Partner or Rival: Entry deterrence with multi-markets

Pak Yee Lee
Hong Kong Baptist University
Department of Economics
Kowloon, Hong Kong

March 14, 2016

Abstract

This paper examines the incentives for predatory pricing within multi-markets. The model considers an incumbent monopolist facing the threat of entry in one of the markets where an entrant and an incumbent compete in price. It was shown that an incumbent may be able to defend its monopoly position even when there is a cost disadvantage and it produces an inferior quality. I have also provided conditions under which the incumbent accommodates entry. The accommodation conditions require that (1) the quality of the entrant's product be sufficiently high, or (2) the entrant has a sufficiently low marginal cost. A surprising result of the analysis is that forbidding firms to use predatory pricing may reduce welfare.

1 Introduction

Over the past three decades, high-technology consumer electronics have become ubiquitous and essential in daily life. The success of an electronics product does not rest only on the product itself, but also upon the support of complementary goods. For example, the existence of App Store helps Apple maintain its dominance over the media player market. The ability to run third party's software makes IPod not only a media player, but also a portable computer platform, and hence allows Apple to compete in the personal digital assistant market. On the other hand, the failure of Sega as a producer of home TV-game consoles illustrates the importance of complementary products. In 1994, by launching the ultra high-tech game console Saturn$^{\mathrm{TM}}$, Sega, the second largest game console producer at the time, posed a difficult question to the major game developers in the market. It was very costly for developers to create a game software to fit the high hardware standard of the Saturn$^{\mathrm{TM}}$. Game developers were therefore reluctant to enter the market, contributing to the final failure of the console. As complementary products are crucial to the survival of the main "complemented" product, it is important to study entry in the complementary products markets and evaluate the different strategies available to an incumbent producer of the complemented product. What strategy will the incumbent in a market use to leverage its power into a complementary market?

The usual response to this question in the literature is tying or bundling (Choi and Stefanadis [2006], Kovac [2005], Nalebuff [2004] and Whinston [1990]). According to the literature, tying provides a mechanism whereby a firm with monopoly power in one market can use the leverage provided by this power to foreclose sales in, and thereby monopolize, a second market (Whinston [1990]). An incumbent with multiple products can always bundle its products and sell at a lower price. Is tying the only mechanism for an incumbent to deter entry?

Predatory pricing can be one of the options for a firm to protect its monopoly position in its complementary market. It is the practice of a firm to sell its products or services at a low price, intending to drive rivals out of the market, or to create barriers to entry for potential rivals. Nowadays, pricing below marginal cost is regarded as predatory and it is banned by antitrust law.

We show that an incumbent who has a safe monopoly power in a market can deter entry into a complementary market without tying the two products. By lowering the price (below the rival's cost) in the (potentially) competitive complementary market, a more flexible tool, the incumbent can always deter entry and maintain its monopoly power in both markets. The idea is that a firm which is on the outside and intends to enter the complementary market faces a credible threat by the incumbent of the complemented product to lower the price below its cost. This threat of predatory pricing is credible because if the transaction volumes in the two markets are interdependent, the incumbent is able to reduce the price in the complementary market and to raise the price by the same amount in the complemented market. A multi-market structure gives the incumbent the power to transfer its profit from one market to another. Pricing below the rival's cost plays two roles: it creates new demand on the complemented good market, and squeezes the single-product entrant out of the complementary market. Unlike the case of predatory pricing in a market without complementary products, in the multi-market case entry can be deterred at an infinitesimal cost. Firm can simply undercuts its price in one market and raise its price in another market. Losses in one market then can be recouped in the other market. The incumbent firm needs not have either a cost advantage or a quality advantage, so long as (1) it has market power in one of the markets and (2) the transaction volumes in the two markets are positively correlated. Predatory pricing can be used by a firm to extend its monopoly power in one market into another competitive market.

Interestingly, accommodation can also be the optimal strategy; incumbents occasionally give up their monopoly power in one market and allow new firms to enter. In a multi-market setting, demand in one market depends on the demand in the other market and hence the quality of its complementary product. An incumbent would like to have the entrant firm produce a product which could provide strong support to the complemented market. Whether the incumbent deters or accommodates entry depends on the difference between the gain from monopolizing both markets and the gain from having the entrant as its alliance partner in one of the markets. Thus, the threat of predatory pricing is only carried out when the incumbent is unable to take advantage from the entrant. We find that if (1) the entrant's marginal cost is sufficiently low or (2) the entrant produces a sufficiently high quality product, then an incumbent accommodates entry.

We regard our results as helping to explain why in some cases low-cost firms stay out of the market in the absence of entry barrier. For example, Epson exited the market of digital range finder cameras while Leica, the leading lens maker of range finder camera, launched its own digital range finder camera afterwards. Our paper also sheds light on why Apple allows firms to develop iPhone applications which compete with its own.

While predatory pricing has always been viewed as welfare decreasing, it is not necessarily so with multi-market contact. On the negative side, predatory pricing deters competitive pricing and allows the incumbent to squeeze potential entrants out. As a result, there is a risk that the firm in the market produces at a higher cost than the potential entrants. Raising the price in the monopoly market after predatory pricing has no benefit to the consumers. On the positive side, the threat of predatory pricing could save the potential entrant's entry costs (e.g., in R&D effort), and it could stop a low cost entrant from squeezing a high-quality incumbent out of the market.

Our paper relates to a growing body of literature studying entry deterrence with multi-market contact. Of the previous work, Farrell and Katz (2000) and Kovac (2005) are closest to ours. Farrell and Katz (2000) study the innovation incentive facing an incumbent who wants to integrate the supply of the complementary product. They show that given its monopoly power in the provision of one component, the incumbent can always force its rival to charge a lower price. Hence, integration can inefficiently reduce the R&D incentive when an incumbent extracts rent from the independent firm. We extend their work by allowing consumers to have heterogeneous preferences for quality. Unlike Farrell and Katz (2000), our model focus on entry deterrence, rather than on extending monopoly power in the complementary market. We demonstrate that even with the ability to control the market of the complementary product, an incumbent may accommodate a strong entrant. Kovac (2005) constructs a model of bundling with entry deterrence. In his model, a multiproduct firm in one market bundle its products in order to prevent the inferior entrant from entering the complementary market. The paper also demonstrates that monopoly power in the first market is not required. Our paper differs in two fundamental ways. First, rather than bundling as an entry tool, we illustrate the role of predatory pricing as an effective threat. Second, we show that an inferior incumbent also has the ability to defend its monopoly position.

Another line of related research considers bundling as a deterrence tool with multi-market contact. Choi and Stefanadis (2001) and (2006) find that bundling can distort the specialization decision of entrants and reduce the entrants' incentive to invest in R&D. Salop and Scheffman (1983), and Krattenmaker and Salop (1986) demonstrate that raising the rival's cost increases the predator's profit and it can be a very effective entry barrier. Nalebuff (2004) shows that bundling in a two-sided market serves as an entry deterrence strategy as well as an effective tool for price discrimination. Carlton and Waldman (2002) show that bundling can be used to deter future entry in a dynamic model.

Finally, our paper connects with the antitrust literature. Areeda and Turner (1975) suggest that pricing below cost can be an abuse of monopoly power when an incumbent produces multiple products and advocate banning pricing below cost. Edlin (2002) further argues that pricing above cost can also be a case of predatory pricing. Evans and Noel (2005) find that pricing below cost is endemic in the video game, PC software and payment card markets, and it is not necessarily an indicator of predatory pricing with two-sided markets. Evans and Schmalensee (2005) argue that price equal to marginal cost is not the appropriate standard with multi-market contact. In this paper, we look formally at the case where predatory pricing results in a welfare loss in one market and a welfare gain in another market. Our analysis delineates when the gain is larger than the loss, and thus when predatory pricing should be allowed.

The paper is organized as follows. Section **??** develops the model. Section 3 solves the pricing subgame with multi-market contact by assuming that incumbent and entrant set the product prices simultaneously. Equilibrium is defined for three cases: (1) when the incumbent has a cost disadvantage, but a quality advantage; (2) when the incumbent has a disadvantage both in cost and quality and (3) when the incumbent has a cost disadvantage only. Section 4 studies the entrant's entry choice in response to the threat of predatory pricing. Section 5 examines antitrust policy and the welfare effects of predatory pricing. Section 6 concludes. Proofs are relegated to the appendix.

2 The Model

are two firms ($i \in \{I, E\}$) and two product markets ($j \in \{A, B\}$) . Firm I is the uncontested monopolist in market B. Initially, firm I is also an incumbent monopolist in market A and firm E is a potential entrant of market A. The two products are perfect complements. One can think of the camera body as the product in market A and the lens as the product in market B. Consumers only receive utility from consuming both products in a fixed proportion and thus there is no stand-alone value to the product in any market. Here we assume the products are consumed in a one-to-one relationship and each consumer buys at most one unit of product from each market. To simplify the exposition, assume the utility of the consumers solely depends on the quality of the product in market A. The quality of product A of firm I and of the other firm (firm E) can take two values, $q^i \in \{q_L, q_H\}$ with $q_H > q_L$. Firm I's quality level is exogenous and publicly known. If entry takes place, then firms compete in quality as well as price.

We capture the incumbent's advantage by postulating that the potential entrant faces a fixed development cost F_E. Even though in some cases a license is essential (for example, a license has to be granted to create games for Nintendo systems), here we assume that no license is required for firm E to introduce a product in market A, and firm E is free to enter the market. Firm i has a constant marginal cost of production c_i in market A, with $q^i \geq c_i \geq 0$.

In this paper, we are particularly interested in understanding how does the nature of multi-market give the right to the incumbent to protects its market even with a cost disadvantage. Therefore, we assume the entrant, firm E has a lower marginal cost of production ($c_I > c_E$). To focus on the entry choice in market A, and without loss of generality, we assume that firm I's marginal cost of production in market B is zero. All costs are publicly known. Equilibrium entry requires firm E to enter if and only if its expected profit exceeds the entry cost.

The timing of the game as follows: First, firm E decides whether to enter market A and then nature determines firm E's quality level in market A (q_H with probability γ and q_L with probability $1-\gamma$). Second, if E has entered, firms compete in price in market A and firm I simultaneously sets the price in market B. If firm E stays out, firm I sets prices in both markets.

