Locating Emergency Facilities: Targeting Efficiency and Cost-Effectiveness.

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Abstract
The delivery of most public services involves direct contact between the service facility and the target population. The location of emergency facilities is one of those public facilities in which the proximity to the target population is very important. Often communities in a service area desire a closer location of facilities such as ambulance stations and fire stations in their service area. Due to scarcity of resources, a limited number of facilities tend to be located to serve larger settlement areas and this often leads to efficiency and effectiveness problems in terms of service delivery. This paper showed efficient and effective ways of locating emergency facilities. A new method is developed based on the *p*-median problem. This method is tested using simulated data and a number of literature problems. The testing and comparison of this method with other heuristic methods of solving the *p*-median problem demonstrate its effectiveness.

Introduction

Facility location problems form an important class of industrial optimization problems. These problems typically involve the optimal location of facilities. A facility is just a physical entity that assists with the provision of a service or the production of a product. Examples include: ambulance depot, emergency care centers, firestations, workstations, schools, libraries etc. The objective may involve factors such as cost, distance or service utilization. The optimization problems are complicated with the need to meet a number of specified constraints. These constraints may relate to safety, available resources, level of service, time, etc.

The optimization problems are usually grouped into two categories namely service and manufacturing industries. In the service industries, the location of emergency facilities (ambulance, fire station, emergency centers) affects significantly on the safety and well-being of the community. The safety and well-being of the community depends directly or indirectly on the response time of the emergency facilities. The objective is to minimize the average response time (time between the receipt of a call and the arrival of emergency vehicle). The minimization of the response time measures the performance of emergency facilities. The performance of these facilities can be improved by either improving the existing location of emergency facilities or increasing the number of facilities. However, increasing the number of facilities is generally limited or impossible due to capital constraints. It is therefore important to locate emergency facilities effectively and efficiently.

The important way to measure the efficiency and effectiveness of emergency facility is by evaluating the average distance between the customers and the facilities. When the average distance decreases, the accessibility of the facilities increases and this will decrease the average response time. This is known as the p-median problem, which was introduced by Hakimi (1964) and is defined as: determine the location of p facilities to minimize the average (total) distance between demands and their closest facility.

The p-median problem is computationally difficult to solve by exact methods because the problem is *NP*-hard on general networks as shown by Kariv and Hakimi (1979). However, solutions from the p-median model are considered efficient since they bring the facility locations into closer proximity of the users. The difficulty of solving the p-median problem by exact method has led researchers to consider sub optimal solutions generated by heuristic approaches. Heuristics for solving the p-median problem have been discussed in Daskin (1995), Maranzana (1964), Teitz and Bart (1968) and Densham and Rushton (1992).

This paper discusses three new heuristic methods for solving the p-median problem. These methods are motivated by the desire to eliminate outliers from having strong influence over the final solution given by the heuristics. These heuristics will also improve the delivery of emergency medical care by properly locating emergency facilities in an area.

In these heuristics, the facility location problem can be formulated as a network optimization problem as follows. The geographical region is partitioned into a number of subregions and a corresponding graph is constructed, each node of this graph represents a subregion and each link of the graph represents the fact that the corresponding regions share a boundary. This gives us a structural model. Non-structural information is added as weights on the nodes (reflect expected demand in region) and the links (reflect travel time). Usually the nodes of the network represent possible location of facilities. An efficient reduction method is then used to address the problem of outliers.

Computational results, based on 400 random uniformly generated problems, show that the heuristics gives a good performance when compared with the optimal. Motivated by their performance the best heuristic is further compared with the 400 random problem and the well known existing *p*-median heuristics giving better solution in most cases

2. The *P*-Median Model and Emergency Facilities

The criterion for finding a good location for emergency facilities requires the improvement of the response times to the emergency calls. The response time depend on the distance between the emergency facilities and the emergency sites.

Thus, the aim of locating emergency facilities is to locate these facilities such that the average (total) distance travelled by those who visit or use these facilities is minimized. This measures the effectiveness and efficiency of the emergency facilities. Thus, the utility derived from using those facilities increases as the distance between them decreases. That is as travel distances increases, facility accessibility decreases and the effectiveness of the facility located decreases giving rise to increase response time. The *p*-median problem measures this effectiveness. It is clear that people tend to travel to the closest facility regardless of the distance or time travelled. A good way to achieve that is by the application of the *p*-median problem.

