *Allocation of access and veto*

With horizontal coordination, the equilibrium levels of investment (e_1^c,e_2^c,e_3^c) are given by $(\frac{(1+\lambda)}{2}-\frac{k}{3})\alpha_1^{'}=c_1^{'}(e_1^c)$, $(\frac{(1-\lambda)}{2}-\frac{k}{3})\alpha_2^{'}(e_1^c,e_2^c)+\frac{(3-\mu)}{6}\beta_2^{'}(e_2^c,e_3^c)=c_2^{'}(e_2^c)$ and $\frac{(3-\mu)}{6}\beta_3^{'}(e_2^c,e_3^c)=c_3^{'}(e_3^c)$ and the total surplus is $S^C(e_1^c,e_2^c,e_3^c)$.

(i) Effect of access and veto on the incentive to invest

For a given level of investment, access increases the outside option of agent 1 (without changing 2's). This increases agent 1's ex-post share of surplus (π_1) in her bargaining with the other agents, and reduces 2's and 3's. Therefore access *increases* 1's ex-ante investment and *decreases* 2's. But it does not affect 3's level of investment because 3 is only concerned by horizontal synergy and, with access only, 1 does not veto A_2 and cannot influence the output of horizontal synergy between 2 and 3.

Veto by 1 reduces the outside option of the coalition between 2 and 3. It gives additional bargaining power to 1 in her bargaining with the other agents, and increases her ex-post share of surplus while decreasing 2 and 3's. But the level of 1's investment is unaffected by veto because she does not have any leverage on β (with veto only, she does not *access* A_2 and cannot influence the output of the synergy between A_2 and B). So the effect of veto is essentially to *decrease* 2 and 3's investment. The effect of access and veto on the agents' incentive to invest is summarized in TABLE 1 below.

Effect of λ, μ on incentives	λ			μ		
	Agent 1	*Agent 2*	*Agent 3*	*Agent 1*	*Agent 2*	*Agent 3*
Incentive to invest	↑	↓	-	-	↓	↓

TABLE 1

With non-coordination, there is no horizontal synergy, 3 does not invest, veto does not play any role and only access influences 1 and 2's investment.

(ii) Effect of access and veto on the incentive to coordinate

Agents 2 and 3's incentives to coordinate are given by constraints C2 and C3, which translate into:

$$2\text{: } \left(\frac{(1+\lambda)}{2}-\frac{k}{3}\right)\alpha^{c}+\frac{(3-\mu)}{6}\beta^{c}-c_2(e_2^{c})>\frac{(1+\lambda)}{2}\alpha^{nc}-c_2(e_2^{nc})$$

$$3\text{: } \frac{(3-\mu)}{6}\beta^{c}-c_3(e_3^{c})>0$$

where $\alpha^{nc}=\alpha(e_1^{nc},e_2^{nc})$

For a given level of investment, veto by 1 decreases 2 and 3's incentive to coordinate because it reduces the output of their horizontal coordination. On the other hand, access does not affect 3's incentive to coordinate (because it does not impact directly the output of horizontal coordination). The impact on 2's incentive is less straightforward. If the output of vertical synergy is higher with horizontal coordination than without (which is unlikely because of the trade-off effect), access will raise 2's incentive to coordinate, otherwise it will lower it.

But the above analysis on the direct effect of access and veto on agents' incentives to coordinate is highly incomplete. It does not take into account the fact that access and veto do change agents' equilibrium investments and hence indirectly affect their incentive to coordinate. Trying to assess the overall effect is highly intractable for general α and β. The next section considers the case of linear α and β.

3. Optimal allocation of rights with linear synergies

We assume that $\alpha(e_1,e_2)=e_1+e_2$ and $\beta(e_2,e_3)=e_2+e_3$ [14]. Moreover, the allocation of agent 2's investment between the two kinds of synergies is described by a parameter ρ ($\rho\in[0,1[$ [15]), whereby a share ρe_2 is spent on vertical synergy while $(1-\rho)e_2$ is spent on horizontal synergy[16]. Alternatively, given that the marginal productivity of the agents has been set to 1 in this model, ρ represents the *relative* marginal productivity of agent 2's effort on vertical synergy (α) compared to horizontal synergy (β).

The production function for the three agents is thus:

$$R(e_1,e_2,e_3)=(1-k)(e_1+\rho e_2)+((1-\rho)e_2+e_3)$$

Finally we make the technical assumption that the private cost of investment for each agent i is $c_i(e_i)=\frac{1}{2}(e_i)^2$.

3.1. First Best

The equilibrium investment of the agents is $(e_1^*,e_2^*,e_3^*)=(1,1,0)$ and the resulting first best surplus is $S_{FB}^{NC}=1$. If agents 2 and 3 decide to coordinate, equilibrium investments become $(e_1^{**},e_2^{**},e_3^{**})=(1-k,1-\rho k,1)$ (where $k<1$ to get interior solutions) and the resulting first best surplus is $S_{FB}^{C}=\frac{k^2(1+\rho^2)}{2}-k(1+\rho)+\frac{3}{2}$. When $k<\frac{\rho+\sqrt{2\rho}+1}{\rho^2+1}$, $S_{FB}^{C}>S_{FB}^{NC}$, the agents would decide to (horizontally) coordinate and S_{FB}^{C} would be realized. If $k\geq\frac{\rho+\sqrt{2\rho}+1}{\rho^2+1}$, $S_{FB}^{C}\leq S_{FB}^{NC}$, the agents would not (horizontally) coordinate and S_{FB}^{NC} would prevail.

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[14] This production function is similar to the one used by Whinston (2001, 2003) where the marginal returns on investment are set equal to 1.

[15] If $\rho=1$, agent 2 will only invest in vertical synergy, not in horizontal synergy.

[16] If there was no 'conflict' on agent 2's investment, she could invest e_2 on vertical *and* horizontal synergy. It can be shown that in that case, the surplus of coordination is always greater thant the surplus in non-coordination, agents 2 and 3 always want to coordinate and non-integration is always the optimal control structure.

3.2. Second Best

In the non-coordination scenario, the ex-post shares of surplus are: $\pi_1 = \frac{1}{2}(1+\lambda)(e_1 + e_2)$, $\pi_2 = \frac{1}{2}(1-\lambda)(e_1 + e_2)$ and $\pi_3 = 0$. Anticipating this, each agent i will maximize her *ex-ante* share of surplus $S_i = \pi_i - c_i(e_i)$, resulting in equilibrium levels of investment $(e_1^{nc}, e_2^{nc}, e_3^{nc}) = (\frac{1}{2}(1+\lambda), \frac{1}{2}(1-\lambda), 0)$, shares of surplus $S_1^{NC} = \frac{(1+\lambda)(3-\lambda)}{8}$, $S_2^{NC} = \frac{(1-\lambda)(3+\lambda)}{8}$, $S_3^{NC} = 0$, and a total surplus $S^{NC} = \frac{3-\lambda^2}{4}$.