2.1 The Demand Structure

A continuum of potential consumers is differentiated by a parameter θ which is assumed to be uniformly distributed on the interval $(0, 1]$. The parameter θ can be interpreted as the intensity of preference for quality of the product in market A (or the marginal utility of quality). A type θ consumer decides whether to buy the product from firm I or firm E (if firm E has entered) or not to buy in market A; the consumer also decides whether to buy the product from firm I in market B. Let k_j^θ be the purchase decision of a type θ consumer in market j, $k_A^\theta \in \{I, E, N\}$ and $k_B^\theta \in \{I, N\}$. To capture the complementarity of products in the simplest possible way, we assume that consumers only benefit from consuming both products. The utility U_θ of a consumer type θ is defined as follows:

$$U_\theta = \begin{cases} \theta q_I - p_I - p_B & \text{if } k_A^\theta = I \text{ and } k_B^\theta = I \\ \theta q_E - p_E - p_B & \text{if } k_A^\theta = E \text{ and } k_B^\theta = I \\ -p_B & \text{if } k_A^\theta = N \text{ and } k_B^\theta = I \\ -p_I & \text{if } k_A^\theta = I \text{ and } k_B^\theta = N \\ -p_E & \text{if } k_A^\theta = E \text{ and } k_B^\theta = N \\ 0 & \text{if } k_A^\theta = N \text{ and } k_B^\theta = N \end{cases}$$

where p_i is the price of the product in market A from firm i and p_B is the price of the product in market B. Each consumer buys the product combination which provides him with the highest utility and buys nothing if the utility derived from the products does not cover the total price of the products.

The quality of the product in market A plays a critical role on the demand for the product in market B. The higher the quality of the product, the higher the utility of consumers from consuming both products. Quality in market A positively influences the consumers' willingness to pay for the product in market B. Thus, both firms benefit from a high quality product in market A. If firm I is the low-quality firm and it can always sell its product at a very low price (or negative price) in market A, then firm B may be deterred from entering and introducing the high quality. In such a case firm I will extract profit solely from market B. We will allow firm I to sell the market A product at a negative price. This may be profitable because the incumbent may recoup the loss with a higher price in market B. Coupons and market B discounts after a purchase in market A are common examples of zero or negative prices that are observed in practice.

Let $\theta_{N_{IE}}$ be the consumer that, after buying in market B, is indifferent between buying from I and E in market A. It must be

$$\theta_{N_{IE}} q_I - p_I - p_B = \theta_{N_{IE}} q_E - p_E - p_B$$

and hence

$$\theta_{N_{IE}} = \frac{p_I - p_E}{q_I - q_E}.$$

Let θ_{N_I} be the consumer that is indifferent between buying from I in both markets and not buying any product. It is

$$\theta_{N_I} = \frac{p_I + p_B}{q_I}.$$

Let θ_{N_E} be the consumer that is indifferent between not buying any product and buying from E in market A (and I in market B). It is

$$\theta_{NE} = \frac{p_E + p_B}{q_E}.$$

Thus, if $q_I > q_E$, all consumers of type $\theta > \max\{\theta_{N_{IE}},\ \theta_{N_I}\}$ buy both products from firm I, while all consumers of types $\theta < \theta_{N_{IE}}$ and $\theta > \theta_{N_E}$ buy from firm E in market A. Thus, the demand for good B is $Q_B = \max\{0, 1 - \max\{\theta_{N_{IE}}, \theta_{N_I}\}\} + \max\{0, \theta_{N_{IE}} - \theta_{NE}\}$.

If, on the other hand, $q_I < q_E$ then all consumer types $\theta < \theta_{N_{IE}}$ and $\theta > \theta_{N_I}$ buy both products from firm I, while all consumers $\theta > \max\{\theta_{N_{IE}}, \theta_{N_E}\}$ buy from firm E in market A. Thus, the demand for good B is $Q_B = \max\{0, \theta_{N_{IE}} - \theta_{N_I}\} + \max\{0, 1 - \max\{\theta_{N_{IE}}, \theta_{NE}\}\}$.[1] The following graphs show all the possible demand configurations for good B.

We solve the game by backward induction, considering first the equilibrium in the pricing game and then the entry decision of firm E.

3 Equilibrium in the Pricing Subgame

In this section, we consider the case in which firm E has entered market A. There are three possible cases: (1) the incumbent offers a high-quality product and the entrant enters with low-quality, i.e. $q_I = q_H > q_E = q_L$, (2) the incumbent offers a low-quality product and the entrant enters with high-quality, i.e. $q_I = q_L < q_E = q_H$ and (3) the incumbent and the entrant offer the same quality, i.e. $q_I = q_E = q_L$ or $q_I = q_E = q_H$. In the following subgame, there exists an equilibrium which firm I always undercuts the price, however, both firm I and firm E receive a higher profit if firm I agrees to leave the market and firm E agrees to take the market when the the marginal cost of firm I is above the threshold values.[2] For the analysis that follows, we assume that if there are two possible equilibria, the Pareto dominant equilibrium prevails. In this equilibrium, firm I does not always price below the entrant's marginal cost.[3]

[1] $\theta_{N_{IE}} > \theta_{N_E}$ implies $\theta_{N_{IE}} > \theta_{N_I}$

[2] $\pi_I^*(p_I^*,\ p_E^*,\ p_B^*) = \frac{(q_E - c_E)^2}{9q_E} \geq \pi_I^*(p_I^{**}, p_E^{**}, p_B^{**}) = \frac{(q_I - c_I)^2}{4q_I}$ when c_I is above the threshold values $(c_1,\ c_2,\ c_3)$.

[3] On the other hand, if the Pareto-inferior equilibrium is always played in the pricing game, then the incumbent always prices below the entrant's marginal cost and takes over

We begin by considering the first case, $q_I = q_H > q_E = q_L$. There are two different types of subgame perfect equilibria. In the first type of equilibrium, the entrant does not sell (i.e., it sells a zero quantity) in market A, because the incumbent's strategy is to engage in a price war after entry. In the second type of equilibrium, the incumbent accommodates entry; the incumbent gives up market A and the entrant supplies the entire market. This second type of equilibrium only exists if the entrant has a cost advantage in market A.

Before formalizing this result in the next proposition, define:

$$c_1 = q_H - \frac{2}{3}(q_L - c_E)\sqrt{\frac{q_H}{q_L}}.$$

Proposition 1 *Suppose firm E has entered market A and $q_I = q_H > q_E = q_L$. There are two possibilities: (1) If $c_I \leq c_1$, there exists a subgame perfect equilibrium outcome of the pricing game in which firm I prices below firm E's marginal cost in market A (predatory pricing equilibrium), (2) If $c_I > c_1$, then there exist a subgame perfect equilibrium outcome in which firm I sells zero quantity in market A. (accommodation equilibrium)*

Proof. See the Appendix.

A few remarks are worth making here. First, whether the incumbent engages in a price war after entry depends on the production costs of incumbent and entrant. There is a threshold level of the incumbent marginal cost, c_1, above which the incumbent accommodates and below which it deters entry.

Second, as the entrant's marginal cost c_E gets smaller, the needed threshold value of c_I gets smaller. When $q_I > q_E$, giving up market A is profitable only when the entrant produces at a sufficiently lower cost than the incumbent. In such a case, the low cost entrant with a low quality product is able to sell to a larger number of consumers than the high cost incumbent with a high quality product. By leaving market A to the entrant, the incumbent is able to induce more consumers to get on board and buy its product in market B and hence the profit of the incumbent in that market increases. On the contrary, predatory pricing is very costly to the incumbent when the entrant's cost is very low. Returning to the camera example, the incumbent who has introduced a camera body with a higher image resolution and more features is willing to exit the camera body market only if the entrant sells

---

the whole market, regardless of the entrant's cost level and quality type. In this case, it is optimal for the entrant to stay out of the market.

the camera body at a much lower price. The camera body with more features reaches fewer consumers, as some features of the camera body are not of much value to the general public and are only useful to the professional photographers. Many consumers do not value the quality difference much. If the incumbent has a high production cost for the extra camera body features, then it would find profitable to lock-in more consumers to its camera lens by letting the entrant sell a cheaper camera body with fewer features.

Third, for $c_E = 0$ there exist cost levels of the incumbent ($c_I \leq q_H - \frac{2}{3}\sqrt{q_H q_L}$) for which I deters entry of the potential entrant. In contrast, for $c_I = 0$ firm I always deter entry and the second type of equilibrium never exists. Therefore, market power in market B may prevent some lower cost entrant from entering market A.

Now we turn to the case where the entrant has a quality advantage over the product provided by the incumbent. Suppose the entrant has developed breakthrough advances in the existing imaging technology and is able to produce a camera body with advanced image technologies that make a difference to consumers. Consumers are now willing to spend more on their camera (both the lens and the camera body) with the latest development. This quality advantage increases the chances that the entrant will enter market A. Before presenting the proposition with this result, define c_2 as

$$c_2 = q_L - \frac{2\,(q_H - c_E)}{(3q_H - q_L)} q_L.$$

Proposition 2 *Suppose firm E has entered market A. Let $q_I = q_L < q_E = q_H$. There are two possibilities: (1) If $c_I \leq c_2$, there exists a subgame perfect equilibrium outcome of the pricing game in which firm I shares market A with firm E (predatory pricing and sharing equilibrium), (2) If $c_I > c_2$, then there exists a subgame perfect equilibrium outcome in which firm I sells zero quantity in market A. (accommodation equilibrium)*

Proof. See the Appendix.

The cost advantage gives the entrant the ability to undercut the incumbent and attract even those consumers who have little value for the quality differentials of the products. The quality advantage further strengthens the competitiveness of the entrant by increasing the consumers willingness to pay for the products in both markets. This allows the incumbent to raise the price in market B. Therefore, in order to lock more consumers in the market and generate higher profits, the incumbent always has an incentive to accommodate entry. When the incumbent has a sufficiently low marginal cost of production $c_I \leq c_2$, it has an incentive to share the market with the entrant. The incumbent with a low quality product enlarges its consumer base in market B by serving the consumers in market A who do not value quality much, while the entrant with a high quality product increase the consumers' willingness to pay on the other product. Keeping a positive market share becomes costly to the incumbent if $c_I > c_2$. In this case, letting the entrant serve the entire market is preferable. When the incumbent's cost increases, the loss from sharing the market by selling the product below cost (predatory pricing) will be higher and outweighs the gain from a larger consumer base. Referring again to our example, the release of a new generation camera body by the entrant prompts consumers to set aside more of their money for the camera body and lens. It provides room for the lens price to go higher and hence generates more profit for the incumbent through greater lens sales. If the incumbent has a sufficiently low marginal cost, then it will reach more consumers by producing the old generation camera body and sharing the market with the entrant. Otherwise, the incumbent will leave the camera body market to the entrant. Either way, a low cost entrant with higher technology is always able to enter and is viewed by the incumbent as a partner.