The *p*-median problem consists of determining the location for *p* emergency facilities to minimize the weighted distance between emergency (demand) points and their closest new emergency facility. The following authors such as Serra and Marinov, 1998; Mirchandani, 1980; Berlin *et al.,* 1976; Paluzzi, 2004; Carson and Batta; 1990 etc. use the *p*-median problem to locate emergency facilities.

We present the mathematical model for the *p*-median problem by defining the following notations as follows:-

$I = \{1,....,m\}$, the set of demand locations,

$J = \{1,...,n\}$, candidates sites for facilities,

$d_{ij} =$ the shortest distance between location *i* and location *j*,

$x_{ij} =$ 1 if the customer at location *i* is allocated to facility at location *j,* 0 otherwise,

$y_j =$ 1 if a facility is established at location *j,* 0 otherwise,

$p =$ the number of facilities to be established,

$a_i =$ the population at the demand node *i.*

The mathematical formulation of a *p*-median problem can be specified as follows,

$$\text{Min} \sum_i \sum_j a_i d_{ij} X_{ij} \ , \qquad (1)$$

subject to

$$\sum_{j \in J} x_{ij} = 1, \quad \forall i \in I \qquad (2)$$

$$\sum_{j \in J} y_j = p \qquad (3)$$

$$x_{ij} \le y_j \quad \forall i \in I, \forall j \in J \qquad (4)$$

$$y_j \in \{0,1\} \ , \ x_{ij} \in \{0,1\} \qquad (5)$$

The objective (1) is to minimize the total distance from customers or clients to their nearest facility. Constraint (2) shows that the demand of each customer or client must be met. From constraint (3), the number of facilities to be located is *p.* Constraint (4) shows that customers must be supplied from an open facility, and constraint (5) restricts the variables to 0,1 values.

Several extensions have been proposed for the p-median based models to improve their efficiency (Daskin *et al*., 1988). Extensions to the *p*-median problem that account for its stochastic nature has been given by Fitzsimmons (1973), Weaver and Church (1985) and Swoveland *et al.* (1973).

3. Solution Methods for the *P*-Median Problem

The *p*-median problem is a computationally difficult problem to solve (the problem is *NP*-hard on general networks). Most solution methods are heuristic based because of the large number of variables and constraints that arise for a medium sized network. The heuristics are based on: genetic algorithms, simulated annealing, tabu search, node partitioning, node insertion, node substitution and various hybrids (see Hosage and Goodchild (1986); Golden and Skiscism (1986); Glover (1990)). Some of these heuristics together with Lagrangian relaxation are briefly discussed below.

3.1 Lagrangian Relaxation

Lagrangian relaxation is based on the principle that removing constraints from a problem makes the problem easier to solve. Generally, Lagrangian relaxation removes a constraint and solves the revised problem but introduces a penalty for violating the removed constraint. The solution procedure for solving the problem is stated below.

The Lagrangian relaxation for the *p*-median is given as

$$L(\lambda) = \min \sum_i \sum_j d_{ij} x_{ij} + \sum_i \lambda_i \left(1 - \sum_j x_{ij} \right) \tag{6}$$

subject to constraints (3)-(5).

The expression $r_j = \sum_i \min\{0, d_{ij} - \lambda_i\}$ (7)

is used to minimize the objective function (6) for the fixed values of the Lagrange multipliers and we then set

$$x_{ij} = \begin{cases} 1, \text{if } y_j = 1 \text{ and } d_{ij} - \lambda_i < 0 \\ 0, \text{otherwise} \end{cases} \tag{8}$$

The lower and upper bounds of the objective function are determined by using the variables of modified and unmodified problems respectively. The next step involves the use of subgradient optimization to update the value of the Lagrange multipliers by using the equation below. See Daskin (1995) for detail procedure.

$$\lambda_i^{m+1} = max\left\{0, \lambda_i^m - t^m \left(\sum_j x_{ij}^m - 1 \right)\right\} \tag{9}$$

$$t^m = \frac{A^m (UB - L^m)}{\sum_i \left\{ \sum_j x_{ij}{}^m - 1 \right\}^2} \qquad (10)$$

where

$A^m =$ a constant on the mth iteration,

$t^m =$ the stepsize at the mth iteration of the Lagrangian procedure,

$UB =$ the best (smallest) upper bound on the P-median objective function,

$L^m =$ the value of the objective function using the solution obtained from the relaxed problem,

$x_{ij}{}^m =$ the optimal value of the allocation variable at the mth iteration.