With non-coordination, the surplus is decreasing in λ (and is obviously independent from μ). Thus the optimal surplus of non-coordination will be achieved when 1 does not have access to A_2 ($\lambda = 0$): $\hat{S}^{NC} = \frac{3}{4} = S^{SQ}$. When giving (any level of) access to 1, the increase of surplus generated by 1's *increased* investment is more than compensated by the decrease of surplus generated by 2's *decreased* investment, so that the overall effect on the surplus is negative.

With coordination, the equilibrium levels of investments are: $e_1^c = \frac{1}{2}(1+\lambda) - \frac{1}{3}k$, $e_2^c = \frac{2}{3} - \frac{1}{2}\lambda - \frac{1}{6}\mu - \frac{1}{3}\rho k$, $e_3^c = \frac{1}{2} - \frac{1}{6}\mu$.

To ensure interior solutions and positive investment by 1 and 2 we need $k < \frac{3}{2}(1+\lambda)$ and $k < \frac{4-\mu-3\lambda}{2\rho}$, which is graphically represented in Figure 4.

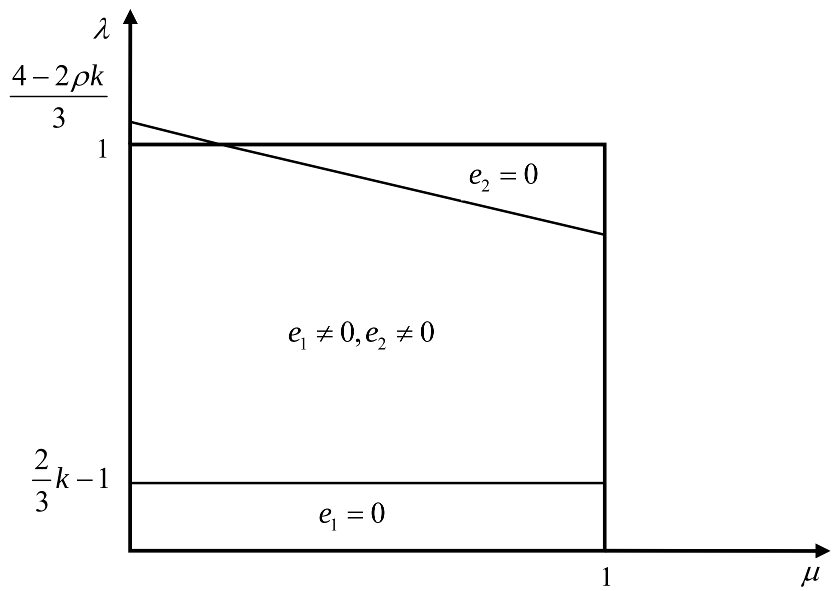

Figure 4

When k increases, the line $\lambda = \frac{2}{3}k - 1$ moves up which expands the area where $e_1 = 0$, and the line $\lambda = \frac{4-\mu-2\rho k}{3}$ moves down which also expands the area where $e_2 = 0$. The area where $e_1 \neq 0, e_2 \neq 0$ is thus reduced. When the trade-off effect is very large, this may reduce agent 1's incentive to invest so much that she stops investing. That may also reduce agent 2's incentive to invest so she stops investing.

When $k \geq \frac{3}{2}(1+\lambda)$, $e_1 = 0$ and there is no vertical synergy. Horizontal synergy prevails with equilibrium investments $e_2^c = e_3^c = \frac{3-\mu}{6}$ and the total surplus is $S^C = -\frac{1}{36}\mu^2 - \frac{1}{6}\mu + \frac{3}{4}$. The optimum surplus is then reached when $\mu = 0$ and is $\hat{S}^C = \frac{3}{4} = S^{SQ}$. The optimum allocations of rights are partial access ($\lambda < \frac{2}{3}k - 1, \mu = 0$) or non-integration ($\lambda = 0, \mu = 0$) (any point in the area below the line $\lambda = \frac{2}{3}k - 1$ is suboptimal beside those corresponding to $\mu = 0$). Our purpose being to study the effect of control rights on horizontal synergy *in presence* of vertical synergy, this case is not very interesting. In fact, given that 1 and 2 belong to the same firm, the case where 1 and 2 would stop their vertical synergy to implement an horizontal synergy is unlikely to happen. Hence (and to ensure a broader set of possible allocations of rights), we will focus on the case where $k < \frac{3}{2}(1+\lambda), \forall \lambda$, i.e. $k < \frac{3}{2}$.

When $k \geq \frac{4-\mu-3\lambda}{2\rho}$, $e_2 = 0$ and there is no horizontal synergy. Non-coordination is chosen, vertical synergy prevails with equilibrium investments $e_1^{nc} = \frac{1+\lambda}{2}, e_2^{nc} = \frac{1-\lambda}{2}$, and $\hat{S}^{NC} = \frac{3}{4} = S^{SQ}$. The optimum allocation of rights is partial (or full) veto ($\lambda = 0, \mu > 4 - 2\rho k$). The only value of k that would get this area empty is $k = 0$.

The total surplus of coordination takes the following form:

$$S^C = -\frac{1}{4}\lambda^2 + \lambda(\frac{1}{3}\rho k - \frac{1}{3}k + \frac{1}{12}) + \frac{1}{12}\lambda\mu - \frac{1}{36}\mu^2 + \mu(\frac{1}{9}\rho k - \frac{5}{36}) + (\frac{5}{18}\rho^2 k^2 + \frac{5}{18}k^2 - \frac{7}{9}\rho k - \frac{2}{3}k + \frac{43}{36})$$

To conduct the analysis, we first take the simple case where $\rho = 1/2$, then will generalize to any ρ.

- *When* $\rho = 1/2$

LEMMA 1. *When* $\rho = 1/2$, *(i) if* $\mu = 0$, *there is always a synergy trap,* $\forall \lambda$ *(ii) if* $\mu = 1$ *there is an efficiency trap when* $\lambda \leq 1/2$

Proof. In the Appendix

When agent 1 does not have any veto on A_2, agents 2 and 3 tend to coordinate 'too much'. Note first that the surplus of coordination decreases with k and the likelihood of synergy traps is greatest for 'higher' values of k: when k is relatively 'high', the surplus of coordination is lower, i.e. is more likely to be lower than the status-quo (which does not depend on k). Moreover when $\mu = 0$

and k is not too small the *ex-ante* surplus of coordination is lower than the ex-ante surplus of status-quo for 1: her ex-post surplus is obviously lower (1 gets the full trade-off effect without gaining from the horizontal synergy) which drives her to underinvest, and this lower level of investment does not compensate for the significantly lower level of ex-post surplus[17]. Thus when k is sufficiently high so that agents 2 and 3's incentives to coordinate are positive but their ex-ante share of surplus in coordination is close to their share in status-quo, the *total* ex-ante surplus will be lower and we will have a synergy trap. With no veto, there are values of k for which horizontal coordination has an overall negative effect, which decomposes into a positive effect for 2 and 3 and an even greater negative effect on 1. Since 2 and 3 take the decision (to coordinate or not), they do not internalize the negative effect and coordinate nevertheless.