Note that if the entrant has zero marginal cost, $c_E = 0$ then $c_2 = (q_H - q_L)q_L/(3q_H - q_L)$; thus, there exist some low cost incumbents that share the market with the entrant. In other words, an entrant with a quality advantage and a zero production cost is not able to take over the entire complementary market if the incumbent's cost is low.

We now consider the case in which the two firms have the same quality level $q \in \{q_H, q_L\}$, that is the entrant does not add any new feature to the camera body or the new features are not compelling and do not add any value to the existing camera body. Firms then purely compete in price and cost determines firms' survival. The following threshold value of c_I will be used in the next proposition:

$$\begin{aligned} c_3(q) &= \frac{q + 2c_E}{3} \\ &= q - \frac{2}{3}(q - c_E). \end{aligned}$$

Proposition 3 *Suppose firm E has entered market A. Let* $q_I = q_E = q \in \{q_H, q_L\}$. *(1) If* $c_I \leq c_3(q)$*, or equivalently* $q \geq 3c_I - 2c_E$*, there exists a subgame perfect equilibrium outcome of the pricing game in which firm I sets its price below firm E's marginal cost in market A (predatory pricing equilibrium), (2) If* $c_I > c_3(q)$*, or equivalently* $q < 3c_I - 2c_E$*, then there exists a subgame perfect equilibrium outcome in which firm I sells zero quantity in market A. (accommodation equilibrium)*

Proof. See the Appendix.

When incumbent and entrant have the same quality, if the quality level is sufficiently high (above $q^* = 3c_I - 2c_E$), then in the equilibrium of the pricing subgame the entrant does not sell and the incumbent sells to the entire market. On the other hand, if the quality level is below q^*, then the incumbent gives up market A and prefers not to produce in that market. The cost of taking over market A by setting p_I lower than p_E while keeping $p_I + p_B$ optimal is high when $p_E < c_I$. In such a case, it is profitable to let the entrant to serve the whole complementary market rather than to suffer a loss from serving the market by itself. This can only occur if the entrant has a sufficiently large cost advantage ($3c_I \geq 2c_E$). The incumbent free rides on the low cost entrant; the entrant charges a price lower than the incumbent's marginal cost and as a result the incumbent enjoys a larger base of consumers in market B. Note that the range of cost parameters under which the incumbent accommodates entry is larger if both firms have a low rather than a high-quality product. This can be illustrated with the telecommunication industry in the UK. Mobile phone are mainly sold by the telecommunication companies at a low price even without bundling the mobile phone with the telecommunication service. The telecommunication companies have an incentive to lower the price of the mobile phone and induce more consumers to pay for telecommunication services. Such predatory pricing also leads the mobile phone manufacturers to sell the mobile phones to the telecommunication companies rather than selling them directly to the customers. (Phones on the Nokia online website sold at a price two to three times higher than the price at the telecommunication stores.) The following graphs show the timeline of the game and the result.

Table 1 shows the equilibria in each pricing subgame.

Equilibrium in the Pricing Subgame			
q_I	q_H	q_L	$q \in \{q_H, q_L\}$
q_E	q_L	q_H	$q \in \{q_H, q_L\}$
Predatory pricing equilibrium ($p_I^* < c_I$)			
Condition	Always hold		
p_E^*	$\geq q_L - (q_H - c_I)\sqrt{\frac{q_L}{q_H}}$	$\geq q_H - (q_L - c_I)\sqrt{\frac{q_H}{q_L}}$	$\geq c_I$
p_I^*	$\leq c_E$	$\leq c_E - (q_H - q_L)$	$\leq c_E$
p_B^*	$\frac{(q_H+c_I)}{2} - p_I^*$	$\frac{(q_L+c_I)}{2} - p_I^*$	$\frac{(q+c_I)}{2} - p_I^*$
Q_E^*	0	0	0
Q_I^*	$\frac{(q_H-c_I)}{2q_H}$	$\frac{(q_L-c_I)}{2q_L}$	$\frac{(q-c_I)}{2q}$
Q_B^*	$\frac{(q_H-c_I)}{2q_H}$	$\frac{(q_L-c_I)}{2q_L}$	$\frac{(q-c_I)}{2q}$
Predatory pricing and sharing equilibrium			
Condition		$c_I > q_L - \frac{2(q_H-c_E)}{(3q_H-q_L)} q_L$	
p_E^*		$\frac{(q_H-q_L+2c_E+c_I)}{3}$	
p_I^*		$\frac{(q_L-q_H+c_E+2c_I)}{3}$	
p_B^*		$\frac{(2q_H+q_L-2c_E-c_I)}{6}$	
Q_E^*		$\frac{(q_H-q_L+c_I-c_E)}{3(q_H-q_L)}$	
Q_I^*		$\frac{[(q_H-q_L)(q_L-c_I)-2(c_Iq_H-c_Eq_L)]}{6q_L(q_H-q_L)}$	
		$\frac{(q_L-c_I)}{2q_L}$	
Accommodation equilibrium			
Condition	$c_I \leq q_H - \frac{2(q_L-c_E)}{3}\sqrt{\frac{q_H}{q_L}}$	$c_I \leq q_L - \frac{2(q_H-c_E)}{(3q_H-q_L)} q_L$	$c_I \geq \frac{q+2c_E}{3}$
p_E^*	$\frac{(q_L+2c_E)}{3}$	$\frac{(q_H+2c_E)}{3}$	$\frac{(q+2c_E)}{3}$
p_I^*	$> \frac{(3q_H-2q_L+2c_E)}{3}$	$> \frac{(q_H+2c_E)}{3}$	$> \frac{(q+2c_E)}{3}$
p_B^*	$\frac{(q_L-c_E)}{3}$	$\frac{(q_H-c_E)}{3}$	$\frac{(q-c_E)}{3}$
Q_E^*	$\frac{(q_L-c_E)}{3q_L}$	$\frac{(q_H-c_E)}{3q_H}$	$\frac{(q-c_E)}{3q}$
Q_I^*	0	0	0
Q_B^*	$\frac{(q_L-c_E)}{3q_L}$	$\frac{(q_H-c_E)}{3q_H}$	$\frac{(q-c_E)}{3q}$

Table2.1: Equilibria in the pricing subgame.

The cost and quality differences between firms determine whether it is optimal for the incumbent to engage in predatory pricing. When the potential entrant is able to improve significantly on the incumbent's complementary product, either with a higher quality, or a lower cost product, the incumbent will profit from allowing entry. By accommodating entry and forcing the entrant to sell at a low price in the complementary product market, the incumbent reaps the benefit in the complemented good market, where it remains a monopolist, by charging a higher price. Charging a higher price is possible because of the increase in demand that follows from the technology improvement introduced by the entrant. However, the fact that the entrant has a lower production cost or introduces a higher quality does not automatically imply that it is optimal for the incumbent to accommodate entry. Accommodation is profitable to an incumbent only if the gain from the demand increase in the complemented good market outweighs the loss from leaving all or part of the complemented good market to the entrant.

4 Market Entry Game

In this section, we analyze the entrant's incentive to enter market A. Firm E earns a positive profit in market A when firm I doesn't undercut its price after entry; there are three different cases when this happens. If it produces high quality, firm E obtains a payoff $\pi_{E_M}(q_H)$ if firm I does not sell in market A and it obtains a payoff $\pi_{E_D}(q_H)$ if it shares the market with firm I. When firm E has a low quality product, then it obtains a payoff $\pi_E(q_L)$ when firm I does not produce in market A. On the other hand, if firm I plays the predatory pricing equilibrium, firm E receives a zero payoff.

$$\pi_{E_M}(q_H) = \frac{(q_H - c_E)^2}{9q_H}.$$

$$\pi_{E_D}(q_H) = \frac{(q_H - q_L + c_I - c_E)^2}{9\,(q_H - q_L)}.$$

$$\pi_E(q_L) = \frac{(q_L - c_E)^2}{9q_L}.$$

The next proposition describe the equilibrium in the subgame after firm E has entered market A. Define the following values. The following threshold values represent the minimum probability of being a high-quality firm required for firm E to earn a non negative expected profit regarding different cost levels.

$$\gamma^* = \frac{F_E}{\pi_{E_M}(q_H)}$$

$$\gamma^{**} = \frac{F_E}{\pi_{E_D}(q_H)}$$

$$\gamma^{***} = \frac{F_E - \pi_E(q_L)}{\pi_{E_M}(q_H) - \pi_E(q_L)}$$

Proposition 4 *Suppose predatory pricing (i.e., price below marginal cost) is possible. There exists a subgame perfect equilibrium outcome of the entry game in which firm E enters market A and firm I introduces a high quality product in the following scenarios: (i) When firm I has a marginal cost* $c_I \in [c_3(q_H), c_1]$, *if* $\gamma \geq \gamma^*$; *and (ii) when firm I has a marginal cost* $c_I > c_1$, *if* $\gamma \geq \gamma^{***}$. *There exists a subgame perfect equilibrium outcome of the entry game in which firm E enters market A and firm I introduces a low quality product in the following scenarios: (i) When firm I has a marginal cost* $c_I \in [c_2, c_3(q_L)]$, *if* $\gamma \geq \gamma^{**}$; *and (ii) when firm I has a marginal cost* $c_I > c_3(q_L)$, *if* $\gamma \geq \gamma^{***}$.

Proof. See the Appendix.

When the incumbent is a high-quality firm with a low cost (below c_2), it finds it profitable to charge a low price for the complementary product in market A. (The incumbent may even offer the product for free, or give a cash rebate to the consumers when they buy the complemented product in market B.) This attracts the largest consumer base in market B (the complemented product market) and since the incumbent is a monopolist in market B, it can extract consumers' surplus from that market. The entrant stays out of market A in this case. As the production cost of the incumbent increases, predatory pricing becomes more costly and the entrant becomes more helpful in boosting the demand in market B by pricing at a relatively low price in market A. Therefore, the entrant is more likely to enter market A. When the incumbent is a low-quality firm, the probability thresholds are different and the entrant is more likely to enter market A, but the general interpretation of the results is similar.

In this paper, we have assumed that the quality level of the entrant is exogenous and independent of R&D (the fixed cost in our model). However, if the strategy of improving the quality level by investing on R&D were available, the entrant would have an incentive to increase its expenditure on R&D. Therefore, the threat of predatory pricing would play two roles, it would drive a relatively high cost entrant out of the market and hence would encourage the entrant to increase the expenditure on R&D, in an attempt to build competitive and develop better quality product.

