Bond premia, monetary policy and exchange rate dynamics

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[DRAFT]

Abstract

A common assumption in macroeconomics and finance is that an observed short-term government or reference interest rate is risk-free, but theory and evidence suggest otherwise. This paper departs from that assumption and allows for a wedge between the observed short-term rate and the risk-free rate − a short-term bond premium. Incorporating a bond premium helps to explain: why measures of risk that price bonds don't price currencies and vice versa; why empirical tests of uncovered interest parity fail; and why exchange rates are 'too smooth' relative to risk-free rates implied by asset prices. The exchange rate responses to conventional monetary policy - via the risk-free interest rate - and to unconventional monetary policy - via the bond premium - are shown to be qualitatively similar, but to reflect different mechanisms, with implications for the nature of international spillovers.

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1 Introduction

The risk-free rate means different things to different people. To some it is an investment with no chance of financial loss. To others it is the rate that has no correlation with the market portfolio. In much of theoretical macroeconomics and finance, it is the rate of time preference of the representative investor or borrower. In empirical work, a short-term government rate, or a readily-available short-term reference rate, is typically used as a proxy.

Although observed short-term government rates are often assumed to be risk-free, theory and evidence suggest otherwise. First, although government default risk is usually low relative to other rates, (Della Corte et al. 2015) find relative default risk to be significant in explaining exchange rate behaviour. Second, investors demand lower yields on short-term assets that perform well in bad times or that can be sold in liquid markets in bad times, because those assets help the holder to smooth consumption. Krishnamurthy and Vissing-Jorgensen (2012), Amihud et al. (2005) and Duffie (1996) show that short-term safety premia in US Treasuries interest rates are empirically important. Third, there is evidence that monetary policy intervention in short-term money markets affects short-term bond premia. Nagel (2014) shows that, empirically, short-term liquidity premia are correlated with the policy rate - the opportunity cost of holding central bank cash. Canzoneri et al. (2007) show that risk-free rates constructed from common specifications of preferences tend to be negatively correlated with the real Federal Funds rate, a result they link empirically to monetary policy. Both results imply that the short-term bond premium is positively correlated with the policy rate. Fourth, from the home investor's perspective, foreign interest rates reflect exchange rate revaluation risk (Lustig and Verdelhan 2007). Finally, the highly volatile risk-free interest rates implied by equity prices (Brandt et al. 2006, Cochrane and Hansen 1992, Hansen and Jagannathan 1991), are in contrast to relatively persistent observed short-term monetary policy or government bill rates.

This paper departs from the assumption that observed short-term government bond interest rates are risk free. The risk-free rate is defined by investors' willingness to forego a unit of consumption today to consume one plus the risk-free rate next period. Since investors' preferences are not observed, neither is the risk-free rate. Allowing for a wedge between the observed rate and the unobserved risk-free rate − a short-term 'bond premium' − helps to explain three empirical exchange rate anomalies: (i) why measures of risk that price domestic assets do not price currencies and vice versa (Sarno et al. 2012, Burnside 2012); (ii) why tests of uncovered interest parity (UIP) fail - the forward premium puzzle (Engel 2016, Fama 1984 and references therein); and (iii) why exchange rates are 'too smooth' relative to risk-free rates (Chien et al. 2015, Brandt et al. 2006). The paper builds on the application of risk corrections to exchange rate models employed by Lustig and Verdelhan (2007) and Backus et al. (2001).

Including a short-term bond premium raises the question, Does monetary policy influence the risk-free rate or the premium component of the observed short-term interest rate? When a monetary policy tightening raises the risk-free interest rate, the exchange rate response follows Dornbusch (1976): the home currency initially appreciates to eliminate all future excess returns on home bonds, and then depreciates to offset the higher home interest return, period by period.

In contrast, Canzoneri et al. (2007) make a case that a monetary policy tightening raises the short-term bond premium. A policy-induced rise in the observed short-term interest rate should slow activity, slow expected consumption growth, and depress the risk-free rate. Therefore the short-term bond premium − the wedge between the observed rate and the risk-free rate − must rise. In an exchange rate model with short-term bond risk, I show that, when policy intervention raises the short-term premium in the foreign bond market, while the pricing of risk in the currency market is initially unchanged, then for no-arbitrage conditions to hold in the currency market, the foreign currency premium must fall, appreciating the foreign currency. The exchange rate response to monetary policy follows a pattern qualitatively similar to the traditional Dornbusch (1976) response to a change in the risk-free rate, but the the nature of the adjustment mechanism is different. The traditional view is a price response to expected risk-free returns; when monetary policy alters the bond market premium, the adjustment involves building or shedding of risk. The difference has implications for the nature of international spillovers.

Sarno et al. (2012), Burnside (2012) show that measures of risk that price stock and bond returns don't price currency returns, and that measures of risk that price currency returns don't price stock and bond returns. When allowing for short-term bond premia, I show that, when markets are incomplete, a short-term bond premium cannot be both priced symmetrically the bond and currency market (reflected in the bond premium, but not the exchange rate) and be priced asymmetrically in the bond and currency market (reflected in the currency premium and the exchange rate). As in Chien et al. (2015), Lustig and Verdelhan (2007) and Backus et al. (2001), the currency premium reflects asymmetries in the pricing of risk by home and foreign investors. Here, the bond premium captures asymmetric pricing short-term bond risk as well as asymmetric pricing of currency revaluations risk. Currency 'excess returns' reflect both the currency premium and those additional bond risks. When markets are complete, all bond risk, including currency revaluation risk, is reflected in the bond premium. With symmetrical pricing of risk, the currency premium is zero. That is consistent with the empirical findings of Sarno et al. (2012) and Chen and Tsang (2013) that, while yield curve factors (that span bond premia) do little to explain currency returns, they help to explain currency 'excess returns' that include bond risk.

In the model with short-term bond premia, the forward premium puzzle can be viewed as an identification problem that arises because the risk-free rate is not observed. A higher foreign bond premium is not offset by foreign currency depreciation because the premium compensates the holder for risk. When the interest differential reflects only the bond premium, for example when markets are complete, the model predicts disconnect between exchange rate movements and observed interest differentials. Using moments reported by Canzoneri et al. (2007), the short-term bond premium is calculated to bias tests of UIP on a magnitude consistent with the empirical literature on the forward premium puzzle. I show that short-term bond risk, combined with monetary policy intervention in the short-term bond markets (Canzoneri et al. 2007), provide the persistent and transitory drivers required to fit exchange rate dynamics more broadly (Engel 2016). In contrast to the model proposed in Engel (2016) a volatile risk-free rate and a persistent policy response can generate the required dynamics without the need for a volatile exogenous bond premium.

Brandt et al. (2006) argue that either exchange rates are 'too smooth', or the degree of international risk sharing is high.[1] Risk-free rates implied by equities are very volatile (Hansen and Jagannathan 1991). Brandt et al. (2006) show that, if exchange rates reflect relative risk-free rates, then either exchange rates should be considerably more volatile than they are, or home and foreign risk-free rates must be correlated, implying a high degree of risk-sharing.[2]. A model that incorporates short-term bond premia provides an additional interpretation. When policy intervention holds the observed short-term rate steady, but risk-free rates are volatile, then the wedge between teh two - the bond premium - must be volatile and negatively correlated with the risk-free rate. When variation in that premium reflects policy intervention rather than a change in the fundamental price of risk, asymmetric pricing of risk in the bond markets relative to the currency market gives rise to a currency premium. For no-arbitrage to hold in the currency market, the foreign currency premium must rise when the foreign bond market premium is compressed. The higher foreign risk-free rate appreciates the foreign currency, but that is offset by a higher foreign currency premium that depreciates the foreign currency. By stabilising the observed rate, monetary policy isolates the exchange rate from the effects of volatile risk-free rates, providing an additional interpretation of why exchange rates are 'too smooth' relative to highly volatile risk-free rates implied by equity prices. In contrast, under those assumptions, variation in the underlying risk characteristics of short-term bonds are reflected in the currency premium and in the exchange rate.

The next section derives an exchange rate asset price model that incorporates short-term bond premia. Section 3 interprets, through the lens of the model, the empirical disconnect between symmetric pricing of risk reflected in the bond premium and asymmetric pricing of risk reflected in the currency premium. Section 4 uses the model to interpret the forward premium puzzle, and draws on the empirical literature to inform on the potential contribution of short-term bond premia to the failure of tests of UIP. Section 6 considers the case when monetary policy stabilises the observed interest rate and provides an additional interpretation of why exchange rates are smooth relative to the volatile risk-free rates implied by equity prices. Section 7 concludes.

[1] Complete risk-sharing is rejected empirically (Backus and Smith 1993). Kose et al. (2003) show that cross-country consumption correlations did not increase in the 1990s, despite financial integration. More recently, employing a different empirical approach, Flood and Matsumoto (2009) find that consumption growth rates have converged, suggesting that international risk sharing has improved during a period of globalization.

[2] (Chien et al. 2015) provide a segmented markets interpretation.

2 An exchange rate asset price model with risk

This section derives an exchange rate asset price model that incorporates short-term bond premia. The model encompasses the standard exchange rate asset price model (Engel and West 2010, 2005, Dornbusch 1976), but allows for a broader set of interest rate-exchange rate dynamics. In the standard, sticky-price open economy model, exchange rate dynamics are driven by uncovered interest parity (UIP). UIP equates the expected returns on home and foreign short-term bonds:

$$q_t = -r_t^d - \lambda_t + E_t q_{t+1} \tag{1}$$

where q_t is the logarithm of the real exchange rate, defined as the value of the foreign currency in units of home currency, r_t^d is the home-foreign real short-term interest differential and E_t indicates expectations at time, t. The expected foreign currency 'excess return' λ_t is defined as $\lambda_t = E_t \Delta q_{t+1} - r_t^d$.