When $\mu = 1$, this is the opposite. If k is sufficiently small, the ex-ante share of surplus for 1 is always higher (for all values of λ) than her share in status-quo. When the agents 2 and 3 incentives to coordinate are negative (i.e. their share of surplus in coordination is lower than their share in non-coordination) but close to zero, they will *not* coordinate even though the total ex-ante surplus is positive. Here, 2 and 3 do not internalize the *positive* effect of horizontal coordination on 1.

Overall, with 'no veto' to 1, agents 2 and 3 tend to coordinate 'too much' while with full veto to 1, they do not coordinate enough[18]. We now turn to the characterization of an 'optimal' control structure (i.e. the structure which achieves the *second-best*) where the above effect will be important.

PROPOSITION 1. *There exists* $\tilde{k} \geq 1/2$ *and* $\tilde{\mu} \in [0,1]$ *such that the optimal control structure is (i) partial access* ($\hat{\lambda} = \tilde{\lambda}(k), \hat{\mu} = 0$) *when* $k \leq 1/2$ *(ii) non-integration* ($\hat{\lambda} = 0, \hat{\mu} = 0$) *when* $1/2 \leq k \leq \tilde{k}$*, and (iii) partial veto* ($\hat{\lambda} = 0, \hat{\mu} \geq \tilde{\mu}$) *when* $k > \tilde{k}$*. With the first two structures, coordination takes place and the surplus is higher than the status-quo; with the last one, non-coordination prevails and the surplus is equal to the status-quo*

[17] Only when k is sufficiently low will the ex-ante share of surplus become positive if λ is high enough.
[18] A similar result has been highlighted by Hart and Holmstrom (2002) in a different context.

Proof. In the Appendix

NB. $\tilde{k}, \tilde{\lambda}(k), \tilde{\rho}$ take the following values: $\tilde{k} = \frac{38 - 2\sqrt{161}}{25} \cong 0.505$, $\tilde{\lambda}(k) = \frac{1}{6} - \frac{1}{3}k$,

$$\tilde{\rho} = \frac{212 + 2\sqrt{161}}{75} - 12\sqrt{\frac{19\sqrt{161}}{101250} + \frac{11179}{270000}} \cong 0.65$$

With the two inefficiencies described above, there are two sorts of incentives in our framework. The agents' *incentive to invest*, determined by their *ex-post* share of surplus and their *incentive to coordinate* determined by their *ex-ante* share of surplus.

Access's main effect is to raise agent 1's incentive to invest but to lower agent 2's incentive (agent 3's investment is not impacted). For each agent the marginal effect of λ on the surplus is the product of the marginal effect of λ on her investment level (reflecting her incentive to invest) with the marginal return of her investment ($\frac{\partial S}{\partial \lambda} = \sum_{i=1,2} \frac{\partial S}{\partial e_i} \frac{\partial e_i}{\partial \lambda}$). Allocating access to 1 raises 1's investment (and therefore her investment cost) but reduces her (net) marginal return (the marginal return on 1's investment is $1-k$, independent of λ, while her investment increases). It also reduces 2's investment but increases her marginal return (her marginal return is $1-\rho k$, also independent of λ, while her investment decreases). Agent 1's marginal effect of λ is not clear (it can be negative if her marginal return is driven down to a negative value) while agent 2's effect is negative. When k is sufficiently low, agent 1's marginal effect is positive (her marginal return is positive) and there is an optimal value of λ for which the two effects compensate, the marginal impact of λ equals zero, and the surplus is optimal. But when k becomes higher, the marginal return of 1 gets more affected (2 gets affected by a factor ρ only), the positive (or negative) effect of 1 does not compensate for the negative effect of 2, the marginal impact of λ on the total surplus is negative and $\lambda = 0$ is optimal.

When the trade-off effect is small, allocating access to 1 increases her incentive and that compensates for the reduced incentive of 2. When k gets larger, allocating access to 1 'hurts' 2 more than it 'helps' 1 and non-integration is better. Finally, when k is large enough, veto kicks in.

Veto's main effects are *(i)* to reduce agents 2 and 3's incentive to invest (without increasing agent 1's incentive) and *(ii)* to reduce their incentive to coordinate. The first effect means that veto always decreases the surplus *of coordination*, so an optimal control structure will have $\mu = 0$ when coordination takes place. But with $\mu = 0$ there is always a synergy trap (from *Lemma* 1) beyond a certain value of k (let's call it $\hat{k}$): agents 2 and 3 coordinate even though it is not optimal. To avoid that, allocating veto when $k > \hat{k}$ is optimal because it reduces agents 2 and 3's incentive to coordinate.

Overall, access and veto have different and independent effects on the two kinds of incentives, and a separate allocation of access and veto optimizes surplus. Loosely speaking, synergy traps happen when the agents' incentives to invest are too low compared to their incentives to coordinate. Access increases agent 1's incentive to invest. In absence of veto, it also decreases 3's incentive to coordinate as well as 2's, when k is small, and hence may prevent synergy traps created by agent 1 underinvestment. Veto reduces agents 2 and 3's incentives to coordinate without lowering agent 1's incentive to invest, so it prevents synergy traps created by agents 2 and 3 excessive incentive to coordinate.

The effects of access and veto are summarized in TABLE 2 below.

Effect of λ, μ on	λ			μ		
incentives	*Agent 1*	*Agent 2*	*Agent 3*	*Agent 1*	*Agent 2*	*Agent 3*
Incentive to invest	↑	↓	-	-	↓	↓
Incentive to coordinate	-	↓ (low k) ↑ (high k)	↓	-	↓	↓

TABLE 2

In Figure 5 below we compare the second best obtained to the first-best and to 'pure' control structures: access, veto, ownership, and non-integration.