We have shown how a firm may protect its position in a complementary market by leveraging its monopoly power in the complemented market. Intuitively, a firm can also profitably use predatory pricing to extend its monopoly power from one market to another, provided the following three conditions hold: (1) the two markets are complementary; (2) the firm is in a safe market and (3) it can freely enter another market. The logic is straightforward: the firm can sell its product below cost and drive the incumbent out of the market, while increasing its profit in the safe market because of an increased demand.

5 Welfare Implications

So far, our analysis has focused on the impact of predatory pricing on the entrant's quality choice and on entry deterrence. Now we would like to answer our last question: If predatory pricing erects an entry barrier, should the antitrust authority prohibit it? Under which circumstances is charging a price below marginal cost welfare reducing?

Predatory pricing has an effect on welfare if and only if entry is deterred by the incumbent. Recall that the incumbent only deters entry when its cost is sufficiently low. Therefore, in this section, we only consider the case when the fixed entry cost is sufficiently small ($F_E < \pi_{E_D}(q_H)$, so that $\gamma > \gamma^{**}$) and the incumbent is not willing to leave the entire market to the entrant if predatory pricing is possible (i.e., the case when $c_I < c_3(q_L)$ if the incumbent's product is of low quality, and the case when $c_I < c_1$ if the incumbent's product is of high quality).

Define the following values.

$$c_I^*(q) = q - \frac{2\sqrt{5}}{3\sqrt{3}}(q_L - c_E)\sqrt{\frac{q_H}{q_L}} < c_1$$

$$c_I^{**}(q) = q - 2\sqrt{\frac{2q}{3}\left[\gamma\frac{5(q_H - c_E)^2}{18q_H} + (1-\gamma)\frac{5(q_L - c_E)^2}{18q_L} - F_E\right]}$$

Proposition 5 *If firm I introduces a high quality product, predatory pricing is socially optimal and $W_{pre} > W_{ban}$ in the following scenarios: (i) When* *firm I has a marginal cost,* $c_I \in [c_3(q_H), c_I^*(q_H)]$*; and (ii) when firm I has a marginal cost,* $c_I \leq \min[c_I^{**}(q_H), c_3(q_H)]$*. If firm I introduces a low quality product, predatory pricing is socially optimal and* $W_{pre} > W_{ban}$ *in the following scenarios: (i) When firm I has a marginal cost,* $c_I \in [c_2, c_I^*(q_L)]$*; and (ii) when firm I has a marginal cost,* $c_I \leq \min[c_I^{**}(q_L), c_2]$*.*

Proof. See the Appendix.

In the single market case, an incumbent may set low prices today and sacrifice current profit in order to gain a monopoly profit in the future. This practice improves consumer welfare in the short run, but in the future, supra-competitive pricing reduces welfare. Determining the welfare effect of predatory pricing is more complicated in the context of multi-markets. With uncontested market power in a safe market, the incumbent can always sacrifice profit in the complementary competitive market by pricing below cost, while raising price and profit in the safe market. Pricing low effectively serves as an entry barrier to the potential entrant. Such a practice works only if the markets are (perfect) complementary. Price cutting in the competitive market raises consumer welfare, since the incumbent does not raise the price in the safe market raise by the same amount as the price cut in the competitive market. However, this is not the end of the story. The objective of the price cutting is to deter a potential entrant from entering the competitive market. If some very low cost potential entrants are deterred by the incumbent through price cutting, consumer's welfare may be lower when pricing below marginal cost by the incumbent is allowed. Then, this type of price cutting can be regarded as "predatory" and should be banned by law.

If pricing below cost is prohibited, leaving the market is the only option for an incumbent that faces a lower cost entrant. When products are complementary and consumers value quality differently, production cost will be the sole factor deciding who stays in the market. A high quality incumbent may be driven out of the market for the complementary product by a low quality entrant. Moreover, the entrant has to pay a fixed R&D cost of entry. Thus, if the cost of production of the incumbent is only marginally higher than the cost of the entrant, a law banning pricing below marginal cost may reduce welfare. In such an instance, the antitrust authority should not intervene in the market; pricing below marginal cost should be permitted.

6 Concluding Remarks

Most of the literature on entry deterrence with multi-market contact has focused on bundling. On the contrary, we have argued that an incumbent with multi-market contact can successfully deter potential entry without bundling its products by using predatory pricing. We have shown that entry can be deterred at essentially zero cost in a complementary market when the incumbent is a monopolist in another "complemented" product. The incumbent needs not to have either a cost advantage or a quality advantage in the complementary market. This implies that a low cost entrant may be deterred by a weak incumbent, resulting in a welfare loss. This result explains why strong entrant firms may be unable to enter some markets, like Epson in the rangefinder market. We suggest that the antitrust authority should prohibit such predatory pricing.

However, our model also finds that a law always banning predatory pricing with multi-market contact is not socially optimal. When markets are complementary and pricing below cost has been prohibited by antitrust law, cost becomes the only factor to determine the fate of an entrant. A cost advantage gives the entrant enough power to drive the incumbent out of the market regardless of the incumbent's quality level. Taking quality into account, welfare may be lower with a low cost entrant than with a higher cost incumbent who produces a higher quality. Therefore, we suggest that the quality difference between firms should be taken into account when determining if pricing below cost should be deemed "predatory" and hence should be illegal.

Our analysis shows that, because of predatory pricing, firms never coexist when products are vertically differentiated, markets are complementary and the incumbent has a sufficiently high production cost. In future research, it would be interesting to study predatory pricing in a model of horizontally differentiated goods. We believe that predatory pricing would be less effective. An entrant can do better, and hence enter more easily, if it can horizontally differentiate its product and attract consumers that have preference for its brand. In our model, nature determines the incumbent and entrant's quality levels. It would be interesting to endogenize quality choice. In order to reduce the probability of being deterred by the incumbent, the entrant has an incentive to spend more money on R&D and improve its product quality. The higher the quality the incumbent produces, the higher the entrant's incentive to invest. It is then likely that the incumbent will produce a lower quality product than the entrant. Finally, future research could also attempt to model how predatory pricing affects the incentive to merge.

Appendix

Proof of Proposition 1

1. If $1 > \theta_{N_{IE}} > \theta_{N_E}$ and $\theta_{N_{IE}} > \theta_{N_I}$, then $Q_B = 1 - \theta_{N_E}$, $Q_{A_I} = 1 - \theta_{N_{IE}}$ and $Q_{A_E} = \theta_{N_{IE}} - \theta_{N_E}$Suppose $1 > \theta_{N_{IE}} > \theta_{N_E}$ and $\theta_{N_{IE}} > \theta_{N_I}$ holds, firms maximizes their profits by choosing the optimal price in the markets, p_I, $p_E$and p_B and their profit functions are given as follow:

$$\begin{aligned} \pi_I(q_H) &= (p_I - c_I)(1 - \theta_{N_{IE}}) + p_B(1 - \theta_{N_E}) \\ &= (p_I - c_I)\left(1 - \frac{p_I - p_E}{q_H - q_L}\right) + p_B\left(1 - \frac{p_E + p_B}{q_L}\right). \end{aligned} \tag{1}$$

$$\begin{aligned} \pi_E(q_L) &= (p_E - c_E)(\theta_{N_{IE}} - \theta_{N_E}) \\ &= (p_E - c_E)\left(\frac{p_I - p_E}{q_H - q_L} - \frac{p_E + p_B}{q_L}\right). \end{aligned} \tag{2}$$

Maximizing equation 1 and 2 yield the optimal price in the markets:

$$p_I^* = \frac{3q_L(q_H - q_L) + c_I(q_H + q_L) - 2q_H(c_I + c_E)}{6q_H}.$$

$$p_E^* = \frac{2c_I q_H + c_E q_L}{3q_H}.$$

$$p_B^* = \frac{3q_L q_H - c_I q_L - 2c_E q_H}{6q_H}.$$

This gives $\theta_{N_{IE}} > \theta_{N_E}$ to exist if and only if $\theta_{N_{IE}} < \theta_{N_I}$, which violates the assumption. Thus, this is not the equilibrium.

2. If $\theta_{N_{IE}} > 1$ and $1 > \theta_{N_E}$, then $Q_B = 1 - \theta_{N_E}$, $Q_{A_I} = 0$ and $Q_{A_E} = 1 - \theta_{N_E}$

   When $\theta_{N_{IE}} > 1$ and $1 > \theta_{N_E}$ holds, firms maximizes their profits by choosing the optimal price in the markets, p_E and p_B and their profit functions are given as follow:

$$\begin{aligned} \pi_I(q_H) &= p_B(1 - \theta_{N_E}) \\ &= p_B\left(1 - \frac{p_E + p_B}{q_L}\right). \end{aligned}$$

$$
\begin{aligned}
\pi_E(q_L) &= (p_E - c_E)\left(1 - \theta_{N_E}\right) \\
& (p_E - c_E)\left(1 - \frac{p_E + p_B}{q_L}\right).
\end{aligned}
$$

Maximizing $\pi_{I_1}(q_H)$and $\pi_{E_1}(q_L)$ and yield the optimal price in the markets:

$$
p_E^* = \frac{q_L + c_E - p_B^*}{2} = \frac{q_L + 2c_E}{3}.
$$

$$
p_B^* = \frac{q_L - p_E^*}{2} = \frac{q_L - c_E}{3}.
$$

and this gives firms the profit $\pi_I^*(q_H)$ and $\pi_E^*(q_H)$, where

$$
\pi_I^*(q_H) = \pi_E^*(q_L) = \frac{\left(q_L - p_E^*\right)^2}{4q_L} = \frac{\left(q_L - c_E\right)^2}{9q_L}.
$$

For $\theta_{N_{IE}} > 1$ to hold, $p_I^* > (q_H - q_L) + p_E^*$

3. If $1 > \theta_{N_I} > \theta_{N_{IE}}$ and $\theta_{N_E} > \theta_{N_{IE}}$, then $Q_B = 1 - \theta_{N_I}$, $Q_{A_I} = 1 - \theta_{N_I}$ and $Q_{A_E} = 0$

   Consider configuration 3. For $Q_{A_E} = 0$, firm I must sells its product in market A at a price, $p_I^* \leq c_E$ (that is the minimum price for firm E to make a non negative profit). In this case, firm I maximizes its profit by choosing the optimal price in market B, p_B and its profit function is given as follow:

$$
\begin{aligned}
\pi_I(q_H) &= (p_I + p_B - c_I)\left(1 - \theta_{N_I}\right) \\
&= (p_I + p_B - c_I)\left(1 - \frac{p_I + p_B}{q_H}\right).
\end{aligned} \tag{3}
$$

Maximizing equation 3 yields the optimal price in market B:

$$
p_I^* + p_B^* = \frac{q_H + c_I}{2}.
$$

and it gives firm I an optimal profit $\pi_I^*(q_H)$, where

$$\pi_I^*(q_H) = \frac{(q_H - c_I)^2}{4q_H}.$$

In this case, $1 > \theta_{N_I} > \theta_{N_{IE}}$ and $\theta_{N_E} > \theta_{N_{IE}}$ holds.