To a first-order approximation, the expected value of λ_t is zero: the expected depreciation of the home currency, $E_t \Delta q_{t+1}$, offsets a higher home-foreign interest return r_t^d. Empirically, currencies with high interest rates (relative to other currencies and relative to average) tend to have expected excess returns on their short-term bonds (Engel 2016). On average, they do not depreciate by enough to offset the higher interest return, and often appreciate. Excess returns could reflect a variety of factors, including non-rational expectations, infrequent portfolio adjustment and risk premia. When the 'excess return' is interpreted as risk, as in this paper, then equation (1) can be read: the expected depreciation of the currency, $E_t \Delta q_{t+1}$, is equal to relative *risk-adjusted* returns $(r_t^d + \lambda_t)$.

This paper derives a two-equation partial equilibrium model comprised of (1) and the following equation that expresses the observed interest rate differential as relative risk-free returns, net of a foreign bond premium:

$$r_t^d = (r_t^f - r_t^{f*}) - \lambda_t^R \tag{2}$$

where r_t^f and r_t^{f*} are the home and foreign risk-free interest rates respectively, and λ_t^R is the foreign (relative to home) short-term bond market premium. In allowing for a short-term bond market premium, λ_t^R, the model will depart from the assumption that observed short-term returns are risk-free. The bond premium compensates for expected losses and risk (eg liquidity risk, Nagel 2014, Duffie 1996, Amihud and Mendelson 1991 and currency revaluation risk Lustig and Verdelhan 2007), and may reflect policy intervention in the short-term bond markets (Nagel 2014, Canzoneri et al. 2007).

The baseline two-equation model allows for incomplete markets, whereby home and foreign investors' pricing of risk may differ because of different pricing kernels (different discount factors) or intervention in short-term bond markets. Within that general framework, the complete markets case and the role of central bank intervention in short-term bond markets (Canzoneri et al. 2007, Nagel 2014) are considered. The main model features and dynamics are discussed below. Derivations are shown in Appendix A.

2.1 One-period risk-free bonds

The starting point is a standard asset pricing model (eg. Cochrane 2001, Chapter 1). The risk-free rate, r_t^f, is defined by the home investor's rate of time preference: his willingness to give up a unit of consumption today to consume $(1 + r_t^f)$ units of consumption next period. The 'stochastic discount factor' (SDF), M_{t+1}, of the home investor is:

$$M_{t+1} = E_t \beta U_{C,t+1}/U_{C,t} = \frac{1}{1+r_t^f} \qquad (3)$$

where β is the home investor's subjective discount factor, $U'_{C,t}$ is the marginal utility of consumption, and E_t indicates expectations at time t. The results do not depend on a particular specification of the SDF, so there is no reason to specify a utility function, but simply to postulate that it exists. The risk-free rate is lower when people save more because they are patient (β), they are averse to varying consumption across time (inter-temporal substitution), they are averse to varying consumption across states (risk aversion), or they expect consumption growth to be volatile (precautionary savings).

Following Lustig and Verdelhan (2007), I assume that the SDF and gross asset returns are conditionally log-normal (see Appendix A). Taking the logarithm of equation (3),

$$log M_{t+1} = -r_t^f$$

Similarly, the foreign real, risk-free interest rate, r_t^{f*}, is defined by the foreign investor's rate of time preference:

$$log M_{t+1}^* = -r_t^{f*} \qquad (4)$$

where M_{t+1}^* is the foreign investor's SDF, β^* is the foreign investor's subjective discount factor.

2.2 The bond premium

The home investor's pricing (Euler) equation for the home bond is:

$$1 = E_t[M_{t+1}(1 + r_t)] \qquad (5)$$

Equation (5) equates the cost of buying a unit of home bond this period to the expected, discounted return on the bond at time $t + 1$. The logarithm of the expected return can be expressed as the risk-free rate plus a risk correction (see Cochrane 2001 for a general exposition, Lustig and Verdelhan 2007 for an application to exchange rates):

$$log E_t(1 + r_t) = r_t^f - cov_t(m_{t+1}, r_t) \qquad (6)$$

where $m_{t+1} = log M_{t+1}$. If the ex-post payoff, $E_t(1 + r_t)$, is known with certainty, at time t, and the home bond has a very short maturity, then $cov_t(m_{t+1}, r_t) = 0$. That is the standard assumption in the literature (for example, Lustig and Verdelhan 2007). Although the contracted interest rate is known with certainty ex-ante, the value of the ex-post payoff may not be known for a variety of reasons, including potential losses from default or from selling the bond before maturity (liquidity risk), or changes in the risk-free rate over the holding period (term premium). In fact, $cov_t(m_{t+1}, r_t) = 0$ is a very special case.

More generally, the pricing equation (5) can be written in terms of the ex-ante contracted interest rate, r_t^c and a variable, Z_{t+1} that reflects uncertainty about the ex-post payoff:

$$1 = E_t[M_{t+1}(1 + r_t^c)Z_{t+1}] \tag{7}$$

Z_t captures uncertainty regarding losses from default or from selling the bond before maturity, and uncertainty regarding the evolution of the risk-free rate over the holding period. The log pricing equation (6) can be written in the more general form:

$$logE_t(1 + r_t^c) \sim r_t^c = r_t^f - logE_t(Z_{t+1}) - cov_t(m_{t+1}, z_{t+1}), \tag{8}$$

The second term on the right hand side captures expected losses and the final term on the right hand side of (8) is a risk correction. The risk correction increases the yield on assets with payoffs that are positively correlated with consumption growth (negatively correlated with consumption utility growth). Holding such assets assets makes consumption more volatile; the higher yield compensates the holder for consumption risk. Appendix A provides examples in which expected losses (or gains) and risk corrections drive a wedge between the contracted rate and the risk-free rate.

Similarly, the contracted return in the foreign bond market can be expressed as the foreign investor's risk-free rate, r_t^{f*}, plus expected losses and a risk correction that is priced according to the foreign investor's log SDF, m_t^*:

$$logE_t(1 + r_t^{c*}) \sim r_t^{c*} = r_t^{f*} - logE_t(Z_{t+1}^*) - cov_t(m_{t+1}^*, z_{t+1}^*) \tag{9}$$

Combining (8) and (9), and assuming that contracted rates reflect the expected value of payoffs, the observed short-term home-foreign interest differential can be expressed as the relative risk-free return plus a relative bond premium, λ_t^R:

$$\begin{aligned} r_t^d &= r_t^c - r_t^{c*} \\ &= (r_t^f - r_t^{f*}) - \underbrace{(logE_t(Z_{t+1}) - logE_t(Z_{t+1}^*) + cov_t(m_{t+1}, z_{t+1}) - cov_t(m_{t+1}^*, z_{t+1}^*))}_{\text{bond premium, } \lambda_t^R} \end{aligned} \tag{2'}$$

The bond premium is the wedge between the observed, contracted rate on the bond and the unobserved risk-free rate. It reflects the pricing of bonds in the home and foreign bond markets. The terms home investor and foreign investor are used to mean the representative, risk-neutral investor in the home and foreign bond market, respectively.

2.3 Uncovered interest parity and currency revaluation risk

When comparing bonds denominated in different currencies, we need to account for currency revaluation risk. The home investor's pricing equation for the foreign short-term bond is:

$$Q_t \;=\; E_t[M_{t+1}(1+r_t^{c*})Z^*_{t+1}Q_{t+1}], \tag{10}$$

where Q_t is the real exchange rate (value of the foreign currency in units of home currency). Equation (10) equates the cost of buying one unit of the foreign bond this period, Q_t, to the expected, discounted return on the foreign bond at $t+1$, in home currency terms. The log of the pricing equation is:

$$r_t^{c*} = r_t^f - logE_t(Z^*_{t+1}) - E_t\Delta q_{t+1} - cov_t(m_{t+1}, z^*_{t+1}) - cov_t(m_{t+1}, \Delta q_{t+1}) - \tfrac{1}{2}var(\Delta q_{t+1}), \tag{11}$$

where $q_t = log(Q_t)$. From the perspective of the home investor, the foreign bond premium reflects expected losses, including expected depreciation of the foreign currency, $E_t\Delta q_{t+1} + \frac{1}{2}var(\Delta q_{t+1})$, and risk corrections that reduce the yield on bonds that are expected to perform well (eg safe-haven currencies) when the marginal utility of consumption rises. Holding such bonds makes consumption less volatile (Lustig and Verdelhan 2007).

Combining the home investor's pricing equation for home short-term bonds (8) with the home investor's pricing equation for foreign bonds (11), gives the UIP condition that equates the expected return on the home bond to the expected return on the foreign bond:

$$\begin{aligned} q_t \;=\; & -r_t^d \\ & - \underbrace{(logE_tZ_{t+1} - logE_tZ^*_{t+1} + (cov_t(m_{t+1}, (z_{t+1} - z^*_{t+1})) - cov_t(m_{t+1}, \Delta q_{t+1}) - \tfrac{1}{2}var(\Delta q_{t+1}))}_{\text{`Excess return', } \lambda_t} \\ & + E_t(q_{t+1}) \end{aligned} \tag{1'}$$

The currency 'excess return' reflects the home investor's pricing of home and foreign bond risk. Constructing λ_t that way assumes that the home investor dominates the currency market, eg. the foreign country is small and has a floating exchange rate. Conversely, it is useful to think of the home investor as the global investor.