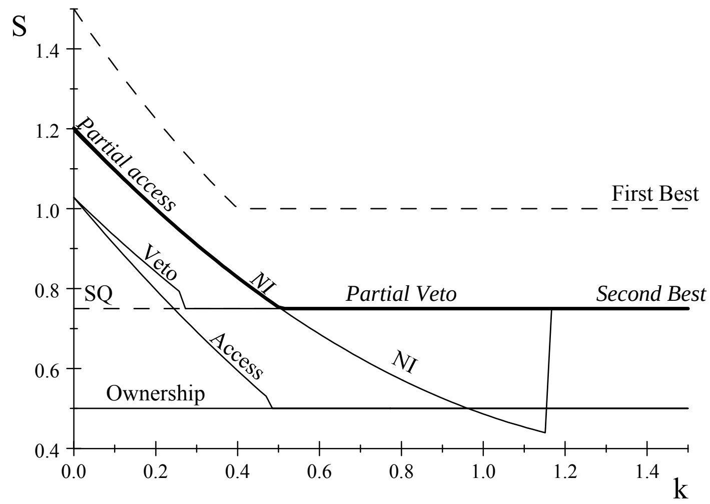

Figure 5

Overall, giving access (but not veto) to 1 increases surplus because it compensates the negative effect of trade-off on her incentive to invest, but as the trade-off effect gets larger it creates a synergy trap (without veto, agents 2 and 3 have too much incentive to coordinate because they do not internalize the effect on 1): to avoid it, giving veto (but not access) to 1 is necessary as it prevents agents 2 and 3 to coordinate and optimizes the surplus in non-coordination.

So far, we have considered that the effect of agent 2's investment on vertical and horizontal synergies is identical ($\rho = 1/2$). We now generalize our analysis for any ρ .

- *With any* ρ

When the relative marginal productivity of agent 2's investment between vertical and horizontal synergies varies, the optimal allocation of rights changes. For example, if ρ is (very) small, most of agent 2's productivity goes to horizontal synergy. The 'loss' on vertical synergy resulting from the trade-off effect is small and more than compensated by the 'gain' of horizontal coordination for any value of k . If ρ is small enough, there may be cases where the surplus of coordination is always higher than the surplus of non-coordination (there is no synergy trap) and agents 2 and 3 always want to coordinate. In that 'ideal' case there is no need for veto. On the other hand, if ρ is very high, most of agent 2's productivity goes to vertical synergy and this may affect the optimal allocation of rights. In fact, the generalization of ρ gives rise to the following *Proposition*.

PROPOSITION 2. *There exists* $k_0, k_1, k_2, \rho_1, \rho_2$ *such that (i) When* $\rho < \rho_1$*, the optimal control structures are partial access (*$\hat{\lambda} = \tilde{\lambda}(k,\rho), \hat{\mu} = 0$*) if* $k \le k_0$ *and NI if* $k > k_0$ *(ii) when* $\rho_1 \le \rho < \rho_2$*, partial access if* $k \le k_0$*, NI if* $k_0 \le k < k_1$*, and partial veto (*$\hat{\lambda} = 0, \hat{\mu} \ge \tilde{\mu}(k_1)$*) if* $k \ge k_1$ *(iii) when* $\rho \ge \rho_2$*, partial access if* $k < k_2$*, and partial veto (*$\hat{\lambda} = 0, \hat{\mu} \ge \tilde{\mu}(k_2)$*) if* $k \ge k_2$

Proof. In the Appendix

NB. $k_0, k_1, k_2, \rho_1, \rho_2, \tilde{\lambda}(k,\rho)$ take the following values: $k_0 = \frac{1}{4(1-\rho)}$,

$k_1 = \frac{7\rho - \sqrt{9\rho^2 + 84\rho - 4} + 6}{5\rho^2 + 5}$, $k_2 = \frac{26\rho - 3\sqrt{26}\sqrt{8\rho - \rho^2 - 1} + 26}{28\rho^2 - 16\rho + 28}$, $\rho_1 = \frac{10\sqrt{2} - 14}{3}$,

$\rho_2 = \frac{44 - \sqrt{151}}{63}$, $\hat{\lambda}(k,\rho) = \frac{1}{6} - \frac{2}{3}k(1-\rho)$

A graphical expression of *Proposition 2* is sketched in Figure 6 below.

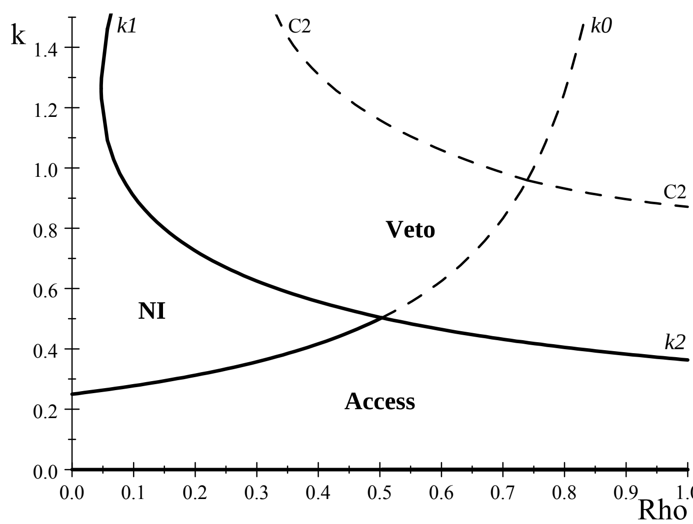

Figure 6

If $\rho < \rho_1 (\cong 0.05)$, there is no synergy trap and no need for veto. For 'intermediate' values of ρ the situation is identical to the one described in *Proposition 1* (where $\rho = 1/2$) and we see the existence of partial access, non-integration and partial veto as k increases.

When ρ is high, most of agent 2's productivity goes to vertical synergy and suffers from the trade-off effect. So her marginal productivity is lower. The marginal effect of λ for 2 is smaller and the positive effect on 1 dominates. Hence the marginal effect of λ on the total surplus is always positive and non-integration is never optimal.

In fact, the amount of access necessary to optimize the surplus of coordination increases with ρ. But here again, when k is 'too high', access will not be sufficient to compensate the trade-off effect, the surplus of coordination will become lower than the status-quo, and veto will be required.

The amount of veto necessary to avoid inefficient coordination increases and then decreases with ρ. This is the result of two effects: veto is triggered by the existence of a synergy trap (which is determined by $\tilde{k}$) *and* by the incentive of agent 2 or 3 *not* to coordinate.

When ρ increases, the surplus of coordination decreases because a larger part of 2's productivity is subject to the trade-off effect: this reduces her ex-post share of surplus and hence her incentive to invest (without affecting 1 and 3's incentives to invest), which results in a lower surplus of coordination. This in turn results in an increased likelihood for a synergy trap, i.e. a synergy trap happens 'earlier' (with lower values of k): when ρ increases, $\tilde{k}$ decreases.