Now there are two sets of equilibrium candidates (configuration 2 and 3). We first consider the first set of equilibrium candidates (i.e. configuration 3), In this configuration, Firm I undercuts its price to firm E's marginal cost in market A and profit for firm I is $(q_H - c_I)^2/4q_H$. If firm I deviates and exits market A (equivalently setting its price above p_E^* in market A and this is the best possible deviation), then firm E chooses its monopoly price in the market to maximizes its profit and firm I obtains a profit equal to $(q_L - p_E^*)^2/4q_L$. This deviation is profitable if and only if $(q_L - p_E^*)^2/4q_L > (q_H - c_I)^2/4q_H$. In other words, firm I has no profitable deviation if $p_E^* \geq q_L - (q_H - c_I)\sqrt{q_L/q_H}$. Given $p_I^* \leq c_E$, entrant has no profitable deviation in this configuration.

We next consider the second set of equilibrium candidates (configuration 2). Firm I leaves market A for firm E and each firm receives a profit for$(q_L - c_E)^2/9q_L$. If firm I deviates and undercuts its price in market A (that is setting its price below c_E) in market A and this is the best possible deviation, then firm I chooses its price in market B as a best response to p_E^* and firm I obtains a profit equal to $(q_H - c_I)^2/4q_H$. This deviation is not profitable if $c_I > c_1$. There is no profitable deviation for firm E.

We now establish that there are two equilibria. If $c_I \leq c_1$, then there is only an equilibrium exists, where

$$\begin{aligned} p_I^* &\leq c_E, \\ p_E^* &\geq q_L - (q_H - c_I)\sqrt{q_L/q_H} \text{ and} \\ p_I^* + p_B^* &= (q_H + c_I)/2 \end{aligned}$$

On the other hand, if $c_I > c_1$, there are two subgame perfect equilibria exist, where

$$\begin{aligned} p_I^* &\leq c_E, \\ p_E^* &\geq q_L - (q_H - c_I)\sqrt{q_L/q_H} \text{ and} \\ p_I^* + p_B^* &= (q_H + c_I)/2 \end{aligned}$$

$$
\begin{aligned}
p_I^{**} &> (3q_H - 2q_L + 2c_E)/3, \\
p_E^{**} &= (q_L + 2c_E)/3 \text{ and} \\
p_B^{**} &= (q_L - c_E)/3
\end{aligned}
$$

We assume only the second type of equilibrium will be played. This makes sense. Both firm I and firm E receive a higher profit if firm I agrees to leave the market and firm E agrees to take the market. The second type of equilibrium is Pareto optimal given that $\pi_I^*(p_I^*,\ p_E^*,\ p_B^*) = \frac{(q_L - c_E)^2}{9q_L} \geq \pi_I^*(p_I^{**}, p_E^{**}, p_B^{**}) = \frac{(q_H - c_I)^2}{4q_H}$ when $c_I > c_1$. ■

Proof of Proposition 2

Consider firm E introduces a product with better quality, $q_E > q_I$. If consumers have bought product from firm B, all consumers of type $\theta > \max\{\theta_{M_{IE}}\ ,\ \theta_{M_E}\}$.buys the product from firm E in market A satisfied the following conditions:

1 The utility one gets from the product of firm E is higher than that from the product of firm I, i.e. $\theta_{M_{IE}} = \frac{p_E - p_I}{(q_E - q_I)}$

2 The utility one gets from the product of firm E is higher than the utility from buying nothing, i.e. $\theta_{M_E} = \frac{p_E + p_B}{q_E}$

Hence for all consumers of type $\theta < \theta_{M_{IE}}$ and $\theta > \theta_{M_I}$ buys the product from firm I in market A, where $\theta_{M_I} = \frac{p_I + p_B}{q_I}$[4]. We consider the optimal prices in each of the possible demand functions for product in market B which listed below:

$$
Q_B = \begin{cases}
1 - \theta_{M_I} & \text{if } 1 \geq \theta_{M_{IE}} > \theta_{M_E} \text{ and } \theta_{M_{IE}} > \theta_{M_I} \\
1 - \theta_{M_I} & \text{if } \theta_{M_{IE}} \geq 1 \text{ and } 1 > \theta_{M_I} \\
1 - \theta_{M_E} & \text{if } 1 > \theta_{M_E} > \theta_{M_{IE}} \text{ and } \theta_{M_I} > \theta_{M_{IE}} \\
0 & \text{otherwise}
\end{cases}
$$

1. If $1 \geq \theta_{M_{IE}} > \theta_{M_E}$ and $\theta_{M_{IE}} > \theta_{M_I}$, then $Q_B = 1 - \theta_{M_I}$, $Q_{A_I} = \theta_{M_{IE}} - \theta_{M_I}$ and $Q_{A_E} = 1 - \theta_{M_{IE}}$

   Suppose $1 > \theta_{M_{IE}} > \theta_{M_E}$ and $\theta_{M_{IE}} > \theta_{M_I}$ holds, firms maximize their profits by choosing the optimal price in the markets, $p_{I,}$ $p_E$and p_B and

[4] $\theta_{M_{IE}} > \theta_{M_E}$ implies $\theta_{M_{IE}} > \theta_{M_I}$

their profit functions are given as follow:

$$\begin{aligned}
\pi_I(q_L) &= (p_I - c_I)\,(\theta_{M_{IE}} - \theta_{M_I}) + p_B\,(1 - \theta_{M_I}) \\
&= (p_I - c_I)\left(\frac{p_E - p_I}{q_H - q_L} - \frac{p_I + p_B}{q_L}\right) + p_B\left(1 - \frac{p_I + p_B}{q_L}\right).
\end{aligned} \tag{4}$$

$$\begin{aligned}
\pi_E(q_H) &= (p_E - c_E)\,(1 - \theta_{M_{IE}}) \\
& (p_E - c_E)\left(1 - \frac{p_E - p_I}{q_H - q_L}\right).
\end{aligned} \tag{5}$$

Maximizing equation 4 and 5 yield the optimal price in the markets:

$$\begin{aligned}
p_I^* &= \frac{q_L - q_H + c_I + p_E^*}{2} \\
&= \frac{q_L - q_H + c_E + 2c_I}{3}.
\end{aligned}$$

$$\begin{aligned}
p_E^* &= \frac{q_H - q_L + c_E + p_I^*}{2} \\
&= \frac{q_H - q_L + 2c_E + c_I}{3}.
\end{aligned}$$

$$\begin{aligned}
p_B^* &= \frac{q_H - p_E^*}{2} \\
&= \frac{2q_H + q_L - 2c_E - c_I}{6}.
\end{aligned}$$

and this gives firms the profit $\pi_I^*(q_L)$ and $\pi_E^*(q_H)$, where

$$\begin{aligned}
\pi_I^*(q_L) &= \frac{(q_L - c_I)^2}{4q_L} + \frac{(q_H - q_L + c_I - p_E^*)^2}{4(q_H - q_L)} \\
&= \frac{(q_L - c_I)^2}{4q_L} + \frac{(q_H - q_L + c_I - c_E)^2}{9(q_H - q_L)}
\end{aligned}$$

$$\pi_E^*(q_H) = \frac{(q_H - q_L - c_E + p_I^*)^2}{4(q_H - q_L)} = \frac{(q_H - q_L + c_I - c_E)^2}{9(q_H - q_L)}.$$

$\theta_{M_{IE}} > \theta_{M_E}$ and $\theta_{M_{IE}} > \theta_{M_I}$ only hold if and only if the following condition is satisfied:

$$(q_H - q_L)(q_L - c_I) - 2(c_I q_H - c_E q_L) > 0$$

or, equivalently, if and only if

$$c_I < q_L - \frac{2(q_H - c_E)q_L}{(3q_H - q_L)} = c_2$$

2. If $\theta_{M_{IE}} \geq 1$ and $1 > \theta_{M_I}$, then $Q_B = 1 - \theta_{M_I}$, $Q_{A_I} = 1 - \theta_{M_I}$ and $Q_{A_E} = 0$

   Consider both configuration 3. For $Q_{A_E} = 0$, firm I must sells its product in market A at a price, $p_I < p_E - (q_H - q_L) = c_E - (q_H - q_L)$ (all consumers are indifferent between buying from firm E and firm I at this price and firm E makes a non negative profit) and it implies $\theta_{M_{IE}} = 1$ and firm I maximizes its profit by choosing the optimal price in market B, p_B and its profit function is given as follow:

$$\pi_I(q_L) = (p_I + p_B - c_I)(1 - \theta_{M_I}) = (p_I + p_B - c_I)\left(1 - \frac{p_I + p_B}{q_L}\right) \tag{6}$$

   Maximizing equation 6 yields the optimal price in market B:

$$p_I^* + p_B^* = \frac{q_L + c_I}{2}.$$

   and it gives firm I an optimal profit $\pi_I^*(q_H)$, where

$$\pi_I^*(q_L) = \frac{(q_L - c_I)^2}{4q_L}.$$

   In this case, $\theta_{M_{IE}} \geq 1$ and $1 > \theta_{M_I}$ holds.

