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Revisiting the Fiscal Theory of Sovereign Risk from the DSGE View *

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Abstract

We revisit Uribe's 'Fiscal Theory of Sovereign Risk' advocating that there is a trade-off between stabilizing inflation and suppressing default. We develop a class of dynamic stochastic general equilibrium model with nominal rigidities and compare two de facto inflation stabilization policies, optimal monetary policy and optimal monetary and fiscal policy with the minimizing interest rate spread policy which completely suppress the default. Under the optimal monetary and fiscal policy, not only the nominal interest rate, but also the tax rate work to minimize welfare costs through stabilizing inflation. Under the optimal monetary both inflation and output gap are completely stabilized although those are fluctuating under the optimal monetary policy. In addition, volatility on the default rate under the optimal monetary policy is considerably lower than one under the optimal monetary policy. Thus, there is not the SI-SD trade-off. In addition, while the minimizing interest rate spread policy makes inflation rate severely volatile, the optimal monetary and fiscal policy stabilize both the inflation and the default. A trade-off between stabilizing inflation and suppressing default is not so severe what pointed out by Uribe.

Keywords: Sovereign Risk; Optimal Monetary Policy; Fiscal Theory of the Price Level

JEL Classification: E52; E60; F41; F47

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1 Introduction

Uribe[18] advocates that if the central bank ' s policy is to peg the price level, government gives up its ability to inflate away the real value of nominal public liabilities and default on the public debt is inevitable. Or, if it is to peg the nominal interest rate, government obtains its ability to suppress default on the public debt while the price level is no longer stabilized. This argument is consistent with our at glance. Through a series of default scare stemming from Greek debt crisis, the SI-SD trade-off (SI-SD Trade-off) precisely found by Uribe (2006) will be emphasized more and more.

However, we find that there is not necessarily such the SI-SD trade-off or even if there is such a trade-off, it is not so much as emphasized by Uribe[18], by revisiting Uribe[18]'s fiscal theory of sovereign risk (FTSR) from dynamic stochastic general equilibrium (DSGE) with nominal rigidities view which is established by Woodford[19]. That is, we develop a class of DSGE model with nominal rigidities and find that stabilizing inflation is not inconsistent with suppressing default. Further, we have a policy implication that by stabilizing inflation, default risk should be stabilized and this policy implication is quite different from Uribe[18]'s.

Uribe[18]'s FTSR seems strongly affects the ECB on conducting monetary policy. Amid sovereign debt crisis, conducting monetary policy seems extremely difficult. Even if Greece did not go into default at that time when huge fiscal deficit is revealed on October 2009, Greece 10-Year Credit Default Swap premium started to soar and reached USD 20,404 on April 2012, the ECB faced on difficulty on conducting monetary policy. While HCPI inflation started to increase from -0.6% on July 2009, ECB's policy interest rate, short run buying operation rate, was left on 1% until April 2011 when HCPI inflation reached 2.8%. The ECB seemed reluctant to stabilize inflation because of sovereign debt problems in Greek and aware of Uribe[18]'s FTSR.

In this paper, we double-check Uribe[18] by developing a class of DSGE models with nominal rigidities and compare the optimal monetary (OM) policy and the optimal monetary and fiscal (OMF) policy with the minimizing the interest rate spread which is difference between it for safty asses and the government debt's yield excluding the default risk, namely, the nominal interest rate for risky assets (MIS) policy in an economy with sovereign risk. Note that the OM and OMP policies correspond to Taylor rule and the price level targeting in Uribe[18] because these polisies are de facto inflation stabilization policy and the MIS policy corresponds to the interest rate peg in Uribe[18] because these policies set the expected default rate to zero or minimize it. While we basically adopt Uribe[18]'s default rule, we turn our attention to fiscal balance which is an exogenous shock in Uribe[18] and we notice that this setting generates Uribe[18]'s policy implication that there is the SI-SD trade-off. We introduce government into our model and equip the default rule following Uribe[18]. The most important thing in our model is endogenized production which is commonplace setting in literature of the OM policy and generates policy implication quite different from Uribe[18]. Thus, the difference on results or policy implication between us and him depends on whither exogenous production or endogenous production.