Equation (1') encompasses the standard asset price model of the exchange rate.[3] If the home risk-adjusted interest rate rises, or is expected to rise, relative to the foreign risk-adjusted rate, the home currency should immediately appreciate à la Dornbusch (1976) and then depreciate to its equilibrium value, over the period of higher home returns. The initial appreciation eliminates all future excess risk-adjusted returns, and the subsequent depreciation offsets the higher interest risk-adjusted payoffs each period, so there is no excess return to holding the home or the foreign asset.

[3] The exchange rate, q_t, can be expressed as a forward sum, as in Engel and West (2010), as $q_t = -R_t - \Lambda_t + E_t\bar{q}_t$, where R_t, the sum of expected relative interest payoffs, is defined as $R_t = E_t\sum_{k=0}^{\infty} r^d_{t+k}$; Λ_t, the 'level' excess return; and $E_t\bar{q}_t$ is the time t expectation of the long-run equilibrium real exchange rate consistent with purchasing power parity (PPP). The level bond premium Λ_t^R and the level excess return, Λ_t, can be expressed in terms of expected losses and risk corrections.

3 Disconnect between measures of risk that price bonds and measures of risk that price currencies

3.1 The currency premium

It is useful to divide the currency excess return, λ_t, into two parts: the bond premium, λ_t^R and the currency premium. Accordingly, the currency premium is the currency excess return (1'), net of the bond premium (2'):

$$\begin{aligned}
\lambda_t^{FX} &= \lambda_t - \lambda_t^R \\
&= cov_t(m_{t+1}^*, z_{t+1}^*) - cov_t(m_{t+1}, z_{t+1}^*) - cov_t(m_{t+1}, \Delta q_{t+1}) - \tfrac{1}{2} var(\Delta q_{t+1}) \\
&= cov_t\big((m_{t+1}^* - m_{t+1}), z_{t+1}^*\big) - cov_t(m_{t+1}, \Delta q_{t+1}) - \tfrac{1}{2} var(\Delta q_{t+1}) \qquad (12)
\end{aligned}$$

Defined this way, the log currency risk premium reflects the difference between the home and foreign investors' pricing of risk on the foreign bond. As in Lustig and Verdelhan (2007), Backus et al. (2001) and Chien et al. (2015), the currency premium reflects asymmetric pricing of risk. The last two terms on the right-hand side of (12) are standard (Lustig and Verdelhan 2007, Obstfeld and Rogoff 1996). The first term on the right-hand side reflects the short-term bond premium.

In a model with a short-term bond premium, the currency premium reflects asymmetric pricing of risk in the bond markets relative to the currency market. Asymmetries in the pricing of risk may reflect differences in the pricing kernels ($m_{t+1} \neq m_{t+1}^*$) of the representative investors, or may reflect monetary policy intervention in the short-term bond markets. Both are discussed in subsequent sections.

Using this definition of the currency premium, the exchange rate (1) can be expressed as its expected future value, the observed interest differential and an excess return that includes the relative bond premium and a currency premium:

$$q_t = -\underbrace{((r_t^f - r_t^{f*}) - \lambda_t^R)}_{\text{Observed interest differential}} - \underbrace{(\lambda_t^R + \lambda_t^{FX})}_{\text{`Excess return'}\lambda_t} + E_t q_{t+1}$$

or as its expected future value, the risk-free interest differential and the currency premium:

$$q_t = -\underbrace{(r_t^f - r_t^{f*})}_{\text{Risk-adjusted differential, } r_t^d} - \underbrace{\lambda_t^{FX}}_{\text{Currency premium, } (\lambda_t - \lambda_t^R)} + E_t q_{t+1}$$

The bond premium does not help to price currency returns, Δq_{t+1}, consistent with the tensions in 2-country term structure models between fitting the yield curves and fitting the exchange rate (Sarno et al. 2012). In contrast, the relative bond premium, λ_t^R, should help to explain excess currency returns, λ_t, consistent with results in Sarno et al. (2012) and Chen and Tsang (2013). Bond risks can be reflected in the bond premium (priced symmetrically) or reflected in the currency premium (priced asymmetrically), but not in both.

3.2 Currency and bond premia: complete markets

It is useful to consider the properties of the currency premium and bond premium when allowing for risk sharing and complete markets. Allowing for incomplete markets, the risk-sharing condition holds in differences, $\Delta q_{t+1} = m^*_{t+1} - m_{t+1}$, but not necessarily in levels (Benigno and Thoenissen 2008).

Substituting the risk-sharing condition into equation (12), the currency premium reflects asymmetries in the pricing of risk that in turn reflect the degree of market incompleteness:

$$\lambda_t^{FX} = cov_t\big((m^*_{t+1} - m_{t+1}), z^*_{t+1}\big) - cov_t(m_{t+1}, (m^*_{t+1} - m_{t+1})) - \tfrac{1}{2}var(m^*_{t+1} - m_{t+1}) \quad (13)$$

As markets become more complete, the foreign investor prices the foreign bond from the same global perspective as the home investor:[4]

$$r_t^{c*} = r_t^{f*} - logE_t(Z^*_{t+1}) - E_t\Delta q_{t+1} - cov_t(m^*_{t+1}, z^*_{t+1}) - cov_t(m^*_{t+1}, \Delta q_{t+1}) - \tfrac{1}{2}var(m^*_{t+1} - m_{t+1}) \quad (14)$$

She requires a return on the foreign bond that compensates for currency revaluation risk as well as local currency-denominated bond risks. Combining (8) and (14), the relative bond premium $\lambda_t^R = r_t^d - (r_t^f - r_t^{f*})$ is:

$$\begin{aligned}\lambda_t^R &= logE_t(Z_{t+1}) + cov_t(m_{t+1}, z_{t+1}) \\ &\quad -\big[logE_t(Z^*_{t+1}) + E_t\Delta q_{t+1} + cov_t(m^*_{t+1}, z^*_{t+1}) + cov_t(m^*_{t+1}, \Delta q_{t+1}) + \tfrac{1}{2}var(\Delta q_{t+1})\big]\end{aligned}$$

Suppose that $m^*_t = m_t + \eta_t$, where η_t reflects the deviation from complete markets.

$$\begin{aligned}\lambda_t^R &= logE_t(Z_{t+1}) + cov_t(m_{t+1}, z_{t+1}) - \tfrac{1}{2}var(\Delta q_{t+1}) \\ &\quad -\big[logE_t(Z^*_{t+1}) + E_t\Delta q_{t+1} + cov_t((m_{t+1}+\eta_{t+1}), z^*_{t+1}) + cov_t((m_{t+1}+\eta_{t+1}), \Delta q_{t+1})\big]\end{aligned}$$

Using the risk-sharing condition, $\Delta q_{t+1} = m^*_{t+1} - m_{t+1}$,

$$\begin{aligned}\lambda_t^R &= logE_t(Z_{t+1}) - logE_t(Z^*_{t+1}) + cov_t(m_{t+1}, z_{t+1}) - cov_t(m_{t+1}, z^*_{t+1}) \\ &\quad -\big[E_t\Delta\eta_{t+1} + cov_t(\eta_{t+1}, z^*_{t+1}) + cov_t((m_{t+1}+\eta_{t+1}), \Delta\eta_{t+1}) - \tfrac{1}{2}var(\Delta\eta_{t+1})\big] \quad (15)\end{aligned}$$

The relative bond premium reflects the risk characteristics of the home and foreign bonds (the first line of 15) and terms that reflect market incompleteness, η_t (the second line of 15).

When markets are complete, the home and foreign investor price risk using the same SDF, and the risk-sharing condition holds in levels ($m_t = m^*_t$, so $\eta_t \to 0$), and all terms on the second line of (15) converge to zero. The risk characteristics of the home and foreign bonds are priced symmetrically in the bond markets and the currency market, and are reflected in the relative bond premium (the first line of 15). Substituting $m_t = m^*_t$ into equation (13), that there is no currency premium when risks are priced symmetrically. When markets are complete, the currency 'excess return' is equal to the bond premium, $\lambda_t = \lambda_t^R$. As before, bond risks can be reflected in the bond premium or in the currency premium, but not in both.

[4] This is the same as equation (11), but priced using m^*_t.

4 The forward premium puzzle

Uncovered interest parity equates the expected returns on home and foreign bonds, and implies a close link between exchange rates and relative interest returns. However, statistical tests of UIP fail consistently across currency pairs and sample periods − an empirical regularity known as the 'forward premium puzzle'.[5] High interest rate currencies have expected 'excess returns' on their short-term bonds (Engel 2016, Fama 1984 and references therein). A large literature[6] has argued that time-varying risk may be the cause of the weak empirical link between currency movements and relative returns.

Tests of UIP usually regress the change in the exchange rate on the observed short-term interest differential:

$$\Delta q_{t+1} = c + \beta r_t^d + \epsilon_{t+1} \tag{16}$$

If the observed interest rate differential, r_t^d, is uncorrelated with ϵ_{t+1}, then we expect to estimate $\beta = 1$. Empirically, however, estimates of β are almost always less than one, and often less than zero.