3. If $1 > \theta_{M_E} > \theta_{M_{IE}}$ and $\theta_{M_I} > \theta_{M_{IE}}$, then $Q_B = 1 - \theta_{M_E}$, $Q_{A_I} = 0$ and $Q_{A_E} = 1 - \theta_{M_E}$

   When $1 > \theta_{M_E} > \theta_{M_{IE}}$ and $\theta_{M_I} > \theta_{M_{IE}}$ holds, firms maximizes their profits by choosing the optimal price in the markets, p_E and p_B and their profit functions are given as follow:

$$\begin{aligned}\pi_I(q_L) &= p_B\left(1 - \theta_{M_E}\right) \\ &= p_B\left(1 - \frac{p_E + p_B}{q_H}\right).\end{aligned}$$

$$\begin{aligned}\pi_E(q_H) &= \left(p_E - c_E\right)\left(1 - \theta_{M_E}\right) \\ &\left(p_E - c_E\right)\left(1 - \frac{p_E + p_B}{q_H}\right).\end{aligned}$$

Maximizing $\pi_I(q_L)$and $\pi_E(q_H)$ and yield the optimal price in the markets:

$$\begin{aligned}p_E^* &= \frac{q_H + c_E - p_B^*}{2} \\ &= \frac{q_H + 2c_E}{3}.\end{aligned}$$

$$\begin{aligned}p_B^* &= \frac{q_H - p_E^*}{2} \\ &= \frac{q_H - c_E}{3}.\end{aligned}$$

and this gives firms the profit $\pi_I^*(q_L)$ and $\pi_E^*(q_H)$, where

$$\pi_I^*(q_L) = \pi_E^*(q_H) = \frac{\left(q_H - p_E^*\right)^2}{4q_H} = \frac{\left(q_H - c_E\right)^2}{9q_H}.$$

Suppose $c_I < c_2$.Now there are three sets of equilibrium candidates. We first consider the first set of equilibrium candidates (i.e. configuration 1). In this configuration, firms share the markets. Firm I and firm E then receive a profit $\left(q_L - c_I\right)^2/4q_L + (q_H - q_L + c_I - c_E)^2/9(q_H - q_L)$ and $(q_H - q_L + c_I - c_E)^2/9(q_H - q_L)$ respectively. If firm I deviates and undercuts its price below $c_E - (q_H - q_L)$ in market A, it then receives a profit $(q_L - c_I)^2/4q_L$. This deviation is never profitable for firm I. Another possible deviation for firm I is to exit market A (equivalently setting its price above p_E^* in market A and this is one of the best possible deviation), it is possible and firm E chooses its monopoly price in the market to maximizes its profit and firm I obtains a maximum profit equal to $(q_H - p_E^*)^2/4q_H = (q_H - c_E)^2/(3q_H - q_L)$ by setting p_I^* as a best response to p_E^*.This deviation is never profitable given that the $(q_H - q_L)(q_L - c_I) - 2(c_I q_H - c_E q_L) > 0$. There is no profitable deviation for firm E.

We next consider the second set of equilibrium candidates (configuration 2), In this configuration, Firm I undercuts its price to firm E's marginal cost in market A and profit for firm I is $(q_L - c_I)^2/4q_L$. If firm I deviates and exits market A (equivalently setting its price above p_E^* in market A and this is one of the best possible deviation), then firm E chooses its monopoly price in the market to maximizes its profit and firm I obtains a profit equal to $(q_H - p_E^*)^2/4q_H$. This deviation is profitable if and only if $(q_H - p_E^*)^2/4q_H > (q_L - c_I)^2/4q_L$. In other words, firm I does not undercut its price if $p_E^* \geq q_H - (q_L - c_I)\sqrt{q_H/q_L}$. One of the possible deviations for firm I is to sells its product at a price, $(q_L - q_H + c_I + p_E^*)/2$ and receives an extra profit $(q_H - q_L + c_I - p_E^*)^2/4(q_H - q_L)$. Such deviation is not profitable if $p_E^* \geq q_H - q_L + c_I$. Now, we consider the possible deviations for firm E. Given $p_I^* \leq c_E - (q_H - q_L)$, firm E has no profitable deviation in this configuration.

We now consider the last set of equilibrium candidates (configuration 3). Firm I leaves market A for firm E and each firm receives a profit for $(q_H - c_E)^2/9q_H$. If firm I deviates and undercuts its price in market A (that is setting its price below $c_E - (q_H - q_L)$) in market A and this is the best possible deviation, then firm I chooses its price in market B as a best response to p_E^* and firm I obtains a profit equal to $(q_L - c_I)^2/4q_L$. This deviation is not profitable if $c_I > c_4$, where

$$c_4 = q_L - \frac{2}{3}(q_H - c_E)\sqrt{\frac{q_L}{q_H}} < c_2.$$

. Now, we consider another possible deviation, which is firm I sets its price equal to $(q_L - q_H + c_I + p_E^*)/2$ and share the market with firm E. Firm I then receives a profit, $(q_L - c_I)^2/9q_L$. Given that the best response for the entrant in configuration 2 and 3 are the same, it is possible for firm I to deviate and this deviation is always profitable when $c_I \geq c_2$ holds. There is no profitable deviation for firm E.

First, we suppose $c_I \geq c_2 > c_4$, in this case we have two equilibria in the subgame, where

$$\begin{aligned} p_I^* &\leq c_E - (q_H - q_L), \\ p_E^* &\geq q_H - (q_L - c_I)\sqrt{q_H/q_L} \text{ and} \\ p_I^* + p_B^* &= (q_L + c_I)/2 \end{aligned}$$

$$\begin{aligned} p_I^{**} &> (q_H + 2c_E)/3, \\ p_E^{**} &= (q_H + 2c_E)/3 \text{ and} \\ p_B^{**} &= (q_H - c_E)/3 \end{aligned}$$

On the other hand, if $c_I < c_2$, we also have two subgame perfect equilibria in the subgame, where

$$\begin{aligned} p_I^* &\leq c_E - (q_H - q_L), \\ p_E^* &\geq q_H - (q_L - c_I)\sqrt{q_H/q_L} \text{ and} \\ p_I^* + p_B^* &= (q_L + c_I)/2 \end{aligned}$$

$$\begin{aligned} p_I^{***} &= (q_L - q_H + c_E + 2c_I)/3, \\ p_E^{***} &= (q_H - q_L + 2c_E + c_I)/3 \text{ and} \\ p_B^{***} &= (2q_H + q_L - 2c_E - c_I)/6 \end{aligned}$$

We assume only the second type of equilibrium will be played. This makes sense. Both firm I and firm E receive a higher profit if firm I agrees to leave the market and firm E agrees to take the market. The second type of equilibrium is Pareto optimal given that $\pi_I^*(p_I^*, p_E^*, p_B^*) \leq \pi_I^*(p_I^{**}, p_E^{**}, p_B^{**})$ when $c_I \geq c_2$ and $\pi_I^*(p_I^*, p_E^*, p_B^*) \leq \pi_I^*(p_I^{***}, p_E^{***}, p_B^{***})$ when $c_I < c_2$. This concludes the proof. ■

Proof of Proposition 3

Suppose firm E has entered market A in the previous stage and if both firms offer the same quality level, firms merely compete in price at this stage and consumers only buy from the firm with the lowest price. There are three different possible cases. We first consider the case in which firm I

sets $p_I < p_E$. Strategies that support this equilibrium are as follows. At time $t = 1$, firm I charges at a price p_I in market A where $p_I < p_E$. Given $p_I^* < p_E$, consumer only buys from firm I and hence firm E makes no sales. In this equilibrium, firm E can deviates and undercuts its price to its cost, c_E. Therefore, firm I always undercuts its price and sets $p_I^* \leq c_E$. In this case, firm E cannot make any profit if it further undercuts its cost. Therefore, firm E always exit when $p_I^* \leq c_E$ and firm I keeps its monopoly position in market A. Now firm I acts as a monopoly with the constraint $p_I^* \leq c_E$, it maximizes its profit by choosing the optimal price in market B, p_B and its profit function is given as follow:

$$\pi_{I_1}(q) = (p_I + p_B - c_I)\left(1 - \frac{p_I + p_B}{q}\right). \tag{7}$$

Maximizing equation 7 yields the optimal price in market B:

$$p_I^* + p_B^* = \frac{q + c_I}{2} = \frac{c_I - 2c_E}{2}. \tag{8}$$

and it gives firm I a optimal profit $\pi_I^*(q)$, where

$$\begin{aligned} \pi_{I_1}^*(q) &= (p_I^* + p_B^* - c_I)\left(1 - \frac{p_I^* + p_B^*}{q}\right) \\ &= \left(\frac{q + c_I}{2} - c_I\right)\left(\frac{q - c_I}{2q}\right) \\ &= \frac{(q - c_I)^2}{4q}. \end{aligned} \tag{9}$$

We next consider the case in which firm I sets $p_I^* = p_E$, here we assume firms share the market equally. Therefore, firms maximize profits by choosing the optimal price in its market. Their profit functions are given as follow:

$$\pi_{I_2}(q) = (p_I - c_I)\{\frac{1}{2}\left(1 - \frac{p_I + p_B}{q}\right)\} + p_B\left(1 - \frac{p_I + p_B}{q}\right) \tag{10}$$

$$= (p_E - c_I)\{\frac{1}{2}\left(1 - \frac{p_E + p_B}{q}\right)\} + p_B\left(1 - \frac{p_E + p_B}{q}\right). \tag{11}$$

$$\pi_{E_2}(q) = (p_E - c_E)\{\frac{1}{2}\left(1 - \frac{p_E + p_B}{q}\right)\}. \tag{12}$$

Maximizing equation 10 and 12 yield the optimal prices:

$$\begin{aligned} p_E^* &= \frac{q + c_E - p_B^*}{2} \\ &= \frac{2q - c_I + 4c_E}{5}. \end{aligned}$$

$$\begin{aligned} p_B^* &= \frac{2q + c_I - 3p_E^*}{4} \\ &= \frac{q + 2c_I - 3c_E}{5}. \end{aligned}$$

and it gives firms the optimal profits $\pi_{I_2}^*(q)$ and $\pi_{E_2}^*(q)$, where

$$\begin{aligned} \pi_{I_2}^*(q) &= (p_I^* - c_I)\{\frac{1}{2}\left(1 - \frac{p_E^* + p_B^*}{q}\right)\} + p_B^*\left(1 - \frac{p_E^* + p_B^*}{q}\right) \\ &= \frac{(2q - c_I - p_E^*)^2}{16q} \\ &= \frac{(2q - c_I - c_E)^2}{25q}. \end{aligned} \quad (13)$$

$$\begin{aligned} \pi_{E_2}^*(q) &= (p_E^* - c_E)\left\{\frac{1}{2}\left(1 - \frac{p_E^* + p_B^*}{q}\right)\right\} \\ &= \left(\frac{p_E^* - c_E}{2}\right)\left(\frac{2q - c_I - p_E^*}{2q}\right) \\ &= \frac{(2q - c_I - c_E)^2}{25q}. \end{aligned} \quad (14)$$

Now consider the case in which firm I sets $p_I > p_E$, that is firm I leaves the market after firm E's entry. Firm E then acts as a monopoly and firms maximize profits by choosing the optimal price in its market. Their profit functions are given as follow:

$$\pi_{I_3}(q) = (p_B)\left(1 - \frac{p_E + p_B}{q}\right). \quad (15)$$

$$\pi_{E_3}(q_H) = (p_E - c_E)\left(1 - \frac{p_E + p_B}{q_H}\right). \tag{16}$$

Maximizing equation 7 and 16 yield the optimal prices:

$$\begin{aligned} p_E^* &= \frac{q + c_E - p_B^*}{2} \\ &\frac{q + 2c_E}{3}. \end{aligned}$$

$$\begin{aligned} p_B^* &= \frac{q - p_E^*}{2} \\ &\frac{q - c_E}{3}. \end{aligned}$$

and the optimal profits are $\pi_I^*(q)$ and $\pi_E^*(q)$, where

$$\pi_{I_3}^*(q) = \frac{(q - p_E^*)^2}{4q} = \frac{(q - c_E)^2}{9q}.$$

$$\pi_{E_3}^*(q) = \frac{(q - c_E)^2}{9q}.$$

Firm E never deviates in the any set of the above equilibrium candidates, as it always makes non-positive profit if it deviates.