An optimal solution is found if the lower bound is equal or very close to the upper bound. Narula *et al.* (1977) and Galvao (1980) and Beasley (1993) have successfully applied the subgradient optimization to solve a number of problems. However for the larger problems tested the computational time is excessively large.

3.2 Heuristics

In this section, we start our discussion by observing that it is an easy task to assign a set of m clients to p facilities J' with fixed locations. We just determine

$$d_{ij_i^*} = \min\{d_{ij}\},\ 1 \le i \le m,\ j \in J' \qquad (11)$$

and assign customer i to facility j_i^*. This gives us a tool for generating possible solutions. The procedure is also useful for determining alternative solutions through exchange of facility locations.

We now use the idea above to describe below three simple heuristics, which are competitive with other methods. These heuristics are also widely used currently to solve the p-median problem.

3.2.1 Myopic Algorithm for the *P*-Median Problem

The myopic heuristic is a greedy type, which, works in the following way. Firstly, a facility is located in such a way as to minimize the total cost for all customers. Facilities are then added one by one until *p* is reached. For this heuristic, the location that gives the minimum cost is selected. The main problem with this approach is that once a facility is selected it stays in all subsequent solution. Consequently, the final solution attained may be far from optimal.

3.2.2 Neighborhood search Heuristic

Maranzana (1964) first proposed the search heuristic. The heuristic is described as follows. We begin with any set of *p* facilities sites. For each facility site, the heuristic identifies the set of demands nodes that constitute the neighborhood around the facility site. Within each neighborhood, the optimal 1-median is found. If any sites have change, the algorithm again finds the 1-median within each neighborhood. We continue this process until there is no change in neighborhood.

3.2.3 Exchange Heuristic

This is one of the first heuristics developed for the *p*-median problem and Teitz and Bart (1968) was the first to propose an exchange heuristic for the *p*–median problem. The heuristic starts by generating a feasible solution and then attempts to improve the current solution through a swapping operation, which relocates a facility to an unused site. The process continues until no further improvements in the objective function value are possible. The solution thus obtained is a local optimum, not a global optimum. Densham and Rushton (1992) have proposed a number of enhancements to the algorithm.

4. New *P*-Median Heuristics for Locating Emergency Facilities

4.1. Reduction Heuristics (*RH1, RH2, RRH*)

As discussed in the previous section, some of the heuristics (myopic in particular) for the *p*-median problem uses all the values of the distance matrix without any modification to solve the problem of extreme values (outliers). In this section, we show how to locate facilities by using a reduction technique to eliminate outliers.

To obtain the initial solution set for the heuristics we first eliminate the extreme values in each column. The number of extreme values to be eliminated depends on the size of the problem being considered. That is, the larger the size, the greater the number of extreme values to be eliminated. The remaining values for each column are then summed and we choose the nodes corresponding to the first *p* nodes of the totals arranged in ascending order, which represent the number of facilities to be located. For instance to locate three facilities we select the nodes corresponding to the first three totals arranged in ascending order of magnitude as the initial set.

We use the initial solution to reduce the original distance matrix by setting the distance values for nodes corresponding to the initial set for both rows and columns to zero. This is done with the assumption that customers at those nodes need no cost in using those facilities if facilities are located there. For *RH1*, the columns of the resulting distance matrix are added and the column (node) corresponding minimum value among the totals of each column is chosen for substituting or exchanging for each node in initial solution. We finally choose the set corresponding to the minimum objective value as the final solution when each customer or demand is assigned to its nearest facility. In the case of *RH2*, all nodes that are not in the initial solution are exchanged one-by-one for the nodes in the initial solution. We then choose the facility set that corresponds to the minimum objective value as the final solution.

However, for both heuristics (*RH1* and *RH2*), we choose the initial set as the final solution if there is no improvement in the objective value after the swapping procedure is complete.

We describe the three new reduction heuristics as follows.

4.2 Reduction Heuristic (*RH1*)

Step 1: Input the number of nodes n and the number of facilities p of the data.

Step 2: Arrange the n values for each column in ascending order and define:

$$\alpha = \begin{cases} p, & if\ n \leq 29 \\ 2p, & if\ 30 \leq n \leq 39 \\ \left(\left\lceil \frac{n}{10} \right\rceil - 1\right)p, & otherwise \end{cases}$$

Where p is the number of facilities and n is the number of nodes.