Subtracting q_{t+1} from both sides of equation (1) and rearranging:

$$\Delta q_{t+1} = r_t^d + \underbrace{\lambda_t + [q_{t+1} - E_t(q_{t+1})]}_{\epsilon_{t+1}}, \tag{17}$$

If r_t^d is correlated with ϵ_{t+1}, then the estimated value, $\hat{\beta}$, in equation (16) is:

$$\hat{\beta} \;=\; \frac{cov(r_t^d, \Delta q_{t+1})}{var(r_t^d)} = \frac{cov(r_t^d, (r_t^d + \epsilon_{t+1}))}{var(r_t^d)} = 1 + \frac{cov(r_t^d, \epsilon_{t+1})}{var(r_t^d)}$$

Assuming rational expectations, the expectational error in square brackets in (17) should be uncorrelated with variables observed at t. However, the risks reflected in λ_t are known at t. Moreover, in the two-equation model (1') and (2'), risk can not be treated as exogenous because λ_t^R and λ_t reflect common premia. For example, both reflect $logE_t(Z_{t+1}^*) - logE_t(Z_{t+1}) - cov_t(m_{t+1}, z_{t+1})$, and in the complete markets case, $\lambda_t^R = \lambda_t$. Those common premia mean that $r_t^d (= (r_t^f - r_t^{f*}) - \lambda_t^R)$ and λ_t are correlated. Estimation of equation (16) will be biased relative to the coefficient of unity expected in a risk-free model. That bias is the result of a measurement problem: we do not observe the risk-free rate. The estimated value of β is:

$$\hat{\beta} \;=\; 1 + \frac{cov(r_t^d, \lambda_t)}{var(r_t^d)} \tag{18}$$

In the limit of complete markets, the model (1') and (2') becomes:

$$\begin{aligned} q_t &= \lambda_t^R - \lambda_t^R + E_t q_{t+1} \\ r_t^d &= -\lambda_t^R \end{aligned}$$

[5] See Engel (2016), Engel (2013), Engel (1996), and Flood and Rose (1996) for reviews.

[6] For example, Della Corte et al. (2015), Sarno et al. (2012), Lustig and Verdelhan (2007), Brennan and Xia (2006), Duarte and Stockman (2005), Obstfeld and Rogoff (2002), Backus et al. (2001), Fama (1984) and references therein.

The observed interest differential reflects only the bond premium. There is complete disconnect between the exchange rate and the interest differential. From equation (18), the estimated value of β is $1 - \frac{var(\lambda_t^R)}{var(\lambda_t^R)} = 0$. When the foreign bond premium is high relative to the home bond premium, the foreign currency does not depreciate to offset the higher return because the higher return compensates the holder for the risk of holding the foreign bond.

In the more general incomplete markets case, what does the empirical literature tell us about $cov(r_t^d, \lambda_t)$? It is useful to break the covariance term in (18) into smaller parts:

$$\hat{\beta} = 1 + \frac{cov(r_t^d, \lambda_t^R)}{var(r_t^d)} + \frac{cov(r_t^d, \lambda_t^{FX})}{var(r_t^d)} \quad (19)$$

$$= 1 + \frac{cov((r_t^f - r_t^{f*}), \lambda_t^R)}{var(r_t^d)} - \frac{var(\lambda_t^R)}{var(r_t^d)} + \frac{cov((r_t^f - r_t^{f*}), \lambda_t^{FX})}{var(r_t^d)} - \frac{cov(\lambda_t^R, \lambda_t^{FX})}{var(r_t^d)} \quad (20)$$

When the unobserved components $(r_t^f - r_t^{f*})$, λ_t^R and λ_t^{FX} are independent then, as in the complete markets case, the only term in equation (20) that is non-zero is $-\frac{var(\lambda_t^R)}{var(r_t^d)}$.[7]

Canzoneri et al. (2007) construct measures of the US real risk-free rate from common specifications of preferences and show that model-implied risk-free rates are negatively correlated with the observed Federal Funds rate. Using the model derived here and the moments reported in Canzoneri et al. (2007), shown in Table 1, we can calculate the first covariance term in equation (19). Assuming no change in foreign variables, Canzoneri et al. (2007)'s moments imply that $\frac{cov(r_t^d, \lambda_t)}{var(r_t^d)}$ contributes -5.0 to -0.8 to the estimation bias, implying estimates of β in the range -4 to 0.2.[8] That range spans the complete markets value of $\hat{\beta} = 0$ and spans most empirical estimates of $\hat{\beta}$ reported in Engel (2016)[9] which range from -1.4 to 0.6 (real terms) and $-2.7 < \hat{\beta} < 0.6$ (nominal terms), in (Fama 1984) $-1.6 < \hat{\beta} < 0.3$ (nominal); and in Sarno et al. (2012) $-5.5 < \hat{\beta} < 0.2$ (nominal).

What do we know about the covariance between the observed interest rate and the currency premium − the final term in (19), or the final two covariance terms in terms in (20)? In the absence of a bond premium only the third term in (20) remains. The currency premium, λ_t^{FX}, is not directly observable. Some term structure papers examine the relationship between currency returns and relative yield curve factors − level, slope and curvature − derived from term structure models. Since yield curve factors span observed returns, if observed returns are strongly correlated with the currency premium, λ_t^{FX}, then yield curve factors would be expected to be strongly correlated with currency returns $\Delta q_{t+1} = (r_t^f - r_t^{f*}) + \lambda_t^{FX}$ and currency excess returns $\lambda_t = \lambda_t^R + \lambda_t^{FX}$.

Chen and Tsang (2013) show that regressions of currency returns $\Delta q_{t+1} = (r_t^f - r_t^{f*}) + \lambda_t^{FX}$ on yield curve factors (which span $r_t^d = (r_t^f - r_t^{f*}) - \lambda_t^R$) provide very little explanatory power. Adjusted R^2 statistics range from 0.02 to 0.11 for Canada, Japan and the UK, suggesting

[7] Munro (2015) reports exchange rate decompositions on the assumption that innovations in the unobserved components are independent.

[8] Although (Canzoneri et al. 2007)'s moments contribute to negative values of β, in partial equilibrium, the model derived here can only generate $0 \leq \hat{\beta} \leq 1$.

[9] Engel estimates a parameter $\beta_q = 1 - \beta$. See Engel (2016), Table 3.

Table 1: Moments from Canzoneri et al (2007)

Model	Reported moments σ^{r_t} (a)	$\sigma^{r_t^f}$ (b)	$corr(r_t, r_t^f)$ (c)	Implied moments $cov(r_t, \lambda_t^{Rh})$ $(d) = a^2 - a * b * c$	Bias $(\hat{\beta} - 1)$ $-d/a^2$
CRRA	2.39	1.66	-0.37	7.2	-1.3
Fuhrer	2.39	31.25	-0.07	10.9	-1.9
Abel	2.39	26.55	-0.36	28.6	-5.0
CC	2.39	1.64	-0.37	7.2	-1.3
CEE	2.39	7.39	-0.09	7.3	-1.3
Abel(iid)	2.39	2.32	0.17	4.8	-0.8

Source: First three columns from Canzoneri et al. (2007). Notes: CC: Campbell and Cochrane, CEE: Christiano, Eichenbaum and Evans. The home bond spread is the data (observed Federal Funds rate, r_t) net of the model risk-free rate, r_t^f. That is, $\lambda_t^{Rh} = r_t - r_t^f$. Recall that λ_t^R is defined as the *foreign* bond premium net of the home bond premium. The covariance between the data and the implied bond premium is calculated as follows: $cov(r_t, \lambda_t^{Rh}) = cov(r_t, (r_t - r_t^f)) = var(r_t) - cov(r_t, r_t^f) = var(r_t) - corr(r_t, r_t^f)\sigma^{r_t}\sigma^{r_t^f}$. Assuming that foreign variables are unchanged, the implied estimation bias is $-cov(r_t, \lambda_t^{Rh})/var(r_t)$.

that the currency premium, λ_t^{FX}, is at best weakly correlated with the relative risk-free rates. The weak relationship also suggests that relative risk-free rates account for a small share of exchange rate variance, implying a high degree of risk-sharing. Viewed from another perspective, for the third term in equation (20) to be material, relative consumption utility growth across countries would need to be correlated with the the currency excess returns. Such evidence has been difficult to find (Backus and Smith 1993). (Flood and Matsumoto 2009) show that relative consumption growth rates have converged across countries over time, suggesting that the variance of relative risk-free rates, so the third term in equation (20) has declined.

The final term in equation (20) reflects covariance between the bond premium and the currency premium. As discussed in Section 3, risks are priced in the bond premium (symmetrical pricing of risk in the bond market and currency market) or in the currency premium (asymmetrical pricing of risk), but not in both. The empirical disconnect between measures of risk that price domestic assets and measures of risk that price currencies, reported by Burnside 2012 and Sarno et al. 2012, is also consistent with the final term in equation (20) being modest. Chen and Tsang (2013) show that yield curve factors help to explain 'excess returns', but that explanatory power is still modest: adjusted R^2 statistics for quarterly excess returns range from 0.06 to 0.16. That result suggests that yield curve factors don't span $\lambda_t = \lambda_t^R + \lambda_t^{FX}$, and that most of the variance in the excess return, λ_t is accounted for by either variation in λ_t^{FX} or by changes in expectations about long-run fundamentals,[10] Sim-

[10] Small changes in expectations about long-run levels can have a large effect on long-horizon sums of expected deviations from long-run levels, and in turn on the exchange rate. That is best understood in terms of the forward-looking representation of the exchange rate in footnote 3.

ilarly, Sarno et al. (2012) report a tension between fitting yield curves and fitting currency movements in a two-country term structure model. That result is consistent with most other two-country term structure models that require an additional factor to fit currency returns, and supports the idea that the final term in equation (20) may be modest.

In summary, the presence of a short-term bond premium can alters the empirical relationship between relative returns and currency movements, in a direction and magnitude consistent with the 'forward premium puzzle'. Intuitively, a higher yield associated with the bond premium is not offset by currency depreciation because the premium compensates the holder for risk.