Moreover, when ρ increases, agents 2 and 3's incentives to coordinate decrease: for 2 this is because her marginal return on horizontal coordination is lower, for 3 because her share of surplus decreases. Thus *ceteris paribus* 2 and 3 *should* require a lower level of veto *not* to coordinate, but this depends on $\tilde{k}$.

The result of these two effects is straightforward for 3: because she is not directly impacted by the level of $\tilde{k}$, she will require a lower level of veto not to coordinate as ρ increases. On the other hand, agent 2 is subject to two opposite effects: she should require a *lower* level of veto but with a lower k she should require a *higher* level of veto. The second effect is more important and, overall, agent 2 will require a higher level of veto not to coordinate as ρ increases.

Overall, for lower ρ, the level of veto required by 2 will be lower than for 3 while with higher ρ, it will be the opposite. Hence, with lower ρ, 2 determines the necessary level of veto and it is increasing with ρ. For higher ρ, agent 3 is determinant and the level of veto decreases with ρ.

3.3. What about ownership?

In our framework, access aims at maximizing the coordination surplus while the purpose of veto is to *avoid* (inefficient) coordination. These two objectives are contradictory and ownership (the allocation of access *and* veto) is never optimal. With 'full' ownership ($\lambda = \mu = 1$), agent 2 would not invest in coordination and the surplus in non-coordination is lower than in status-quo. Partial ownership ($0 < \lambda = \mu < 1$) is always suboptimal, i.e. the surplus in coordination and non-coordination is always lower than the second-best. Figure 7 positions the surplus generated by full ownership, majority ownership ($\lambda = \mu = 3/4$), joint ownership (JO) ($\lambda = \mu = 1/2$) and minority ownership ($\lambda = \mu = 1/4$) in regards to the second best surplus in the case where $\rho = 1/2$.

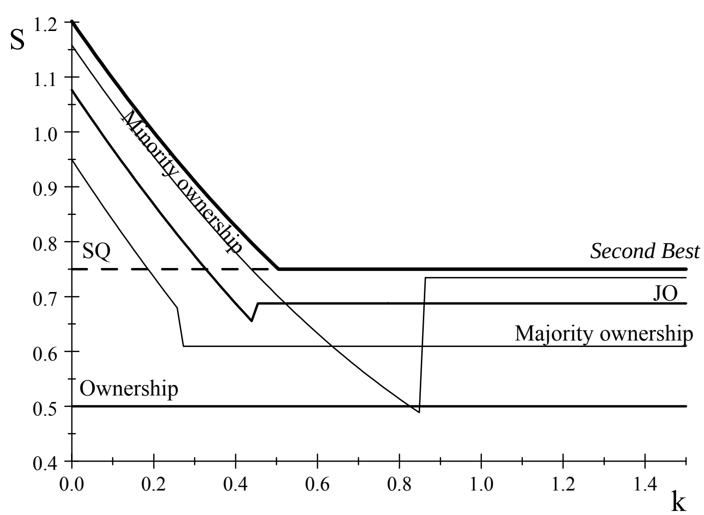

Figure 7

3.4. The case of an outside party

When agent 1 does not have any investment (i.e. she is an *outside party*), should she get any control rights?

With a non-investing party the bargaining game becomes:

$$\begin{cases} v_1 = \lambda e_2; v_2 = 0; v_3 = 0 \\ v_{12} = e_2; v_{13} = \lambda e_2; v_{23} = (1-\mu)(e_2 + e_3) \\ v_{123} = (1-k)\rho e_2 + (1-\rho)e_2 + e_3 \end{cases}$$

Interior solutions to the optimal investment problem require that $k \leq 2/\rho$. Following the same logic as above, we get the following result.

PROPOSITION 3. *When 1 is an outside party, there exists* k_1, k_2 *such that the optimal control structure is NI when* $k \leq k_1$, *partial access* ($\hat{\lambda} = \tilde{\lambda}(k,\rho), \hat{\mu} = 0$) *when* $k_1 < k \leq k_2$, *and no access* ($\hat{\lambda} = 0, \mu$) *when* $k > k_2$. *With the first two structures, coordination takes place and the surplus is higher than the status-quo; with the last one, non-coordination prevails and the surplus is equal to the status-quo*

Proof. In the Appendix

NB. $k_1, k_2, \tilde{\lambda}(k,\rho)$ take the following values: $k_1 = \dfrac{1}{2\rho}$, $k_2 = \dfrac{1}{\rho}$, $\tilde{\lambda}(k,\rho) = \dfrac{4}{3}\rho k - \dfrac{2}{3}$

The idea that it may be optimal for an outside party to receive control rights is not new[19] but the notion that an outside party should get access on an asset that she will not invest on is more intriguing. When 1 is an outside party, horizontal coordination is always ‘positive’ because it does not decrease agent 1’s incentive to invest, so there is no synergy trap. Moreover, agents 2 and 3 always want to coordinate. But the problem here is that there may be too much investment resulting in suboptimal surplus. To reduce agents 2 and 3 investments, access or veto could be used. In our framework, giving veto to agent 1 always decreases the surplus of coordination because it reduces both agents 2 and 3’s incentives to invest. But allocating access to 1 is optimal because it reduces agent 2’s incentive to invest without affecting 3’s incentive. When agent 1 is an outside party, access is useful not when k is

[19] See Rajan and Zingales (1998), Segal and Whinston (2000), and Bel (2006).

small but when it is large. When k is small agent 2's marginal return on investment is positive so the marginal effect of λ on the total surplus is negative (agent 3's incentive to invest is insensible to the level of access given to 1) and there should be no access. When k gets higher, agent 2's marginal return on investment becomes negative and the marginal effect of λ on the total surplus is positive. So access should be given to 1. As k increases further, we reach the point where too much access is given to 1 and this leads 2 to stop investing in coordination (her incentive to invest is equal to zero). At this point non-coordination prevails and is optimized when 1 does not have access: this could be done with NI or partial (or full) veto. Here, access to the outside party is used *not* to increase agent 1's investment (1 does not invest) but to absorb agent 2's excessive incentive to invest and at the same time reduce her marginal return on investment so that her lower investment results in a higher surplus (and when 2's investment is low or equal to zero, there is no need for access). Here again, ownership (even partial) by 1 is never optimal.

4. Conclusion

We have seen the specific roles that access and veto play: access helps in raising the coordination surplus while veto aims at avoiding inefficient coordination. Access (mainly) targets the efficiency of ex-ante investment, veto (mainly) the inefficiency of ex-post bargaining. Access acts on the incentive to invest, veto on the incentive to coordinate. Access and veto prevent synergy traps differently: access by raising the surplus of coordination, veto by preventing coordination to happen. Access is an incentive device, veto a disciplining device[20]. Access is a 'carrot', veto is a 'stick'. To paraphrase Steward and Glassman[21], access is 'soft', veto 'hard'.