Delete the highest α number of values from each column and let the resulting number of demand nodes be equal to n^* (ie. $n^* = n - \alpha$). The value of α depends on the number facilities to be located and the bounds for n are chosen to avoid all the values of the distance matrix from being eliminated.

Step 3: Sum the values for each column after deleting the extreme values, then arrange the total values in ascending order of magnitude, and choose the first p nodes corresponding to the first p totals as the initial set.

Step 4: Use the original weighted distance matrix and set the distance values (for both rows and columns) corresponding to the initial set of facilities to zero and sum the columns of the resulting distance matrix. For example, if the initial set is {1,2,3} then all values in rows and columns 1, 2 and 3 are changed to zero before the summation of each column.

Step 5: Choose the node (or nodes if there is a tie) corresponding to the minimum total value after the values in each column are added. The node(s) corresponding to the minimum value(s) is (are) swapped with every node in the initial set.

Step 6: Choose the set corresponding to the minimum objective value after the substitution procedure as the final solution and compare this with the initial solution. Choose the best solution overall

Reduction Heuristic Two (*RH2*)

For *RH2* (*RH2* is a refinement of *RH1*) Steps 1 to 4 are the same as *RH1* and the remaining steps are outline below as follows.

Step 5: Swap all the nodes, which are not in the initial solution set with the nodes in the initial solution set. For example, we select the node with the lowest number from the non-initial set and substitute for every node in the initial. We continue the process with the next lowest node number until all the nodes not in the initial set are used for swapping with the nodes of the initial set. This will lead to a number of possible solution sets for Step 6. (Note: *RH1* uses only some of the nodes, which are not in the initial solution for swapping nodes with the initial set while *RH2* uses all the nodes, which are not in the initial solution for swapping).

Step 6: Choose the set corresponding to the minimum value as the final solution.

We note from the computational results presented in sections 4 and 5 that the different swapping procedure leads to an improved final solution when compared with *RH1*.

Repeated Reduction Heuristic *(RRH)*

Motivated by the performance of the two new heuristics (*RH1* and *RH2*) we extend *RH2* further and propose a third heuristic, which we term the Repeated Reduction Heuristic (*RRH*).

Starting from the final solution of *RH2*, we let the final solution be initial solution. We then apply the initial solution to reduce the original distance matrix as discussed in Step 4 of *RH1*. We then continue with Step 5 and 6 of *RH2* always using the final solution as initial solution. We repeat Step 4 of *RH1* and Step 5 and 6 of *RH2*. We continue this process until the objective value of the previous solution is the same as the current solution. We then consider the result as the final solution.

We illustrate below the three new heuristics with a simple example.

Illustrative Example

0	82	37	51	100
67	0	78	93	97
74	18	0	20	49
20	87	27	0	66
62	37	51	87	0

To locate two facilities we eliminate the two greatest values in each column and we thus have the following: 67 and 74 in column 1, 82 and 87 in column 2, 51 and 78 in column 3, 87 and 93 in column 4 and 97 and 100 in column 5. Summing the remaining values and arranging in ascending order gives the following nodes with their respective totals in brackets: 2 (55), 3 (64), 4 (71), 1 (82) and 5 (115). We choose nodes 2 and 3 as the initial solution for *RH1*, *RH2* and *RRH*. We therefore set rows and columns 2 and 3 of the data to zero and we have the following table.

0	0	0	51	100
0	0	0	0	0
0	0	0	0	0
20	0	0	0	66
62	0	0	87	0

The resulting totals for the non-zero columns gives node 1 as the node with minimum value. Hence, for *RH1*, we substitute nodes 2 and 3 with node 1 which results in the possible solution sets of {1,3} and {1,2}. We choose {1,2}, since that gives a best value of 75.

In the case of *RH2* and *RRH,* we use all the nodes not in the initial solution for substituting for nodes in the initial solution. This gives the possible solution set as follows: {1,2}; {1,3}; {2,4}; {3,4}; {2,5} and {3,5}. We choose {1,2} as the final solution since it gives a best value of 75. We continue the same process repeatedly for *RRH* and now use {1,2} as its initial solution which finally yields {1,2} as the optimal solution.

We use the same data to locate three facilities. In this case, we eliminate three greatest values in each column and sum the values of the remaining columns giving as initial solution of 1, 2 and 4 by using similar procedure for choosing initial solution for two facilities. Going through the same process and setting rows and columns 1, 2 and 4 to zero we have the following table.