5 The role of monetary policy

This section considers two views of monetary policy: the traditional view that monetary policy moves the risk-free interest rate, and the proposition of Nagel (2014) and Canzoneri et al. (2007) that monetary policy intervention in short-term bond markets influences the premium component of the short-term interest rate. While not the usual assumption, the latter view is consistent with the idea that purchases of long-term bonds influence the term premium (Bernanke (2013) and references therein).

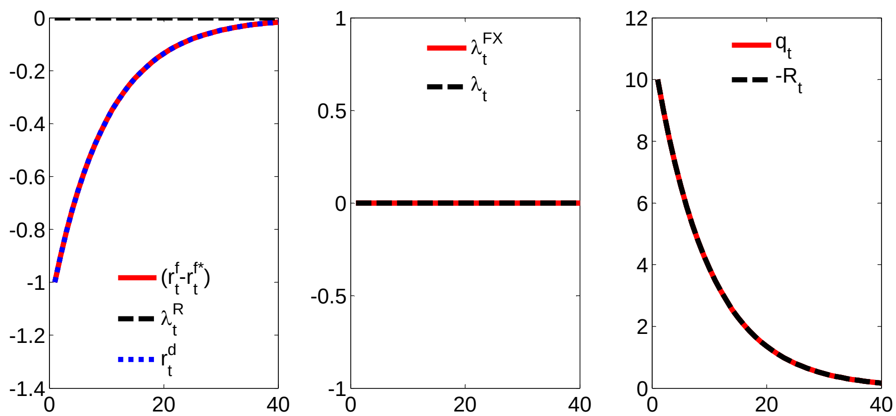
The exchange response to monetary policy are illustrated using impulse response functions. When simulating the responses, relative risk-free rates $r^f - r_t^{*f}$, the price of risk in the bond markets λ_t^R and the price of risk in the currency market λ_t are assumed to be independent, except where indicated. In this partial equilibrium setting, those variables are assumed to follow AR(1) processes with AR(1) coefficients of 0.9 and unit variance shocks. In discussing the adjustment mechanism, it will be useful to think of the exchange rate response in terms of the change in relative risk-free returns and the change in the currency premium, as in equation (13), repeated here for convenience:

$$q_t = -\underbrace{(r_t^f - r_t^{f*})}_{\text{Risk-adjusted differential, } r_t^d} - \underbrace{\lambda_t^{FX}}_{\text{Currency premium, } (\lambda_t - \lambda_t^R)} + E_t q_{t+1}$$

The traditional exchange rate response to a rise in the foreign risk-free rate is illustrated in Figure 1. When the foreign risk-free interest rate, r_t^{f*}, rises, or is expected to rise ($r_t^f - r_t^{f*}$ and r_t^d fall, left-hand panel), the foreign currency should immediately appreciate (q_t rises, red line in right hand panel) to reflect the higher expected future path of the foreign risk-free interest rate, $R_t = E_t \sum_{k=0}^{\infty}(r_{t+k}^f - r_{t+k}^{f*}) \sim \frac{1}{1-0.9}\Delta r_t^f$ (right hand panel), so that it can subsequently depreciate to offset the higher foreign return. The initial appreciation eliminates all future excess returns relative to the long-run equilibrium value of the currency; the subsequent depreciation offsets the higher foreign return, period by period. The magnitude of the initial appreciation reflects both the magnitude of the rise in r_t^{f*} and its expected persistence.

In contrast, in this setup, there is no exchange rate response to an exogenous rise in foreign bond risk. A rise in foreign bond risk is reflected in both the foreign bond market

Figure 1: Response to a rise in the foreign risk-free rate (monetary policy)



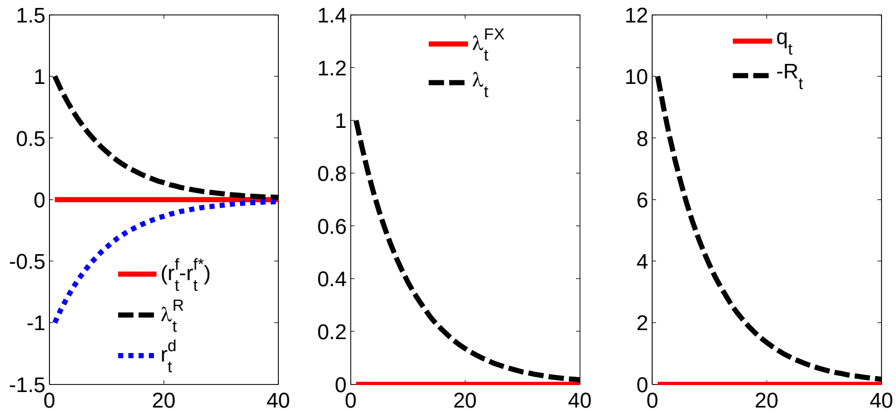
Notes: The vertical axis is in percent per period. The horizonal axis is measured in time periods. Variables: $r_t^f - r_t^{f*}$ is the home-foreign risk-free interest rate differential; λ_t^R is the foreign bond premium relative to the home premium; $r_t^d = r_t^f - r_t^{f*} - \lambda_t^R$ is the home-foreign observed interest rate differential; λ_t is the foreign currency 'excess return'; $\lambda_t^{FX} = \lambda_t - \lambda_t^R$ is the currency premium; $R_t = E_t \sum_{k=0}^{\infty} r_{t+k}^d$ is the expected relative interest rate path; q_t is the real exchange rate (the value of the foreign currency in units of home currency).

(λ_t^R rises, left hand panel of Figure 2) and in the currency market (λ_t rises, centre panel). The higher foreign bond premium increases the observed foreign interest rate, so reduces the observed home-foreign interest rate differential, r_t^d (left hand panel), and increases the expected path of relative returns $-R_t = E_t \sum_{k=0}^{\infty} \lambda_{t+k}^R$ (right hand panel). The currency premium $\lambda_t^{FX} = \lambda_t - \lambda_t^R$ is unchanged. Since the bond premium compensates the holder for risk, the foreign currency does not depreciate to offset the higher return, nor does it initially appreciate. The foreign currency exhibits 'excess returns' equal to the higher foreign bond payoff. In levels, the currency is weak relative to the expected path of relative returns, $-R_t$, as shown in Figure 2.

Now we will turn the traditional assumption that monetary policy moves the risk-free rate on its head, and consider the case when monetary policy intervention in the money market alters the bond premium component of the short-term interest rate. Perhaps surprisingly, the exchange rate response to monetary policy follows a qualitatively similar Dornbusch (1976)-type pattern.

Canzoneri et al. (2007) argue that a monetary tightening (a rise in the observed interest rate) slows consumption growth, so should *reduce* the risk-free rate. Therefore the premium component of the observed rate must rise. While that is not the conventional view of monetary policy operations at the short end of the yield curve, it is a common view of monetary operations at the longer end of the yield curve (Bernanke (2013) and references therein). In the same way that asset purchases at the long-end of the yield curve may depress the term premium, purchases of short-term bonds to achieve the desired policy rate may compress the short-term premium. Canzoneri et al. (2007) show that, empirically, risk-

Figure 2: Response to a rise in the foreign bond premium (priced by all)



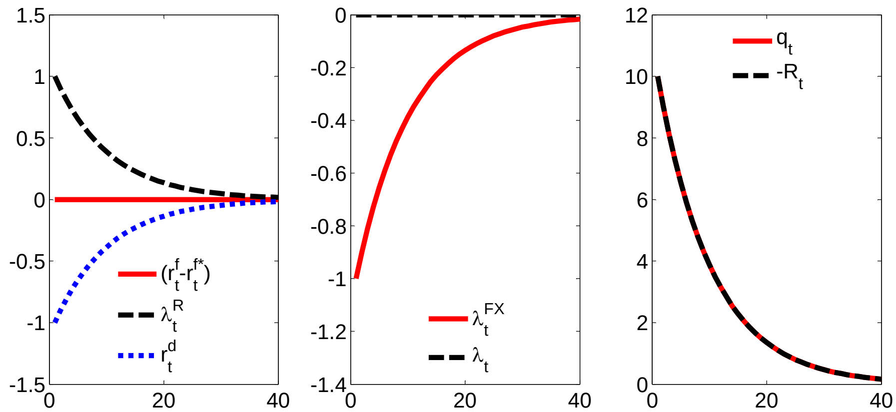
The vertical axis is in percent per period. The horizonal axis is measured in time periods. See footnote to Figure 1 for variable definitions.

free rates implied by common specifications of preferences are negatively correlated with the observed interest rate, implying that the observed rate is positively correlated with the premium. Empirically link that that correlation to monetary policy. Nagel (2014) shows that observed short-term liquidity premia are correlated with policy rates.

Consider the case when a rise in the foreign bond premium λ_t^R (left hand panel of Figure 3) reflects policy intervention in short term bond markets, rather than a fundamental change in the pricing of risk. Assuming that the risk characteristics of the bonds are initially unchanged, the price of risk in the currency market, λ_t (centre panel), is unchanged. The observed foreign interest rate rises, reflecting the higher bond premium (r_t^d falls, left-hand panel, and $-R_t$ rises, dashed-line in the right hand panel). For no-arbitrage conditions to hold in the currency market, the foreign currency premium ($\lambda_t^{FX} = \lambda_t - \lambda_t^R$) must fall (red line in centre panel), reflecting the asymmetric pricing of risk in the bond market relative to the currency market. The fall in the foreign currency premium appreciates the foreign currency, q_t rises (red line in the right hand panel). In contrast to the response to bond risk in Figure 2, where the high interest rate currency was weak relative to the expected interest rate path $-R_t$, here the high interest rate currency follows the expected interest rate path.