Now, consider the following profitable deviations for firm I in the first set of equilibrium candidate. Suppose firm I deviates from the first set of equilibrium candidates and leaves the market, firm I then receives a maximum profit, $(q - p_E^*)^2/4q$. It is possible for firm I to deviate by choosing p_B^* as a best response to $p_E^* = (q + c_E - p_B^*)/2$ and such deviation is not profitable if $p_E^* \geq c_I$. Another possible deviations for firm I is to set its price equal to p_E^* and chooses p_B^* as a best response to $p_E^* = (q + c_E - p_B^*)/2$. This deviation gives firm I a profit, $(2q - c_I - p_E^*)^2/16q$. This deviation is not profitable if $p_E^* \geq c_I$.

We next consider the second set of equilibrium candidates. Firm I can deviates and undercuts its price to $p_I^* \leq p_E^*$. Such deviation is profitable if and only if $\pi_{I_1}^*(q) \geq \pi_{I_2}^*(q)$ and and simple algebra shows that $\pi_{I_1}^*(q) \geq \pi_{I_2}^*(q)$ if and only if $c_I \leq c_3$ where

$$c_3 = \frac{q + 2c_E}{3}$$

Another possible deviation for firm I is to leave the market for the entrant. It is profitable if and only if $\pi^*_{I_3}(q) \geq \pi^*_{I_2}(q)$ and it it is equivalent to $c_I > c_3$. Therefore, if $c_I \leq c_3$, firm I always deviates from the second set of equilibrium candidates and undercuts its price. On the other hand, if $c_I > c_3$, firm I always deviates and leaves the market. Thus, setting exactly the same price as the entrant's is always not a best response of firm I.

We then consider the last set of equilibrium candidates, firm I deviates and undercuts its price below c_E. This deviation is profitable if and only if $\pi^*_{I_1}(q) \geq \pi^*_{I_3}(q)$ and simple algebra shows that $\pi^*_{I_1}(q) \geq \pi^*_{I_3}(q)$ if and only if $c_I \leq c_3$. Another possible deviation for firm I is to set $p^*_I = p^*_E$. Such deviation is profitable if and only if $\pi^*_{I_2}(q) \geq \pi^*_{I_3}(q)$ and it is equivalent to $c_I \leq c_3$.

Therefore, in summary, we have the following equilibria. If $c_I \leq c_3$, then there is only an equilibrium exists, where

$$\begin{aligned} p^*_I &\leq c_E, \\ p^*_E &\geq c_I \text{ and} \\ p^*_I + p^*_B &= (q + c_I)/2 \end{aligned}$$

On the other hand, if $c_I > c_3$, there are two subgame perfect equilibria exist, where

$$\begin{aligned} p^*_I &\leq c_E, \\ p^*_E &\geq c_I \text{ and} \\ p^*_I + p^*_B &= (q + c_I)/2 \end{aligned}$$

$$\begin{aligned} p^{**}_I &> p^{**}_E, \\ p^{**}_E &= (q + 2c_E)/3 \text{ and} \\ p^{**}_B &= (q - c_E)/3 \end{aligned}$$

We assume only the second type of equilibrium will be played. This makes sense. Both firm I and firm E receive a higher profit if firm I agrees to leave the market and firm E agrees to take the market. The second type of equilibrium is Pareto optimal given that $\pi^*_I(p^*_I, p^*_E, p^*_B) \geq \pi^*_I(p^{**}_I, p^{**}_E, p^{**}_B)$ when $c_I > c_3$. ■

Proof of Proposition 4

Suppose firm E has entered market A, and predatory pricing is possible; that is, an incumbent can price below marginal cost and squeeze the entrant out of the market. Consider the case in which firm I introduces a high-quality product.

Suppose firm I has a low marginal cost, $0 \leq c_I < c_3(q_H)$. Firm I always undercuts its price and does not accommodate entry regardless of the quality level of firm E and obtains an expected payoff, π_E^e

$$\pi_E^e = -F_E.$$

.Hence it is always optimal for firm E to stay out of the market.

Now suppose firm I has a marginal cost, $c_I \in [c_3(q_H), c_1]$. Firm I only undercuts its price below its marginal cost and squeeze firm E out of the market if firm E is a low-quality type and firm I finds it profitable to leave the market to firm E with high-quality product. Firm E receives an expected payoff, π_E^e

$$\pi_E^e = \gamma \pi_{E_M}(q_H) - F_E.$$

It is profitable for firm E to enter the market if $\gamma \geq \frac{F_E}{\pi_{E_M}(q_H)}$

If c_I increases and lies above c_1, entering market A is a dominant strategy for firm E, according to proposition 1 and 3, firm I always welcomes any firm E to take over its market and firm E receives an expected payoff, π_E^e

$$\pi_E^e = \gamma \pi_{E_M}(q_H) + (1-\gamma)\pi_{E_M}(q_L) - F_E.$$

Firm E finds it profitable to enter the market if and only if $\gamma > \frac{F_E - \pi_E(q_L)}{\pi_{E_M}(q_H) - \pi_E(q_L)}$

Now consider the case in which firm I introduces a low-quality product. If firm I produces at a low marginal cost (i.e. $c_I \leq c_2$), firm I always prefers to lower its price in the competitive market and deters entry regardless of the type of firm E and firm E obtains an expected payoff, π_E^e

$$\pi_E^e = -F_E.$$

if it enters the market. It is always optimal for firm E to stay out of the market. When firm I has a sufficiently low marginal (i.e., $c_I \in [c_2, c_3(q_L)]$), from Proposition 2 and 3 , firm I finds it profitable to share the market with the entrant if the entrant has an quality advantage over it, otherwise, firm I always undercuts its price. In this case, firm E receives an expected payoff, π_E^e

$$\pi_E^e = \gamma \pi_{E_D}(q_H) - F_E.$$

Firm E finds it profitable to enter the market if $\gamma \geq \frac{F_E}{\pi_{E_D}(q_H)}$.

When $c_I > c_3(q_L)$, firm I always accommodates entry regardless of the type of firm E. Thus, firm E obtains an expected payoff, π_E^e

$$\pi_E^e = \gamma \pi_{E_M}(q_H) + (1-\gamma)\pi_{E_M}(q_L) - F_E,$$

if it enters the market and it is profitable for it to enter the market if $\gamma \geq \frac{F_E - \pi_{E_M}(q_L)}{\pi_{E_M}(q_H) - \pi_{E_M}(q_L)}$.This concludes the proof. ■

Proof of Proposition 5

Suppose predatory pricing is not possible, the entrant always enter the market as it has a cost advantage over the incumbent and leaving the market to the entrant is the only profitable option to the incumbent. Total expected producer surplus when the incumbent leaves the market to the entrant and each firm operates alone in its own market is

$$\begin{aligned} PS_{ban} &= \gamma \int_{\frac{2q_H + c_E}{3q_H}}^{1} \left[p_E^* + p_B^* - c_E\right] \, d\theta + (1-\gamma) \int_{\frac{2q_L + c_E}{3q_L}}^{1} \left[p_E^* + p_B^* - c_E\right] \, d\theta - F_E \\ &= \gamma \frac{2(q_H - c_E)^2}{9q_H} + (1-\gamma)\frac{2(q_L - c_E)^2}{9q_L} - F_E. \end{aligned}$$

where p_E^* is the equilibrium price offered by the entrant in market A. When the incumbent leaves the market to the entrant, total expected consumer surplus is

$$\begin{aligned} CS_{ban} &= \gamma \int_{\frac{2q_H + c_E}{3q_H}}^{1} \left[\theta q_H - p_E^* - p_B^*\right] \, d\theta + (1-\gamma) \int_{\frac{2q_L + c_E}{3q_L}}^{1} \left[\theta q_L - p_E^* - p_B^*\right] \, d\theta \\ &= \gamma \frac{(q_H - c_E)^2}{18q_H} + (1-\gamma)\frac{(q_L - c_E)^2}{18q_L}. \end{aligned}$$

and total welfare is

$$\begin{aligned} W_{ban} &= \gamma \int_{\frac{2q_H + c_E}{3q_H}}^{1} [\theta q_H - c_E] \; d\theta + (1-\gamma) \int_{\frac{2q_L + c_E}{3q_L}}^{1} [\theta q_L - c_E] \; d\theta - F_E. \\ &= \gamma \frac{5(q_H - c_E)^2}{18 q_H} + (1-\gamma) \frac{5(q_L - c_E)^2}{18 q_L} - F_E. \end{aligned}$$

Given predatory pricing is possible, the probability for entrant to have high quality is sufficiently high $\gamma \geq \gamma^{**}$ (i.e. $F_E \leq \gamma \pi_{E_D}(q_H)$) and the incumbent introduces a high quality product, the incumbent takes the entire market if the entrant introduces a low quality product and the incumbent leaves the market to the entrant if the entrant introduces a high quality product. Total expected producer surplus when $c_I \in [c_3(q_H), c_1]$ is

$$\begin{aligned} PS_{pre} &= \gamma \int_{\frac{2q_H + c_E}{3q_H}}^{1} [p_E^* + p_B^* - c_E] \; d\theta + (1-\gamma) \int_{\frac{q_H + c_I}{2q_H}}^{1} [p_I^* + p_B^* - c_I] \; d\theta - F_E. \\ &= \gamma \frac{2(q_H - c_E)^2}{9 q_H} + (1-\gamma) \frac{(q_H - c_I)^2}{4 q_H} - F_E. \end{aligned}$$

where p_I^*, p_E^* and p_B^* are the equilibrium price in each market. Total expected consumer surplus with predatory pricing

$$\begin{aligned} CS_{pre} &= \gamma \int_{\frac{2q_H + c_E}{3q_H}}^{1} [\theta q_H - p_E^* - p_B^*] \; d\theta + (1-\gamma) \int_{\frac{q_H + c_I}{2q_H}}^{1} [\theta q_H - p_I^* - p_B^*] \; d\theta \\ &= \gamma \frac{(q_H - c_E)^2}{18 q_H} + (1-\gamma) \frac{(q_H - c_I)^2}{8 q_H}. \end{aligned}$$

and total welfare is

$$\begin{aligned} W_{pre} &= \gamma \int_{\frac{2q_H + c_E}{3q_H}}^{1} [\theta q_H - c_E] \; d\theta + (1-\gamma) \int_{\frac{q_H + c_I}{2q_H}}^{1} [\theta q_H - c_I] \; d\theta - F_E. \\ &= \gamma \frac{5(q_H - c_E)^2}{18 q_H} + (1-\gamma) \frac{3(q_H - c_I)^2}{8 q_H} - F_E. \end{aligned}$$