0	0	0	0	0
0	0	0	0	0
0	0	0	0	49
0	0	0	0	0
0	0	51	0	0

For *RH1* we have node 5 as the node having the minimum value so we substitute node 5 for nodes 1, 2 and 4 giving us the possible sets of {2,4,5}, {1,4,5} and {1,2,5}. We choose {1,2,5} as the final solution with a best value of 38. In the case of *RH2* and *RRH*, we use nodes 3 and 5, which are not in the initial solution, for substituting nodes 1, 2 and 4. This gives the possible solution sets of {2,3,4}, {1,3,4}, {1,2,3}, {2,4,5}, {1,4,5} and {1,2,5}. We finally choose {1,2,5} as the final solution with a best value of 38. For *RRH* we again use

{1,2,5} as its initial solution and continue the process repeatedly giving us final solution of {1,2,5}.

We describe below how we obtain the values for Myopic, Neighborhood and Exchange heuristics.

For the Myopic heuristic, we do not eliminate any extreme values so we have the following.

0	82	37	51	100
67	0	78	93	97
74	18	0	20	49
20	87	27	0	66
62	37	51	87	0

When we sum all the columns, we have node 3 as the minimum with 193 so for one facility we locate it at node 3. We note that for the *p*-median problem a demand is allocated to the nearest facility. We therefore adjust distance matrix and we have the following.

0	37	37	37	37
67	0	78	78	78
0	0	0	0	0
20	27	27	0	27
51	37	51	51	0

Node 2 gives the minimum value of 101 when the columns of the above matrix added so for two facilities we have nodes 2 and 3 with an objective value of 101.

Similarly, we have adjusted matrix shown below as:

0	37	37	37	37
0	0	0	0	0
0	0	0	0	0
20	27	27	0	27
37	37	51	37	0

Node 1 gives the minimum value when all the columns are added so for three facilities we have nodes 1, 2 and 3 with an objective value of 57.

For the exchange heuristic we arbitrary choose the set {1,2} as the initial solution with the objective value of 75. The nodes not in the initial solution are 3, 4 and 5 so we swap the nodes not in the initial set with those in it. Using node 3 for swapping we have the following sets with their objective value as follows: {2,3}-101 and {1,3}-124. There is no improvement in the objective so we still maintain {1,2} as the initial set. Using node 4 for swapping we consider the following sets: {2,4}-92 and {1,4}-122. Using node 5 gives {2,5}-75 and{1,5}-105. There is no improvement so we choose the initial solution as the final solution resulting in the objective function value of 75 for the location of two facilities

For the location of three facilities, we use the same procedure as above and start with the initial solution set of {1,2,3} with an objective value of 57. Using node 4 which is not in the initial set for swapping we have the following sets with their objective values: {2,3,4}-74; {1,2,4}-55 and {1,3,4}-104. There is an improvement in the objective value from 57 to 55 so we choose {1,2,4} as the new initial set. Node 5 is now used for swapping giving the following sets with their objective values: {2,4,5}-55, {1,4,5}-85 and {1,2,5}-38. There is improvement in the objective value from 55 to 38 so we choose {1,2,5} as the new initial set but using nodes 3 and 4 does not improve the objective value the final solution set is {1,2,5} with an objective value of 38.

We now consider the neighborhood search heuristic. Using {2,3} as the initial set for the location of two facilities, we find *p* subsets and calculate the 1-median for each subset. We find the neighborhood for each facility site resulting in the following subsets: {2,5} and {1,3,4}. We then calculate 1-median for each subset which yield 2 for the first set and 3 for the second set. This does not change the initial facility sites so for the locating of two facilities using neighborhood search we have {2,3} as the facility sites with an objective value of 101.

The final solution depends on the initial solution in this case. For a different initial solution of facilities {1,2}, the neighborhood for the facility located at node 1 is 4 while for the facility located at node 2 the neighborhoods are 3 and 5 so the subsets are {1,4} and {2,3,5}. Finding the 1-median for each subset, we have facilities located at nodes 1 and 3. We continue the process since {1,2} is different from {1,3}. We therefore find the neighborhood for facilities located at nodes 1 and 3 which gives us the following subsets: {1,2,4} and {3,5}. Finding the 1-median for each subset gives the following facilities at nodes 1 and 3. We stop the process because the facility patterns are the same. Therefore, starting with initial facilities at {1,2} we have facilities located at {1,3} with an objective value of 138 which is even greater than the previous result of 101.