Intuitively, the home investor is offered a higher foreign return, but his assessment of foreign economic fundamentals r_t^{f*} and risks λ_t are unchanged. All else equal, the foreign currency must appreciate until the foreign risk-free rate rises (but if anything, it has fallen), or until greater risk offsets the higher return. While the adjustment is the same as a policy-induced rise in the risk-free rate, the adjustment mechanism may be very different. Under the traditional assumption that monetary policy shifts the risk-free rate, the adjustment is a price response to expected risk-free returns. When monetary policy raises the observed interest rate, but the underlying risk characteristics are unchanged, the foreign currency must appreciate until the higher foreign return is matched by either a higher foreign risk-

Figure 3: Response to a rise in the foreign bond premium (monetary policy)

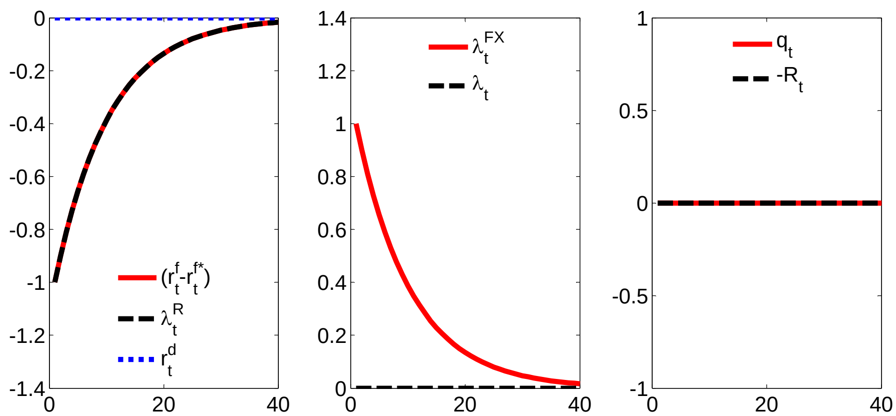


The vertical axis is in percent per period. The horizonal axis is measured in time periods. See footnote to Figure 1 for variable definitions.

free rate (which Canzoneri et al. (2007) argue has fallen) or by greater foreign bond risk, including currency revaluation risk. The adjustment mechanism may involve a build-up or shedding of risk.

Finally, consider the case when monetary policy stabilises the observed interest rate, but the underlying risk-free rate is volatile (Hansen and Jagannathan 1991). When the observed foreign interest rate is held steady at the policy rate, a rise in the underlying foreign risk-free interest rate (red line in left panel of Figure 4) must compress the foreign bond premium, λ_t^R (left panel). In this case, the unobserved components, $(r_t^f - r_t^{f*})$ and λ_t^R are not independent. A rise in the risk-free rate has no effect on the observed policy rate, which is stabilised by policy intervention in the short-term bond markets, nor on the interest rate differential (r_t^d). Since the compressed foreign bond premium reflects foreign monetary policy, rather than the underlying pricing of risk, the home (global) investor's pricing of risk λ_t (centre panel) is assumed to be unchanged. For no-arbitrage to hold in the currency market, the foreign currency premium $\lambda_t^{FX} = \lambda_t - \lambda_t^R$ must rise (centre panel of Figure 4). The increase in the foreign risk-free rate appreciates the foreign currency, but the higher currency premium works in the opposite direction. In this partial equilibrium setup, the combination leaves the exchange rate unchanged. That is in sharp contrast to the traditional Dornbusch (1976) response to a rise in the risk-free rate illustrated in Figure 1: an immediate foreign currency appreciation followed by a gradual depreciation.

Figure 4: Response to a rise in the foreign risk-free rate, with monetary policy



The vertical axis is in percent per period. The horizonal axis is measured in time periods. See footnote to Figure 2 for variable definitions.

6 Exchange rates are "too smooth": a monetary policy explanation

Brandt et al. (2006) show that, although floating exchange rates are volatile, risk-free rates implied by asset prices are much more volatile (Hansen and Jagannathan 1991). If exchange rates reflect relative risk-free interest rates, then either home and foreign risk-free rates are correlated, implying a higher degree of risk sharing than standard estimates, or exchange rates are "too smooth". (Chien, Lustig, and Naknoi 2015) provide a segmented markets explanation.

In a model with a short-term bond premium, monetary policy stabilisation provides an additional explanation. When foreign monetary policy keep foreign short-term interest rates near the policy rate, but the underlying risk-free rate is volatile, there is a strong positive correlation between relative risk-free returns and the foreign currency premium (section 5 and Figure 4). Their opposing effects on the currency leave the exchange rate smooth relative to the path implied by volatile risk-free rates, independent of the degree of risk-sharing.

While the exchange is isolated from variation in the risk-free rate, it reflects variation in the pricing of risk. When monetary policy intervention keeps observed short-term rates near the policy rate, and the underlying risk-free rate is unchanged, then a rise in foreign bond risk is not reflected in the bond market premium λ_t^R. Assuming a floating exchange rate (no policy intervention in the currency market), the rise in foreign bond risk is priced in the currency market (λ_t rises). The currency premium $\lambda_t^{FX} = \lambda_t - \lambda_t^R$ rises, depreciating the foreign currency. Currency movements are driven by the relative risk characteristics of home and foreign bonds, by changes in λ_t^R driven by monetary policy, and by long-run fundamentals such as the terms of trade and relative productivity ((Benigno and Thoenissen 2008).

7 Conclusion

This paper departs from the common assumption that observed short-term government interest rates, or short-term reference interest rates, are risk-free. That departure is in keeping with empirical evidence that premia reflected in short-term Treasuries can be substantial (Feldhütter and Lando 2008, Duffie 1996, Amihud and Mendelson 1991) and that short-term liquidity premia are empirically linked to monetary policy (Nagel 2014 and Canzoneri et al. 2007). The paper builds on the application of risk-corrections to exchange rate models in Lustig and Verdelhan (2007) and Backus et al. (2001).

Including a short-term bond premium − a wedge between the observed interest rate and the unobserved risk-free rate − in an exchange rate asset price model helps to explain three empirical exchange rate anomalies: (i) why measures of risk that price domestic assets do not price currencies and vice versa (Sarno et al. 2012, Burnside 2012); (ii) why tests of uncovered interest parity (UIP) fail - the forward premium puzzle (Engel 2016, Fama 1984); and (iii) why exchange rates are 'too smooth' relative to risk-free rates (Chien et al. (2015), Brandt, Cochrane, and Santa-Clara (2006)). Those results are free of assumptions about preferences and about the degree of risk-sharing.

Including a short-term bond premium raises the question, Does monetary policy move the risk-free interest rate, or the premium component of the observed interest rate? The traditional assumption is that monetary policy influences the risk-free rate. However, monetary policy has been shown to be linked empirically to the short-term premium (Nagel 2014 and Canzoneri et al. 2007). In the model with risk I show that the exchange rate response to monetary policy follows a Dornbusch (1976)-type pattern in both cases, but the adjustment mechanism is very different. In the traditional model, the exchange rate adjustment to monetary policy is a price response to expected returns. In a model with risk, adjustment may involve a building or shedding of risk. The two views have different implications for the nature of international spillovers, and differences in the discounting of future returns in macroeconomic models.

While the short-term bond premium provides an explanation of the forward premium puzzle (expected excess returns on the short-term bonds of high interest rate currencies), Engel (2016) argues that, a model also needs to account for the empirical regularity that a high interest rate currency is strong in level terms. Engel argues that there must be two driving factors. He proposes a volatile liquidity return (bond premium) that dominates exchange rate changes, and a persistent shock to monetary policy that dominates the exchange rate level. The partial equilibrium model derived here provides building blocks for that type of general equilibrium model. The combination of persistent monetary policy that operates through the bond premium (Canzoneri et al. 2007) rather than through the risk-free rate, and volatile underlying risk-free rates (Hansen and Jagannathan 1991), provide potential driving factors. A volatile liquidity return (bond premium) shock is not required: volatile risk-free rates and interest rate smoothing by the monetary authority can generate a volatile bond premium endogenously.

References

Aiyagari, S. R. and M. Gertler (1991). Asset returns with transactions costs and uninsured individual risk. *Journal of Monetary Economics 27*(3), 311–331.

Amihud, Y. and H. Mendelson (1991). Liquidity, maturity, and the yields on us treasury securities. *Journal of Finance 46*, 1411–1425.

Amihud, Y., H. Mendelson, and L. Pedersen (2005). Liquidity and asset prices. *Foundations and Trends in Finance 1*(4), 269–364.

Backus, D., S. Foresi, and C. Telmer (2001). Affine term structure models and the forward premium anomaly. *The Journal of Finance 56*(1), pp. 279–304.

Backus, D. K. and G. W. Smith (1993). Consumption and real exchange rates in dynamic economies with non-traded goods. *Journal of International Economics 35*(3-4), 297–316.

Benigno, G. and C. Thoenissen (2008). Consumption and real exchange rates with incomplete markets and non-traded goods. *Journal of International Money and Finance 27*(6), 926–948.

Bernanke, B. S. (2013). Long-term interest rates.

Brandt, M. W., J. H. Cochrane, and P. Santa-Clara (2006). International risk sharing is better than you think, or exchange rates are too smooth. *Journal of Monetary Economics 53*(4), 671–698.

Brennan, M. J. and Y. Xia (2006). International Capital Markets and Foreign Exchange Risk. *Review of Financial Studies 19*(3), 753–795.

Burnside, C. (2012). Carry trades and risk. In I. W. M. Jessica James and L. Sarno (Eds.), *Handbook of Exchange Rates*, Handbook of Monetary Economics, pp. 283–312. John Wiley & Sons.