Simple algebra shows that $W_{pre} \leq W_{ban}$ if and only if $c_I \geq c_I^*$, where

$$c_I^*(q_H) = q_H - \frac{2\sqrt{5}}{3\sqrt{3}} (q_L - c_E) \sqrt{\frac{q_H}{q_L}} < c_1$$

Incumbent undercuts the price and deters the entry regardless of the type of the entrant when $c_I \leq c_3(q_H)$ and total expected producer surplus is

$$\begin{aligned} PS_{pre} &= \int_{\frac{q_H + c_I}{2q_H}}^{1} [p_I^* + p_B^* - c_I] \; d\theta \\ &= \frac{(q_H - c_I)^2}{4q_H} \end{aligned}$$

Total expected consumer surplus

$$\begin{aligned} CS_{pre} &= \int_{\frac{q_H + c_I}{2q_H}}^{1} [\theta q_H - p_I^* - p_B^*] \; d\theta \\ &= \frac{(q_H - c_I)^2}{8q_H}. \end{aligned}$$

and total welfare with only the incumbent in both markets is

$$\begin{aligned} W_{pre} &= \int_{\frac{q_H + c_I}{2q_H}}^{1} [\theta q_H - c_I] \; d\theta \\ &= \frac{3(q_H - c_I)^2}{8q_H}. \end{aligned}$$

Simple algebra shows that $W_{pre} \leq W_{ban}$ if and only if $c_I \geq c_I^{**}$, where

$$c_I^{**}(q_H) = q_H - 2\sqrt{\frac{2q_H}{3}\left[\gamma \frac{5(q_H - c_E)^2}{18q_H} + (1-\gamma)\frac{5(q_L - c_E)^2}{18q_L} - F_E\right]}$$

Given predatory pricing is possible, the probability for entrant to have high quality is sufficiently high $\gamma \geq \gamma^{**}$ (i.e. $F_E \leq \gamma\pi_{E_D}(q_H)$) and the incumbent introduces a low quality product, the incumbent takes the entire market if the entrant introduces a low quality product and the incumbent leaves the market to the entrant if the entrant introduces a high quality product. Total expected producer surplus when $c_I \in [c_{2,}\ c_3(q_L)]$ is

$$\begin{aligned} PS_{pre} &= \gamma \int_{\frac{2q_H + c_E}{3q_H}}^{1} [p_E^* + p_B^* - c_E] \ d\theta + (1-\gamma) \int_{\frac{q_L + c_I}{2q_L}}^{1} [p_I^* + p_B^* - c_I] \ d\theta - F_E. \\ &= \gamma \frac{2(q_H - c_E)^2}{9q_H} + (1-\gamma)\frac{(q_L - c_I)^2}{4q_L} - F_E. \end{aligned}$$

where p_I^*, p_E^* and p_B^* are the equilibrium price in each market. Total expected consumer surplus with predatory pricing

$$\begin{aligned} CS_{pre} &= \gamma \int_{\frac{2q_H + c_E}{3q_H}}^{1} [\theta q_H - p_E^* - p_B^*] \ d\theta + (1-\gamma) \int_{\frac{q_L + c_I}{2q_L}}^{1} [\theta q_L - p_I^* - p_B^*] \ d\theta. \\ &= \gamma \frac{(q_H - c_E)^2}{18q_H} + (1-\gamma)\frac{(q_L - c_I)^2}{8q_L}. \end{aligned}$$

and total welfare is

$$\begin{aligned} W_{pre} &= \gamma \int_{\frac{2q_H + c_E}{3q_H}}^{1} [\theta q_H - c_E] \ d\theta + (1-\gamma) \int_{\frac{q_L + c_I}{2q_L}}^{1} [\theta q_L - c_I] \ d\theta - F_E. \\ &= \gamma \frac{5(q_H - c_E)^2}{18q_H} + (1-\gamma)\frac{3(q_L - c_I)^2}{8q_L} - F_E. \end{aligned}$$

Simple algebra shows that $W_{pre} \leq W_{ban}$ if and only if $c_I \geq c_I^{***}$, where

$$c_I^*(q_L) = q_L - \frac{2\sqrt{5}}{3\sqrt{3}}(q_L - c_E)\sqrt{\frac{q_H}{q_L}}.$$

Incumbent undercuts the price and deters the entry regardless of the type of the entrant when $c_I \leq c_2$ and total expected producer surplus is

$$
\begin{aligned}
PS_{pre} &= \int_{\frac{q_L + c_I}{2q_L}}^{1} \left[p_I^* + p_B^* - c_I\right] \; d\theta. \\
&= \frac{(q_L - c_I)^2}{4q_L}.
\end{aligned}
$$

Total expected consumer surplus

$$
\begin{aligned}
CS_{pre} &= \int_{\frac{q_L + c_I}{2q_L}}^{1} \left[\theta q_L - p_I^* - p_B^*\right] \; d\theta. \\
&= \frac{(q_L - c_I)^2}{8q_L}.
\end{aligned}
$$

and total welfare with only the incumbent in both markets is

$$
\begin{aligned}
W_{pre} &= \int_{\frac{q_L + c_I}{2q_L}}^{1} \left[\theta q_L - c_I\right] \; d\theta. \\
&= \frac{3(q_L - c_I)^2}{8q_L}.
\end{aligned}
$$

Simple algebra shows that $W_{pre} \leq W_{ban}$ if and only if $c_I \geq c_I^{***}$, where

$$
c_I^{**}(q_L) = q_L - 2\sqrt{\frac{2q_L}{3}\left[\gamma\frac{5(q_H - c_E)^2}{18q_H} + (1-\gamma)\frac{5(q_L - c_E)^2}{18q_L} - F_E\right]}
$$

This concludes the proof. ■

References

Adams, William, J. and Yellen, Janet L. 1976. "Commodity Bundling and the Burden of Monopoly," 90 *Quarterly Journal of Economics* 475-498

Areeda, Phillip. and Turner, Donald F. 1975. "Predatory Pricing and Related Practices Under Section 2 of the Sherman Act.," 88 *Havard Law Review* 697-733

Caillaud, Bernard. and Jullien, Bruno. 2003. "Chicken and Egg: Competition among Intermediation Service Providers," 34 *Rand Journal of Economics* 309-328.

Carlton, Dennis, W. and Waldman, Michael. 2002. "The Strategic Use of Tying to Preserve and Create Market Power in Evolving Industries," 33 *Rand Journal of Economics* 194-220.

Choi, Jay Pil. 2003. "Merger with Bundling in Complementary Markets," 56 *Journal of Industrial Economics* 553-577.

Choi, Jay Pil. and Stefanadis, Christodoulos. 2001. "Tying, Investment, and the Dynamic Leverage Theory," 32 *Rand Journal of Economics* 52-71.

Choi, Jay Pil. and Stefanadis, Christodoulos. 2006. "Bundling, Entry Deterrence, and Specialist Innovators," 79 *Journal of Business* 2575-2594.

Church, Jeffrey. and Gandal, Neil. 1994. "Strategic entry deterrence: Complementary products as installed base," 12 *European Journal of Political Economy* 331-354.

Edlin, Aaron S. 2002. "Stopping Above-Cost Predatory Pricing," 111(4) *Yale Law Journal* 941-992

Evans, David S. and Noel, Michael. 2005. "Defining Antitrust Markets When Firms Operate Two-Sided Platforms," *Columbia Business Law Review* 667-702

Evans, David S. and Schmalensee, Richard. 2005. "The Industrial Organization of Markets with Two-sided Platforms," *NBER Working Paper #11603*

Farhi, Emmanuel. and Hagiu, Andrei. "Strategic Interactions in Two-sided Market Oligopolies," 32 *Rand Journal of Economics* 52-71.

Farrell, Joseph. and Katz, Michael L. 2000. "Innovation, Rent Extraction and Integration in Systems Markets," 48 *Journal of Industrial Economics* 413-432.

Farrell, Joseph. and Katz, Michael L. 2005. "Competition or Predation? Consumer Coordination, Strategic Pricing and Price Floors in Network Markets," 48 *Journal of Industrial Economics* 203-231.

Kovac, Eugen. 2005. "Tying and Entry Deterrence in vertically Differentiated Markets," *CERGE-EI Working Papers* wp266

Lutz, Stefan. 1997. "Vertical Product Differentiation and Entry Deterrence," 65 *Journal of Economics* 79-102.

Milgrom, Paul. and Roberts, John. 1982. "Predation, Reputation and Entry Deterrence," 48 *Journal of Economic Theory* 280-312.

Nalebuff, Barry. 2004. "Bundling as an Entry Barrier," 119 *Quarterly Journal of Economics* 159-188.

Noh, Yong-Hwan. and Moschini, Giancarlo. 2006. "Vertical Product Differentiation, Entry-Deterrence Strategies, and Entry Qualities," 29 *Review of Industrial Organization* 227-252.

Salop, Steven C. and Scheffman, David T. 1983. "Raising Rivals' Costs," 73 *American Economic Review* 267-271.

Schmalensee, Richard. 1984. "Gaussian Demand and Commodity Bundling," 57 *Journal of Business* 211-230.

Stigler, George J. 1963. "United States v. Loew's Inc. : A Note on Block Booking," 152 *Supreme Court Review* 152-157.

Whinston, Michael D. 1990. "Tying, Foreclosure, and Exclusion," 80 *American Economics Review* 837-859.

Whinston, Michael D. 2001. "Exclusivity and Tying in U.S v. Microsoft: What We Know, and Don't Know," 15 *Journal of Economics Perspectives* 63-80.