The initial values that were used in first case to locate two facilities correspond to the facilities identified by myopic algorithm. We note from the results for locating two facilities that the values identified by the myopic algorithm are better initial solution than the other one chosen arbitrary.

For the location of three facilities we arbitrary choose {1,2,3} as the initial solution set. We find the neighborhood for each facility resulting in the following subsets: {1,4}, {2,5} and {3}. The 1-median for the sets {1,4}, {2,5} and {3} are 1, 2 and 3 respectively. This does not change the initial facility sites so for locating three facilities using the neighborhood search heuristic the facilities should located at nodes 1, 2 and 3. This gives an objective value of 57. The results in Table 1 show that the three new heuristics and the Exchange heuristic give the best (optimal) value for the location of two and three facilities while the Myopic and Neighborhood search give a non-optimal value.

Table 1: Results for RH1, RH2, RRH and Myopic, Exchange and Neighborhood Search Heuristics

<table>
<tr><th rowspan="3">Number of Facilities</th><th colspan="4">Solution</th></tr>
<tr><th colspan="2">RH1, RH2, RRH, Exchange</th><th colspan="2">Myopic, Neighborhood Search</th></tr>
<tr><th>Facilities</th><th>Objective Value</th><th>Facilities</th><th>Objective Value</th></tr>
<tr><td>2</td><td>{1,2}</td><td>75</td><td>{2,3}</td><td>101</td></tr>
<tr><td>3</td><td>{1,2,5}</td><td>38</td><td>{1,2,3}</td><td>57</td></tr>
</table>

5. Computational Results

The three new heuristics are implemented in C++ and tested on sets of 400 randomly generated sets of data for a [10, 100] matrix with n ranging from 10 to 50 in steps of ten and p ranging from 2 to 5. That is, for each problem size n and for locating 2, 3, 4 or 5 facilities, 80 uniformly distributed random problems are generated. We compare the results from the heuristics with the optimal values obtained by complete enumeration. This will give an indication of whether the new heuristics can provide a good alternative to the exact solution techniques, which are in many cases complex and expensive to apply. All computations were carried on a personal computer with an Intel Pentium 4 processor, 2.8GHZ and 448MB of RAM.

Table 2 gives the performance of the three new heuristics for locating 2, 3, 4 and 5 facilities. In Table 2 below, we have the average values for using ten, twenty, thirty, forty and fifty nodes.

Table 2: Average Values for the New Heuristics

Number of Nodes (*n*)	**Average Values (%)**		
	RH1	***RH2***	***RRH***
10	2.22	0.79	0.32
20	4.87	1.96	0.72
30	4.38	1.65	0.66
40	4.60	2.27	0.87
50	3.04	1.00	0.49

From Table 2 the average values for *RH1* ranges from 2.22% to 4.87%, *RH2* ranges from 0.79% to 2.27% and the values for *RRH* ranges from 0.32% to 0.87%. The values of *RRH* give values of almost optimal which is good for locating emergency facilities. This might give rise to acceptable response time since the values are almost optimal.

5.1 Comparison of the Repeated Reduction Heuristic (RRH) and Some P-Median Heuristics

Motivated by the performance of *RRH* we compare the heuristic using the 400 random problems described in the previous section and data from the literature. More specifically, for the literature problems, we use the 55-node (Swain; 1971) and a 30-node (Toregas *et al.;* 1971). The 55-node data is given in Colome *et al.* (2003) and has been used by authors such as Daskin (1982, 1983), Colome *et al.* (2003) and Church and Gerrard (2003) for testing location problems.

We compare *RRH* with Myopic algorithm, Exchange Heuristic and the Neighborhood search Heuristic. These heuristics are discussed in Section 3 of this paper and the detail description of the heuristics can be found in Daskin (1995).

We coded the Repeated Reduction Heuristic (*RRH*) in C++ while the results of the other heuristics were obtained from SITATION software (Daskin, 1995). The solution of the heuristics was compared with the optimal solutions, which were determined by the implementation of the Lagrangian relaxation in the SITATION software (Daskin, 1995).

Table 3 and Table 4 show the performance in terms of quality (deviations from the optimal) of *RRH* as compared to the existing *p*-median heuristics using 400 randomly generated test problems. For locating 2, 3, 4 and 5 facilities using 10 to 50 nodes the average deviation from the optimal for Myopic (Mp) ranges from 0.63% to 4.77%; Exchange (Ec) ranges from 0% to

1.83; Neighborhood Search (Nd) ranges from 0.63% to 4.77% and *RRH* ranges from 0% to 1.41%.