Canzoneri, M. B., R. E. Cumby, and B. T. Diba (2007). Euler equations and money market interest rates: A challenge for monetary policy models. *Journal of Monetary Economics 54*(7), 1863–1881.

Chen, Y. and K. Tsang (2013). What does the yield curve tell us about exchange rate predictability? *The Review of Economics and Statistics 95*(1), 185–205.

Chien, Y., H. Lustig, and K. Naknoi (2015). Why are exchange rates so smooth? a segmented asset markets explanation. Working Paper 2015-39, Federal Reserve Bank of St Louis.

Cochrane, J. (2001). *Asset Pricing.* Princeton University Press.

Cochrane, J. H. and L. P. Hansen (1992, June). Asset Pricing Explorations for Macroeconomics. In *NBER Macroeconomics Annual 1992, Volume 7*, NBER Chapters, pp. 115–182. National Bureau of Economic Research, Inc.

Della Corte, P., L. Sarno, M. Schmeling, and C. Wagner (2015). Exchange rates and sovereign risk. Technical report.

Dornbusch, R. (1976). Expectations and exchange rate dynamics. *Journal of Political Economy 84*(6), 1161–76.

Duarte, M. and A. C. Stockman (2005). Rational speculation and exchange rates. *Journal of Monetary Economics 52*(1), 3–29.

Duffie, D. (1996). Special repo rates. *The Journal of Finance 51*(2), 493–526.

Engel, C. (1996). The forward discount anomaly and the risk premium: A survey of recent evidence. *Journal of Empirical Finance* (3).

Engel, C. (2013). Exchange rates and interest parity.

Engel, C. (2016). Exchange rates, interest rates and the risk premium.

Engel, C. and K. West (2010). Global interest rates, currency returns, and the real value of the dollar. *American Economic Review 100*(2), 562–567.

Fama, E. F. (1984). Forward and spot exchange rates. *Journal of Monetary Economics 14*(3), 319–338.

Feldhütter, P. and D. Lando (2008). Decomposing swap spreads. *Journal of Financial Economics 88*(2), 375–405.

Flood, Robert, N. M. and A. Matsumoto (2009). International risk sharing during the globalisation era. Technical report.

Flood, R. P. and A. K. Rose (1996). Fixes: Of the Forward Discount Puzzle. *The Review of Economics and Statistics 78*(4), 748–52.

Hansen, L. P. and R. Jagannathan (1991). Implications of Security Market Data for Models of Dynamic Economies. *Journal of Political Economy 99*(2), 225–62.

Kose, M. A., E. S. Prasad, and M. E. Terrones (2003). How does globalization affect the synchronization of business cycles? *American Economic Review 93*(2), 57–62.

Krishnamurthy, A. and A. Vissing-Jorgensen (2012). The aggregate demand for treasury debt. *Journal of Political Economy* (2).

Lustig, H. and A. Verdelhan (2007). The cross section of foreign currency risk premia and consumption growth risk. *American Economic Review 97*(1), 89–117.

Munro, A. (2015). Exchange rates, expected returns and risk: what can we learn from Asia-Pacific currencies?

Nagel, S. (2014). The liquidity premium of near-money assets. Technical report.

Obstfeld, M. and K. Rogoff (1996). *Foundations of International Macroeconomics*. Cambridge, MA: MIT Press.

Obstfeld, M. and K. Rogoff (2002). *Risk and Exchange Rates*. Cambridge: Cambridge University Press.

Sarno, L., P. Schneider, and C. Wagner (2012). Properties of foreign exchange risk premiums. *Journal of Financial Economics 105*(2), 279–310.

Vayanos, D. (1998). Transaction Costs and Asset Prices: A Dynamic Equilibrium Model. *Review of Financial Studies 11*(1), 1–58.

A Derivations

The risk-free rate The home investor's real stochastic discount factor (SDF), M_{t+1}, between period t and $t+1$ is defined as:

$$M_{t+1} = \beta E_t \frac{U'_{C,t+1}}{U'_{C,t}}$$

where β is the subjective discount factor, and $U'_{C,t}$ is the marginal utility of consumption. Appealing to no-arbitrage and the Fundamental Theorem of Asset Pricing, there is no need to specify the form of the SDF, but simply to postulate that it exists. The home gross risk-free interest rate $1 + r_t^f$ is defined by:

$$\frac{1}{1 + r_t^f} = M_{t+1} \tag{A.1}$$

Following Lustig and Verdelhan (2007), I assume the stochastic discount factor and asset returns to be conditionally log-normal.[11] Define $x_{t+1} = log(X_{t+1})$. If x_{t+1} is normally distributed, then $X_{t+1} = e^{(x_{t+1})}$ is log-normally distributed and $E_t(X_{t+1}) = e^{(E_t(x_{t+1}) + \frac{1}{2}\sigma^2_{x,t})}$.

Taking logs, (A.1) becomes:

$$-r_t^f = logM_{t+1} = E_t m_{t+1} + \tfrac{1}{2} var_t(m_{t+1}) \tag{A.2}$$

Risk corrections: standard approach The home investor's pricing equation for the home short-term bond is:

$$1 = E_t\big(\, M_{t+1}(1 + r_t)\,\big)$$

where r_t, is the expected return on the home bond at time $t+1$, in home currency terms. Taking logs,

$$\begin{aligned} 0 &= logE_t\big(M_{t+1}(1 + r_t)\big) \\ &= log\big(e^{(E_t m_{t+1} + E_t r_t + \frac{1}{2} var(m_{t+1} + r_t))}\big) \\ &= E_t m_{t+1} + \tfrac{1}{2} var(m_{t+1}) + E_t r_t + \tfrac{1}{2} var(r_t) + cov_t(m_{t+1}, r_t) \end{aligned} \tag{A.3}$$

Combining (A.2), (A.3) and $logE_t(1 + r_t) = E_t r_t + \frac{1}{2} var(r_t)$, the home investor's pricing equation for the home short-term bond is:

$$logE_t(1 + r_t) = r_t^f - cov_t(m_{t+1}, r_t)$$

If the ex-post payoff is known with certainty ex-ante, then $cov_t(m_{t+1}, r_t) = 0$.

Risk corrections: allowing for uncertainty However, $cov_t(m_{t+1}, r_t) = 0$ is a special case which assumes certainty about the value of ex-post payoffs, ie., the maturity is very

[11] Alternatively, one can use the definition of covariance, $cov(M, X) = E(MX) - E(M)E(X)$, and take a log approximation.

short, the bond is held to maturity, and there is no risk of loss on the bond. More generally, the home investor's pricing equation (Euler equation) for the home short-term bond can be written:

$$1 \; = \; E_t\big(M_{t+1}(1+r_t^c)Z_{t+1} \big)$$

where r_t^c is the observed, contracted rate on the bond, Z_t, captures uncertainty about the ex-post payoff at time $t+1$. Taking logs,

$$\begin{aligned} 0 &= logE_t\big(M_{t+1}(1+r_t^c)Z_{t+1}\big) \\ &= log\big(e^{(E_t m_{t+1}+E_t r_t^c+E_t z_{t+1}+\frac{1}{2}var(m_{t+1}+r_t^c+z_{t+1}))}\big) \\ &= E_t m_{t+1} + \tfrac{1}{2}var(m_{t+1}) + E_t r_t^c + \tfrac{1}{2}var(r_t^c) + E_t z_{t+1} + \tfrac{1}{2}var(z_{t+1}) \\ &\quad + cov_t(m_{t+1}, r_t^c) + cov_t(m_{t+1}, z_{t+1}) \qquad \text{(A.4)} \end{aligned}$$

Since the contracted rate, r_t^c is known with certainty ex-ante, the term $cov_t(m_{t+1}, r_t^c)$ is zero, and $logE_t(1+r_t^c) \sim r_t^c$. Only systematic risk (covariances with m_{t+k}) is priced, since idiosyncratic risk can be diversified away. Combining (A.2) and (A.4) the home investor's pricing equation for the home short-term bond is:

$$r_t^c \;=\; r_t^f - logE_t(Z_{t+1}) - cov_t(m_{t+1}, z_{t+1}) \qquad \text{(A.5)}$$

Allowing for incomplete markets ($m_t \neq m_t^*$), the equivalent log pricing equation for the foreign bond, from the foreign investor's perspective is:

$$r_t^{c*} = r_t^{f*} - logE_t(Z_{t+1}^*) - cov_t(m_{t+1}^*, z_{t+1}^*) \qquad \text{(A.6)}$$

where r_t^{c*} is the coupon rate on the foreign bond, paid at time $t+1$ in foreign currency, Z_{t+1}^* captures uncertainty about payoffs on the foreign bond, and $r_t^{f*} = -log(E_t M_{t+1}^*)$ is the foreign risk-free rate. Subtracting (A.6) from (A.5) gives equation 2)'.
Examples in which ex-post payoffs are not known with certainty follow.