Now, we review Uribe[18]'s FTSR. By iterating government budget constraint forward and imposing appropriate transversality condition, Uribe[18] shows that the default rate depends on the ratio of the net present value of the real fiscal surplus to real government debt with interest payment. That is, the default rate depends on the ratio of government solvency to the burden of government debt redemption. Thus, a decrease in the fiscal surplus which is exogenous in his setting, government solvency decreases. Facing such a case, if the central bank stabilizes inflation, the burden of government debt redemption cannot be mitigated, the default rate increases. If the central bank give up to stabilize inflation, the burden of government debt redemption can be mitigated by inflation which decreases real government debt and the default is mitigated. This is Uribe[18]'s FTSR hinted by Cochrane[7], Leeper[14] and Woodford[20]'s 'fiscal theory of price level' and the FTSR shows that there is the SI-SD trade-off.

How does endogenized production derive quite different results? First of all, remember that the fiscal surplus is difference between the tax revenue and the government expenditure and tax is levied on the output. Although the government expenditure is practically exogenous, the output is endogenous and the tax rate is one of policy instrumenst under the OMF policy in our paper. The most important thing is that the fiscal surplus not only deeply involves the default rate but also deeply involves the inflation through involving the output gap. That is, stabilizing fiscal surplus not only stabilizes the default rate but also stabilizes both the inflation and the output gap. Note that the OM policy and the OMF policy are de facto inflation stabilization policies because volatility on inflation absolutely occupies welfare costs which stems from households' utility.

Suppose that an increase in productivity which is practically exogenous under the OMF policy where the nominal interest rate and the tax rate are policy instrument. Facing an increase in productivity which applies pressure to increase inflation because productivity shock works as a cost-push shock, the tax rate is cut by the government to lower the marginal cost. In addition, this tax cut applies to increase output gap and cancel pressure to decrease output gap stemming from an increase in a productivity. Inflation-output gap trade off completely dissolved by coping with the central bank whose policy instrument is the nominal interest rate (Because basic mechanism on stabilizing inflation in DSGE models is familiar, we skip to explain why stabilizing output gap brings stabilize inflation). Although an increase in productivity applies pressure to fluctutae the tax revenue thus the fiscal surplus, the tax cut applies stabilizes the tax revenue thus the fiscal surplus. Because fiscal surplus is more stabilized than one under the OM policy where the tax rate is constant over time, the default rate is more stable than one under the OM policy. Under the OM policy, inflation is more fluctuate than under the OMF policy because just the nominal interest rate is available to stabilize inflation under the OM policy. In short, the more stabilized the inflation, the more stabilized the default rate and vice versa. Thus, there is not a necessarily trade-off between stabilizing inflation and suppressing default. Mechanism on the case of an increase in government expenditure is quite smilar-the government operates the tax rate to stabilize the inflation and the default rate is stabilized as a result via stabilizing the fiscal surplus–and inflation is completely stabilized and the default rate is well stabilized. Likewise, there is not a necessarily trade-off between stabilizing inflation.

Finally, we discuss the relationship between our results and those in some previous important works focusing on sovereign risk or crises. Arellano[1] develops a model in which default probability depends on some stochastic process and shows that default is more likely in recessions. He succeeds his model matching with data in Argentina and his assumption on mechanism of default is succeeded by Mendoza and Yue[15] and Corsetti et al.[9]. Mendoza and Yue[15] try to explain negative relationship between output and default which is observed via data. That is, they clarify the reason why sovereign default accompanied by deep recession. Corsetti et al.[9] develop the model with financial intermediaries and show that sovereign risk may give rise to indeterminacy and implies that fiscal retrenchment via government spending cut can help curtail the risk of macroeconomic instability and, in extreme cases, even stimulate economic activity. Their model stems from Curdia and Woodford[10] and is aware of the zero lower bound of nominal interest rate. Corsetti and Dedola[8] develop a model of sovereign debt crisises driven by either self-fulfilling expectations or weak fundamentals and analyze the mechanism by which either conventional or unconventional monetary policy can rule out former. The finding that self-fulfilling debt crises can be ruled out by swapping government debt for monetary liabilities which is one of unconventional monetary policy. Similar to us, Bacchetta, Perazzi and Wincoop[2] develop a class of DSGE model and analyze both conventional and unconventional monetary policy. They find that the central bank cannot credibly avoid a self-fulfilling debt crisis. Okano and Hamano[17] expand the model derived by Uribe[18] to a currency union setting and try to give policy subscription for EMU.