Access and veto are two instruments targeting different contexts[22]. When the value of k is such that the asset (B) of the third agent *decreases* the marginal return of agent 1 in her relationship with 2 (i.e. A_2 and B are *substitutes at the margin*), there is a role for veto and allocating *veto* on A_2

[20] We have seen that in some extreme cases, access may also play a disciplining role, when it is given to an outside party to *reduce* the investment of a productive agent with a negative marginal return.
[21] "Equity is soft, debt hard. Equity is forgiving, debt insistent. Equity is a pillow, debt a sword" (Stewart and Glassman, *Journal of Applied Corporate Finance*, 1988).
[22] See Bel (2006).

to 1 is optimal. When the value of k is such that the presence of B *increases* 1's marginal return (A_2 and B are *complements at the margin*), allocating *access* on A_2 to 1 is optimal. With an outside agent who does not invest, B does not have any impact on 1, A_1 and A_2 are always complementary, and access is always optimal.

In our view, internal organization can be viewed as the nexus of access and veto relationships on the physical, human, financial and organizational assets which compose the firm. Fiat, hierarchy and incentives are flexible ways of organizing the allocation of access and veto to take into account the complex relationships between the assets of the firm and the external assets. On the other hand 'ownership' can be optimal when access *and* veto are jointly needed.

Appendix

- *Proof of Lemma 1*

(i) There is a synergy trap when the surplus of coordination is lower than the surplus in status-quo and the agents decide to coordinate nevertheless. This happens when $S^C < S^{SQ}$ and C2 and C3 are met.

When $\rho = \frac{1}{2}, \mu = 0$, $S^C = \frac{25}{72}k^2 - \frac{1}{6}k\lambda - \frac{19}{18}k - \frac{1}{4}\lambda^2 + \frac{1}{12}\lambda + \frac{43}{36}$ and $S^{SQ} = \frac{3}{4}$. Hence $S^C < S^{SQ}$ when $k > k_T = \frac{6}{25}\lambda - \frac{1}{25}\sqrt{486\lambda^2 + 306\lambda + 644} + \frac{38}{25}$

C2 and C3 are:

$$\frac{4}{9}k + \frac{1}{12}\lambda - \frac{1}{12}k\lambda - \frac{1}{8}k^2 - \frac{25}{72} < 0 \quad \text{(C2)}$$

$$\frac{11}{36}k + \frac{1}{12}\lambda + \frac{1}{12}k\lambda - \frac{5}{36}k^2 - \frac{17}{72} < 0 \quad \text{(C3)}$$

If $\lambda \leq \frac{7}{3} - \sqrt{2}$, C2 is satisfied when $k \leq \frac{16}{9} - \frac{1}{9}\sqrt{9\lambda^2 - 42\lambda + 31} - \frac{1}{3}\lambda$. If $\lambda > \frac{7}{3} - \sqrt{2}$, C2 is always satisfied, $\forall k$. If $\lambda \geq \frac{7}{3}\sqrt{10} - 7$, C3 is satisfied when $k \leq \frac{3}{10}\lambda - \frac{1}{10}\sqrt{9\lambda^2 + 126\lambda - 49} + \frac{11}{10}$. If $\lambda < \frac{7}{3}\sqrt{10} - 7$, C3 is always satisfied, $\forall k$.

To find out when coordination will be decided the two extreme cases where $\lambda > \frac{7}{3} - \sqrt{2}$ or $\lambda < \frac{7}{3}\sqrt{10} - 7$ are straightforward. In the intermediate case where $\frac{7}{3}\sqrt{10} - 7 \leq \lambda \leq \frac{7}{3} - \sqrt{2}$, coordination will take place when $k \leq \frac{3}{10}\lambda - \frac{1}{10}\sqrt{9\lambda^2 + 126\lambda - 49} + \frac{11}{10}$ since $\frac{3}{10}\lambda - \frac{1}{10}\sqrt{9\lambda^2 + 126\lambda - 49} + \frac{11}{10} \leq \frac{16}{9} - \frac{1}{9}\sqrt{9\lambda^2 - 42\lambda + 31} - \frac{1}{3}\lambda$. Overall:

- If $\lambda < \frac{7}{3}\sqrt{10} - 7$, 2 is binding and coordination is done when $k \leq k_C = \frac{16}{9} - \frac{1}{9}\sqrt{9\lambda^2 - 42\lambda + 31} - \frac{1}{3}\lambda$
- If $\lambda \geq \frac{7}{3}\sqrt{10} - 7$, 3 is binding and coordination is done when $k \leq k_C = \frac{3}{10}\lambda - \frac{1}{10}\sqrt{9\lambda^2 + 126\lambda - 49} + \frac{11}{10}$

There is a synergy trap when $k_T < k < k_C$. Since $k_T < k_C, \forall \lambda$, there is always a synergy trap. Its size is $k_C - k_T$. When $\lambda < \frac{7}{3}\sqrt{10} - 7$, it increases with λ. When $\lambda \geq \frac{7}{3}\sqrt{10} - 7$, it decreases with λ.

(ii) There is an efficiency trap when the surplus of coordination is higher than the surplus in status-quo and the agents decide not to coordinate nevertheless. This happens when $S^C > S^{SQ}$ and C2 or C3 is not met.

When $\rho=\frac{1}{2},\mu=1$, $S^C=\frac{25}{72}k^2-\frac{1}{6}k\lambda-k-\frac{1}{4}\lambda^2+\frac{37}{36}$ and $S^{SQ}=\frac{3}{4}$. Hence $S^C>S^{SQ}$ when $k<k_T=\frac{6}{25}\lambda-\frac{1}{25}\sqrt{2}\sqrt{243\lambda^2+216\lambda+398}+\frac{36}{25}$

C2 and C3 are:

$$\frac{5}{12}k-\frac{1}{12}k\lambda-\frac{1}{8}k^2-\frac{1}{9}<0 \quad \text{(C2)}$$

$$\frac{1}{4}\lambda+\frac{1}{12}k\lambda-\frac{5}{36}k^2-\frac{1}{18}<0 \quad \text{(C3)}$$

C2 is *not* satisfied when $k\geq\frac{5}{3}-\frac{1}{3}\sqrt{\lambda^2-10\lambda+17}-\frac{1}{3}\lambda$ and C3 is *not* satisfied when $k\geq\frac{3}{10}\lambda-\frac{1}{10}\sqrt{9\lambda^2+54\lambda+41}+\frac{9}{10}$.