Table 3: Comparison Performance of *RRH* and the Existing Heuristics using the 400 random data

Number of Nodes (*n*)	Number of Facilities (*p*)	*Mp*	*Ec*	*Nd*	*RRH*
10	2	1.47	0.47	1.33	0.08
	3	3.22	0.30	3.01	0.26
	4	4.20	1.34	4.11	0.59
	5	1.74	0	1.74	0.35
20	2	0.63	0.16	0.63	0
	3	2.15	1.32	2.15	0.49
	4	4.31	1.37	4.31	1.14
	5	4.77	1.36	4.77	1.28
30	2	0.77	0.01	0.77	0
	3	2.63	1.10	2.63	0.40
	4	3.53	1.07	3.26	1.06
	5	4.12	0.81	4.12	1.21
40	2	0.92	0.32	0.92	0.28
	3	1.68	1.43	1.68	0.44
	4	2.52	1.83	2.52	1.41
	5	2.55	1.36	2.55	1.38
50	2	0.63	0.37	0.63	0.08
	3	1.86	0.47	1.86	0.47
	4	1.65	0.80	1.65	0.50
	5	2.98	0.82	2.98	0.91

Table 4 gives the average values for 10, 20, 30, 40 and 50 nodes. Comparing *RRH* with the Exchange heuristic using Table 4 there is percentage improvement for 10, 20, 30, 40 and 50 nodes when *RRH* is used to locate 2, 3, 4 and 5 facilities.

Table 4: Average Values for RRH and Existing Heuristics

Number of Nodes	Average Values (%)			
	RRH	***Mp***	***Nd***	***Ec***
10	0.32	2.65	2.55	0.52
20	0.72	2.96	2.96	1.05
30	0.66	2.76	2.69	0.74
40	0.87	1.91	1.91	1.23
50	0.49	1.78	1.78	0.61

The percentage improvements are 0.20% for 10 nodes, 0.33% for 20 nodes, 0.08% for 30 nodes, 0.36% for 40 nodes and 0.12% for 50 nodes. For example, using Table 4 the average

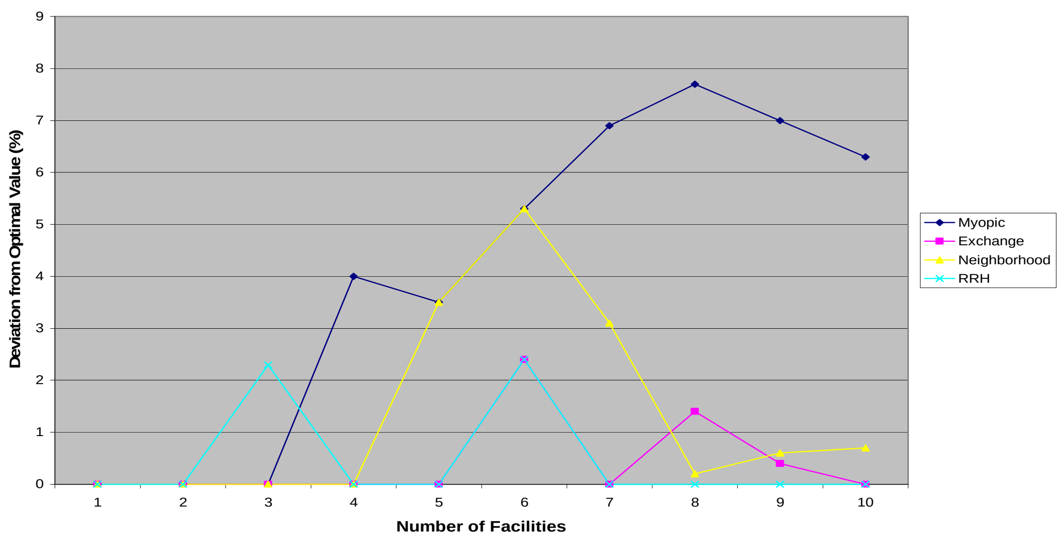
RRH value for 10, 20, 30, 40 and 50 nodes is 0.61%, the average value for Myopic is 2.41%, the average value for Neighborhood Search is 2.37% and the Exchange heuristic's average value is 0.83%.

Table 5 and Figure 1 shows the performance of the new heuristics and the existing ones when the 55-node network is used.

Table 5: Comparison Performance of RRH and Existing Heuristic using 55-node Network.