Default risk Consider a 1-period bond that is contracted at a gross rate $(1+r_t^c)$, payable at $t+1$, but that defaults with a non-zero probability. The pricing equation is

$$1 = E_t[M_{t+1}(1+r_t^c)(1-d_{t+1})]$$

where $d_t \in$ [0,1] captures both the probability of default and loss in the event of default. The log pricing equation is:

$$\begin{aligned} 0 &= log[E_t\big(M_{t+1}(1+r_t^c)(1-d_{t+1})\big)] \\ &= log\big(e^{(E_t m_{t+1}+E_t r_t^c-E_t d_{t+1}+\frac{1}{2}var(m_{t+1}+r_t^c-d_{t+1}))}\big) \\ &= -r_t^f + r_t^c + logE_t(1-d_{t+1}) - cov_t(m_{t+1}, d_{t+1})) \\ r_t^c &= r_t^f - logE_t(1-d_{t+1}) + cov_t(m_{t+1}, d_{t+1}) \end{aligned}$$

The contracted rate is known ex-ante, so $logE_t(1+r_t^c) \sim r_t^c$ and $cov_t(r_t^c, m_{t+k}) = 0$. Defining the default premium as the difference between the contracted rate and the risk-free rate,

$$\begin{aligned} \text{default premium} &\equiv r_t^c - r_t^f \\ &= -log(E_t(1-d_{t+1})) + cov_t(m_{t+1}, d_{t+1}), \end{aligned}$$

The default premium reflects the expected loss and a risk correction that increases the contracted yield if losses from default are expected to be higher when the marginal utility of consumption is expected to rise.
The default premium is part of the bond premium, λ_t^R (equation 2').

Term premium While the assumption that the term premium is small is relevant for overnight securities, for the monthly or quarterly returns that are typically used in empirical exchange rate analysis, term premia may be material. Consider a two-period fixed-rate bond that pays a certain $(1+r_{2,t}^c)^2$ at $t+2$. The pricing equation, $1 = E_t[M_{t+1}M_{t+2}(1+r_{2,t}^c)^2]$, equates the cost of buying the bond today with the expected value of the payoff at $t+2$. The log of the pricing equation is:

$$\begin{aligned} 0 &= logE_t[M_{t+1}M_{t+2}(1+r_{2,t}^c)^2] \\ &= log\left(e^{(E_t m_{t+1} + E_t m_{t+2} + 2E_t r_{2,t}^c + \frac{1}{2} var(m_{t+1} + m_{t+2} + 2r_{2,t}^c))}\right) \\ &= -E_t r_t^f - E_t r_{t+1}^f + 2E_t r_{2,t}^c + cov_t(m_{t+1}, m_{t+2}) \\ 2r_{2,t}^c &= E_t r_t^f + E_t r_{t+1}^f - cov_t(m_{t+1}, m_{t+2}) \end{aligned} \tag{A.7}$$

Since the contracted rate $r_{2,t}^c$ is known with certainty at time t, $cov_t(r_{2,t}^c, m_{t+k}) = 0$. However, the state of the economy, and so the marginal utility of consumption in subsequent periods is not known with certainty, so $cov_t(m_{t+1}, m_{t+2}) \neq 0$. Defining the term premium as the holding-period return on the 2-period bond net of the return on rolling over a 1-period risk-free bond,

$$\begin{aligned} \text{term premium} &\equiv 2r_{2,t}^c - r_t^f - r_{t+1}^f \\ &= -cov_t(m_{t+1}, m_{t+2}) \end{aligned}$$

The term premium compensates the holder for uncertainty about the marginal utility of consumption in the future. To generate a positive term premium, the stochastic discount factor must have negative serial correlation: $cov_t(m_{t+1}, m_{t+2}) < 0$. Negative serial correlation in the risk-free rate means that holding a multi-period bond with a fixed nominal payoff makes consumption more volatile. If the payoff $r_{2,t}^c$ helps to smooth consumption today, the negative serial correlation between m_{t+1} and m_{t+2} means that it is unlikely to help to smooth consumption next period.
The term premium is part of the bond premium, λ_t^R (equation 2').

Liquidity premium The price of a short-term bond can deviate considerably from its hold-to-maturity value because of collateral value, demand and supply, and short-term safety factors.

Consider holding the 2-period bond described above, but with a non-zero probability that the bond will be sold, at $t+1$, to smooth consumption, subject to a liquidation cost. The pricing equation is

$$1 \;=\; E_t[M_{t+1}M_{t+2}(1+r^s_{2,t})^2(1-d_{t+1})]$$

where $(1+r^s_{2,t})$ is the gross yield on the bond that may need to be sold, and d_{t+1} captures both the probability that the bond will be sold at $t+1$ and the expected discount if the bond is sold, relative its hold-to-maturity value. The log of the pricing equation is:

$$\begin{aligned} 2r^s_{2,t} &= E_t r^f_t + E_t r^f_{t+1} - logE_t(1-d_{t+1}) - cov_t(m_{t+1}, m_{t+2}) \\ &\quad + cov_t(m_{t+1}, d_{t+1}) + cov_t(m_{t+2}, d_{t+1}) \end{aligned}$$

The observed, contracted rate on the bond reflects expected risk-free returns, expected losses from selling the bond before maturity, a term premium, and a risk correction that increases the yield on the bond if losses are expected to be greater when the marginal utility of consumption is expected to rise.
Defining the liquidity premium as the yield on the bond that is sold at a discount at $t+1$, net of the yield on the 'liquid' bond (A.7),

$$\begin{aligned} \text{liquidity premium} &\equiv r^s_{2,t} - r^c_{2,t} \\ &= \tfrac{1}{2}(logE_t(1-d_{t+1}) + cov_t(m_{t+1}, d_{t+1})) \end{aligned}$$

the liquidity premium captures the expected loss from selling the bond, $logE_t(1-d_{t+1})$, and a risk correction.[12] If investors are more likely to liquidate bonds to smooth consumption when the marginal utility of consumption rises, the expected loss from selling illiquid bonds is likely to be positively correlated with m_{t+1}. The expected discount d_{t+1} can also be interpreted as a transaction cost associated with selling the bond (see Krishnamurthy and Vissing-Jorgensen (2012), Vayanos (1998) and Aiyagari and Gertler (1991)) or 'specialness'. If a bond is expected to sell at a premium in bad times $E_t d_{t+1} < 0$, for example when the market wants to hold high quality assets and collateral − a 'flight to quality' response − then the yield on the bond will be lower, reflecting its expected liquidity value. Feldhütter and Lando (2008), Duffie (1996) and Amihud and Mendelson (1991) estimate short-term safety factors, to be substantial for US Treasuries. (Nagel 2014) links liquidity premia to monetary policy.
The liquidity premium is part of the bond premium, λ^R_t (equation 2').

Uncovered interest parity The home (global) investor's pricing equation for the foreign short-term bond is:

$$Q_t \;=\; E_t M_{t+1}(1+r^{c*}_t)Z^*_{t+1}Q_{t+1},$$

where Q_t is the real exchange rate (a rise is a depreciation of the home currency). Taking

[12] See Amihud et al. (2005) for a discussion of different types of liquidity risk.

logs,

$$
\begin{aligned}
0 &= logE_t\big(M_{t+1}(1+r_t^{c*})Z^*_{t+1}\frac{Q_{t+1}}{Q_t}\big) \\
&= log\Big(e^{(E_t m_{t+1}+E_t r_t^{c*}+E_t z^*_{t+1}+E_t\Delta q_{t+1}+\frac{1}{2}var(m_{t+1}+r_t^{c*}+z^*_{t+1}+\Delta q_{t+1}))}\Big) \\
&= E_t m_{t+1}+\tfrac{1}{2}var(m_{t+1})+E_t r_t^{c*}+\tfrac{1}{2}var(r_t^{c*})+E_t z^*_{t+1}+\tfrac{1}{2}var(z^*_{t+1})+E_t\Delta q_{t+1} \\
&\quad +cov_t(m_{t+1},z_t^*)+cov_t(m_{t+1},\Delta q_{t+1})+\tfrac{1}{2}var(\Delta q_{t+1}) \\
q_t &= -r_t^f+r_t^{c*}+logE_t(Z^*_{t+1})+E_t q_{t+1}+\tfrac{1}{2}var(\Delta q_{t+1})+cov_t(m_{t+1},z_t^*)+cov_t(m_{t+1},\Delta q_{t+1})
\end{aligned} \tag{A.8}
$$

Subtracting, (A.5) from (A.8) gives the UIP condition (equation 1'):

$$
\begin{aligned}
q_t = &-r_t^d \\
&-\underbrace{(logE_t Z_{t+1}-logE_t Z^*_{t+1}+(cov_t(m_{t+1},(z_{t+1}-z^*_{t+1}))-cov_t(m_{t+1},\Delta q_{t+1})-\tfrac{1}{2}var(\Delta q_{t+1}))}_{\text{`Excess return', }\lambda_t} \\
&+E_t(q_{t+1})
\end{aligned} \tag{A.9}
$$

that equates the expected return on a home bond to expected return on a foreign bond, from the home investor's perspective.
The currency premium is defined as:

$$
\begin{aligned}
\lambda_t^{FX} &= \lambda_t-\lambda_t^R \\
&= (logE_t(Z_{t+1})-logE_t(Z^*_{t+1})+(cov_t(m_{t+1},z_{t+1})-cov_t(m_{t+1},z^*_{t+1})-cov_t(m_{t+1},\Delta q_{t+1})) \\
&\quad -\tfrac{1}{2}var(\Delta q_{t+1})-(logE_t(Z_{t+1})-logE_t(Z^*_{t+1})+cov_t(m_{t+1},z_{t+1})-cov_t(m^*_{t+1},z^*_{t+1})) \\
&= cov_t(m^*_{t+1},z^*_{t+1})-cov_t(m_{t+1},z^*_{t+1})-cov_t(m_{t+1},\Delta q_{t+1})-\tfrac{1}{2}var(\Delta q_{t+1}) \\
&= cov_t\big((m^*_{t+1}-m_{t+1}),z^*_{t+1}\big)-cov_t(m_{t+1},\Delta q_{t+1})-\tfrac{1}{2}var(\Delta q_{t+1})
\end{aligned} \tag{A.10}
$$