Overall, non-coordination will be chosen when $k\geq k_C=\frac{3}{10}\lambda-\frac{1}{10}\sqrt{9\lambda^2+54\lambda+41}+\frac{9}{10}$ since $\frac{3}{10}\lambda-\frac{1}{10}\sqrt{9\lambda^2+54\lambda+41}+\frac{9}{10}\leq\frac{5}{3}-\frac{1}{3}\sqrt{\lambda^2-10\lambda+17}-\frac{1}{3}\lambda$.

There is an efficiency trap when $k_C<k<k_T$. This the case when $\lambda\leq\frac{1}{2}$.

- *Proof of Propositions 1&2 (apply $\rho=\frac{1}{2}$ for Proposition 1)*

(i) The program is: $Max_{\lambda,\mu}S^C \; s.t.\begin{cases}S_2^C>S_2^{NC} & \text{(C2)}\\ S_3^C>S_3^{NC} & \text{(C3)}\end{cases}$, i.e.

$$Max_{\lambda,\mu}[-\frac{1}{4}\lambda^2+\lambda(\frac{1}{3}\rho k-\frac{1}{3}k+\frac{1}{12})+\frac{1}{12}\lambda\mu-\frac{1}{36}\mu^2+\mu(\frac{1}{9}\rho k-\frac{5}{36})$$
$$+(\frac{5}{18}\rho^2k^2+\frac{5}{18}k^2-\frac{7}{9}\rho k-\frac{2}{3}k+\frac{43}{36})]$$

$$s.t.\begin{cases}S_2^C>\frac{(1-\lambda)(3+\lambda)}{8} & \text{(C2)}\\ S_3^C>0 & \text{(C3)}\end{cases}$$

Where

$$S_2^c=\frac{1}{12}\lambda\mu-\frac{1}{8}\lambda^2+(\frac{1}{6}\rho k-\frac{1}{3})\lambda+(\frac{1}{18}\rho k-\frac{5}{18})\mu$$
$$+\frac{1}{24}\mu^2+(\frac{1}{18}\rho^2k^2+\frac{1}{9}k^2-\frac{2}{9}\rho k-\frac{1}{3}k+\frac{13}{18})$$

$$S_3^c=\frac{1}{12}\lambda\mu+(\frac{1}{6}\rho k-\frac{1}{6}k-\frac{1}{12})\lambda+(\frac{1}{9}\rho k-\frac{2}{9})\mu$$
$$+\frac{1}{24}\mu^2+(\frac{1}{9}\rho^2k^2+\frac{1}{9}k^2-\frac{5}{18}\rho k-\frac{1}{6}k+\frac{17}{72})$$

The result is:

$$\begin{cases} \hat{\lambda} = \frac{1}{6} - \frac{2}{3}k(1-\rho), \hat{\mu} = 0 & \text{if } k \le \frac{1}{4(1-\rho)} \text{ and } \rho \le \frac{5}{6} \text{ OR } k \le \frac{5}{4\rho} \text{ and } \rho > \frac{5}{6} \\ \hat{\lambda} = 0, \hat{\mu} = 0 & \text{if } k > \frac{1}{4(1-\rho)} \text{ and } \rho \le \frac{5}{6} \\ \hat{\lambda} = \lambda_u, \hat{\mu} = \mu_u & \text{if } k > \frac{5}{4\rho} \text{ and } \rho > \frac{5}{6} \end{cases}$$

(when $\lambda = \frac{1}{6} - \frac{2}{3}k(1-\rho), \mu = 0$ or $\lambda = 0, \mu = 0$, $\frac{4-\mu-3\lambda}{2\rho} > \frac{3}{2}$, so the constraint $k \le \frac{4-\mu-3\lambda}{2\rho}$ never binds since $k \le \frac{3}{2}$)

In the first two cases, the optimal surplus of coordination is respectively (the third case is non relevant here since non-coordination is always chosen for these values of ρ and k, as will be seen later):

$$\hat{S}^C = \frac{7}{18}\rho^2 k^2 - \frac{2}{9}\rho k^2 + \frac{7}{18}k^2 + \frac{13}{18}\rho k - \frac{13}{18}k + \frac{173}{144}$$

$$\hat{S}^C = \frac{5}{18}\rho^2 k^2 + \frac{5}{18}k^2 - \frac{7}{9}\rho k - \frac{2}{3}k + \frac{43}{36}$$

(a) When is $\hat{S}^C \ge \hat{S}^{SQ}$?

$S^{SQ} = \frac{3}{4} = \hat{S}^{NC}$

When $k \le \frac{1}{4(1-\rho)}$ and $\rho \le \frac{5}{6}$ or when $k \le \frac{5}{4\rho}$ and $\rho > \frac{5}{6}$, $\hat{S}^C \ge S^{SQ}$ if $\rho < 4-\sqrt{15}$ or if $\rho \ge 4-\sqrt{15}$ and $k \le \frac{26\rho - 3\sqrt{26}\sqrt{8\rho-\rho^2-1}+26}{28\rho^2-16\rho+28}$, and $\hat{S}^C < S^{SQ}$ otherwise.

When $k > \frac{1}{4(1-\rho)}$ and $\rho \le \frac{5}{6}$, $\hat{S}^C \ge S^{SQ}$ if $\rho < \frac{10}{3}\sqrt{2} - \frac{14}{3}$ or if $\rho \ge \frac{10}{3}\sqrt{2} - \frac{14}{3}$ and $k \le \frac{1}{5\rho^2+5}(7\rho - \sqrt{9\rho^2+84\rho-4}+6)$, and $\hat{S}^C < S^{SQ}$ if $\frac{1}{5\rho^2+5}(7\rho - \sqrt{9\rho^2+84\rho-4}+6) < k \le \frac{1}{5\rho^2+5}(7\rho + \sqrt{9\rho^2+84\rho-4}+6)$.

(b) When are C2 and C3 met?

When $\lambda = \frac{1}{6} - \frac{2}{3}k(1-\rho), \mu = 0$, C3 is always satisfied and C2 is satisfied when $k < \frac{9\rho - \sqrt{372\rho - 207\rho^2 - 92}+10}{12\rho^2 - 8\rho + 8}$.

When $\lambda = 0, \mu = 0$, C3 is always satisfied. C2 is satisfied when $k < \frac{1}{\rho^2+2}(2\rho - \frac{1}{2}\sqrt{48\rho - 9\rho^2 - 14}+3)$.