Number of Facilities	$\frac{Mp-O}{O}\times 100$	$\frac{Ec-O}{O}\times 100$	$\frac{Nd-O}{O}\times 100$	$\frac{RRH-O}{O}\times 100$
1	0	0	0	0
2	0	0	0	0
3	0	0	0	2.3
4	4.0	0	0	0
5	3.5	0	3.5	0
6	5.3	2.4	5.3	2.4
7	6.9	0	3.1	0
8	7.7	1.4	0.2	0
9	7.0	0.4	0.6	0
10	6.3	0	0.7	0

Fig 1: Comparison of Heuristic Performance using 55-node Network



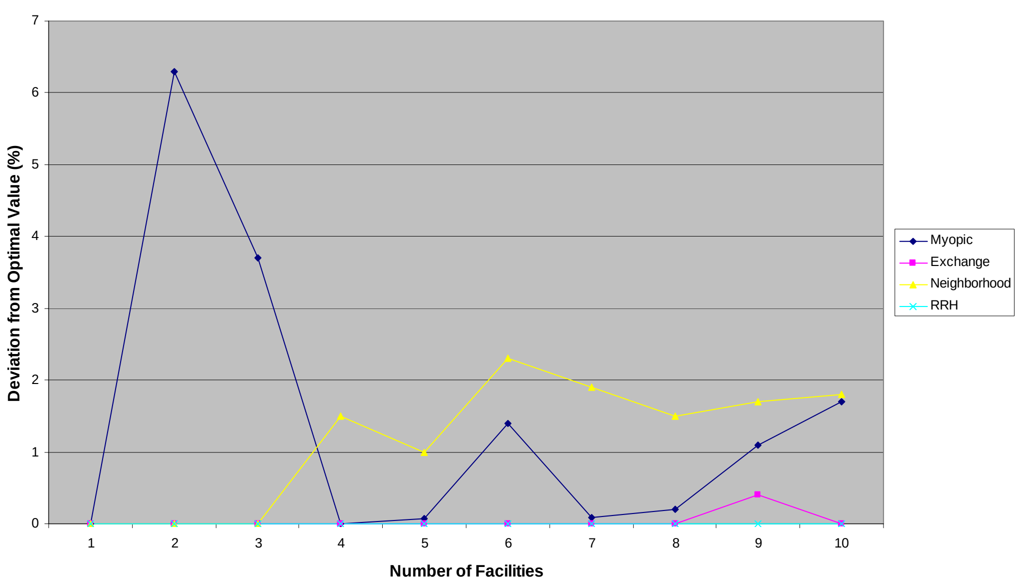
From Table 5 and Figure 1 in terms of quality of solution, the performance measured in terms of the number of optimal solution given is ranked as (from the best to the worst): *RRH*, Exchange Heuristic, Neighborhood Search Heuristic and Myopic Heuristic. However, the new heuristic *RRH* performs better in the location of all facilities with the exception of the location three and six facilities.

Table 6 and Figure 2 shows that when the new and existing heuristics are applied to the test data *RRH* gives optimal values in all cases while the exchange heuristic does not give optimal value in the location of nine facilities.

Table 6: Comparison Performance of RRH and Existing Heuristics using 30-node Network

Number of Facilities	$\frac{Mp-O}{O}\times 100$	$\frac{Ec-O}{O}\times 100$	$\frac{Nd-O}{O}\times 100$	$\frac{RRH-O}{O}\times 100$
1	0	0	0	0
2	6.3	0	0	0
3	3.7	0	0	0
4	0	0	1.5	0
5	0.07	0	1.0	0
6	1.4	0	2.3	0
7	0.09	0	1.9	0
8	0.2	0	1.5	0
9	1.1	0.4	1.7	0
10	1.7	0	1.8	0

Figure 2 : Comparison of Heuristic Performance using 30-node Network



6. Conclusion

In this paper, we introduced three new heuristic methods to locate emergency facilities efficiently and effectively. These heuristics based on the *p*-median problem was tested using 400 random data and compared with well-known existing primary heuristics for the *p*-median problem using the 400 random data and two (55-node and 30-node) literature problems.

The performance of our new heuristics compared with optimal and compared with existing heuristics gives good results in the quality of solution. The best heuristic among the three is less than 1% of the optimal, and, when compared with other heuristics it performs better than the other heuristics. The optimal or the near optimal values of our best new heuristic as compared with the existing suggests that it is efficient and effective to locate the emergency facilities using the new heuristic *(RRH)*.

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