(c) Synergy traps

From the above, we get that there will be synergy traps (ST) when:

(i) $\lambda = \frac{1}{6} - \frac{2}{3}k(1-\rho), \mu = 0$:

$$k > \frac{26\rho - 3\sqrt{26}\sqrt{8\rho - \rho^2 - 1} + 26}{28\rho^2 - 16\rho + 28}, \text{ and}$$

$$k \leq Min[\frac{1}{4(1-\rho)}, \frac{5}{4\rho}, \frac{9\rho - \sqrt{372\rho - 207\rho^2 - 92} + 10}{12\rho^2 - 8\rho + 8}]$$

(ii) $\lambda = 0, \mu = 0$:

$$\rho \geq \frac{10}{3}\sqrt{2} - \frac{14}{3}, \text{ and } k > Max[\frac{4}{1-\rho}, \frac{1}{5\rho^2+5}(7\rho - \sqrt{9\rho^2 + 84\rho - 4} + 6)], \text{ and}$$

$$k \leq Min[\frac{1}{5\rho^2+5}(7\rho + \sqrt{9\rho^2 + 84\rho - 4} + 6), \frac{1}{\rho^2+2}(2\rho - \frac{1}{2}\sqrt{48\rho - 9\rho^2 - 14} + 3)]$$

Hence, the values of k which will trigger a ST are:

- if $\rho < \frac{10\sqrt{2} - 14}{3}$, there is no ST
- if $\frac{10\sqrt{2} - 14}{3} \leq \rho < \frac{44 - \sqrt{151}}{63}$, $k > \tilde{k}_1 = \frac{1}{5\rho^2+5}(7\rho - \sqrt{9\rho^2 + 84\rho - 4} + 6)$
- if $\rho \geq \frac{44 - \sqrt{151}}{63}$, $k > \tilde{k}_2 = \frac{26\rho - 3\sqrt{26}\sqrt{8\rho - \rho^2 - 1} + 26}{28\rho^2 - 16\rho + 28}$

Hence there is a synergy trap when $k > \tilde{k}_i$. To avoid it, we must ensure that non-coordination is chosen beyond this point and that non-coordination is optimal. This will be the case if C2 *or* C3 does not hold from that point and if $\lambda = 0$. It is then easy to determine the minimum $\mu = \tilde{\mu}$ at $k = \tilde{k}$. For example, when $\rho = \frac{1}{2}$,

$$\tilde{k}_1 = \frac{38 - 2\sqrt{161}}{25} \cong 0.50491 \text{ and } \tilde{\mu} = \frac{2}{75}\sqrt{161} - 12\sqrt{\frac{19}{101250}\sqrt{161} + \frac{11179}{270000}} + \frac{212}{75} \cong 0.65$$

Overall, the optimal control structures are:

- when $\rho < \frac{10\sqrt{2} - 14}{3}$, $\hat{\lambda} = \frac{1}{6} - \frac{2}{3}k(1-\rho), \hat{\mu} = 0$ (partial access) if $k \leq \frac{1}{4(1-\rho)}$ and $\hat{\lambda} = 0, \hat{\mu} = 0$ (NI) if $k > \frac{1}{4(1-\rho)}$ (there is no ST and no need for veto)
- when $\frac{10\sqrt{2} - 14}{3} \leq \rho < \frac{44 - \sqrt{151}}{63}$, $\hat{\lambda} = \frac{1}{6} - \frac{2}{3}k(1-\rho), \hat{\mu} = 0$ if $k \leq \frac{1}{4(1-\rho)}$, $\hat{\lambda} = 0, \hat{\mu} = 0$ if $\frac{1}{4(1-\rho)} \leq k < \tilde{k}_1$, and $\hat{\lambda} = 0, \hat{\mu} = \tilde{\mu}(\tilde{k}_1)$ (partial veto) if $k \geq \tilde{k}_1$
- when $\rho \geq \frac{44 - \sqrt{151}}{63}$, $\hat{\lambda} = \frac{1}{6} - \frac{2}{3}k(1-\rho), \hat{\mu} = 0$ if $k < \tilde{k}_2$, and $\hat{\lambda} = 0, \hat{\mu} = \tilde{\mu}(\tilde{k}_2)$ if $k \geq \tilde{k}_2$

- *Proof of Proposition 3*

The proof follows the same lines as for *Propositions 1* and *2*.

(i) The result of the maximization program is now:

$$\begin{cases} \hat{\lambda}=0, \hat{\mu}=0 & \text{if } k \leq \dfrac{1}{2\rho} \\ \hat{\lambda}=\dfrac{4}{3}\rho k-\dfrac{2}{3}, \hat{\mu}=0 & \text{if } \dfrac{1}{2\rho}<k \leq \dfrac{5}{4\rho} \\ \hat{\lambda}=1, \hat{\mu}=0 & \text{if } k>\dfrac{5}{4\rho} \end{cases}$$

$S^{SQ}=\frac{3}{8}=\hat{S}^{NC}$. Here there is no constraint on the level of investment for 1 (since she does not invest), but the constraint on 2 ($k<\frac{4-\mu-3\lambda}{2\rho}$) remains.

When $\lambda=0, \mu=0$, the constraint $k<\frac{4-\mu-3\lambda}{2\rho}$ is not binding since $\frac{1}{2\rho}<\frac{2}{\rho}$. When $\lambda=\frac{4}{3}\rho k-\frac{2}{3}, \mu=0$, $k<\frac{4-\mu-3\lambda}{2\rho}=\frac{1}{\rho}<\frac{5}{4\rho}$, so the constraints binds. When $\lambda=1, \mu=0$, $k<\frac{4-\mu-3\lambda}{2\rho}=\frac{1}{2\rho}<\frac{5}{4\rho}$, so it is impossible.

Hence the result of the maximization problem is:

$$\begin{cases} \hat{\lambda}=0, \hat{\mu}=0 & \text{if } k \leq \dfrac{1}{2\rho} \\ \hat{\lambda}=\dfrac{4}{3}\rho k-\dfrac{2}{3}, \hat{\mu}=0 & \text{if } \dfrac{1}{2\rho}<k \leq \dfrac{1}{\rho} \end{cases}$$

For $k \geq \frac{4-\mu-3\lambda}{2\rho}$, agent 2 will not invest in coordination, and the optimal control structure will be one that maximizes the surplus of non-coordination, i.e. NI or partial veto ($\lambda=0$).

For the two structures above, the constraints C2 and C3 are always satisfied (2 and 3 always want to coordinate, $\forall k, \rho$)

(a) When $k \leq \frac{1}{2\rho}$ and $\lambda=0, \mu=0$, $\hat{S}^{C}>S^{SQ}=\frac{3}{8}, \forall k, \rho$. Hence, there is no synergy trap and coordination is realized.

(b) When $\frac{1}{2\rho} \leq k \leq \frac{1}{\rho}$ and $\lambda=\frac{4}{3}\rho k-\frac{2}{3}$, $\hat{S}^{C} \geq S^{SQ}, \forall k, \rho$. There is no synergy trap and coordination is realized.

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