Here, we make it clear where we are different from them. Except for Corsetti and Dedola[8] and Bacchetta, Perazzi and Wincoop[2], their concerns are how sovereign default macroeconomic dynamics, especially dynamics on output while we focus on how the OMF policy affects the default. Although Corsetti and Dedola[8] and Bacchetta, Perazzi and Wincoop[2] analyze monetary policy, they do not concern about fiscal policy or do not treat it as a stabilization or welfare cost minimization tool. Thus, our concerns is not identical with their concerns and it can be said that we propose fiscal policy to suppress both stabilizing inflation and suppressing default risk while they propose monetary policy to suppress default risk.[1] Okano and Hamano[17] analyze just the OM and the OMF policies and they cannot find that the OMF policy well stabilizes the default rate especially to an increase in government expenditure.

Repeatedly we emphasize that even previous work obtain important implications, neither those previous works do not mention a trade-off between inflation stabilization and preventing default risk. Even Uribe[18] mentions that trade-off, we emphasis that there is not necessarily a trade-off between stabilizing inflation and preventing default risk. Needless to say, neither Uribe[18] nor Corsetti et al.[9] and Mendoza and Yue[15] do not derive this result. Mentioning this trade-off and finding the opposite result is our greatest contribution.

The remainder of the paper is organized as follows. Section 2 develops the model. Section 3 solves the linear-quadratic (LQ) problem, shows the First-order necessary conditions (FONCs) for the policy authorities, and discusses the taxation and non-taxation regimes. Section 4 calibrates the model under both regimes. Section 5 concludes the paper. Appendices provide some empirical evidences.

2 The Model

Following Okano[16] who analyze the OMF policy in a currency union, we introduce firms into Uribe[18]'s model and develop a class of DSGE models with nominal rigidities.[2] Thus, the default mechanism is quite similar to Uribe[18]. We follow Benigno[3] (an earlier working paper version of Benigno[4]) to clarify the households ' choice of risky assets.

The households on the interval $[0, 1]$ and own firms. We adopt Calvo pricing and assume that tax is levied on output and is distorted. Thus, monopolistic power remains and the steady state

[1] Because they do not focus on fiscal policy and their model is not suitable for analyzing fiscal policy while our model can analyze and evaluate the effect of fiscal policy. Government in Arellano[1] does not levy tax on any economic agents, Mendoza and Yue[15], Corsetti et al.[9] and Bacchetta, Perazzi and Wincoop[2] assume lump-sum tax or transfer and assume lump-sum tax. Thus, fiscal policy cannot be analyzed under their setting. In our paper, government handle policy interest rate and tax rate to minimize welfare costs and fiscal policy can be analyzed specifically and we can watch the effects of the OMF policy to default easily. This is our advantage to those authors from a viewpoint of model building.

[2] Okano[16] derives model following Gali[12] and Ferrero[11].

is distorted.

2.1 Households

A representative household's preferences are given by:

$$\mathcal{U} \equiv \mathrm{E}_0 \left(\sum_{t=0}^{\infty} \beta^t U_t \right), \tag{1}$$

where $U_t \equiv \ln C_t - \frac{1}{1+\varphi} N_t^{1+\varphi}$ denotes the period utility, E_t is the expectation conditional on the information set at period t, $\beta \in (0,1)$ is the subjective discount factor, C_t is the consumption index, $N_t \equiv \int_0^1 N_t(i)\,dh$ is the hours of labor and φ is the inverse of the elasticity of labor supply.

The consumption index of the continuum of differentiated goods is defined as follows:

$$C_t \equiv \left[\int_0^1 C_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{2}$$

where $C_t(i)$ is a generic good and $\varepsilon > 1$ is the elasticity of substitution across goods.

The price level is defined as follows:

$$P_t \equiv \left[\int_0^1 P_t(h)^{1-\varepsilon} dh \right]^{\frac{1}{1-\varepsilon}} \tag{3}$$

where $P_t(i)$ is price of a generic good.

By solving cost-minimization problems for households, we have the optimal allocation of expenditures as follows:

$$C_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon} C_t. \tag{4}$$

By aggregating households' budget constraints, we have the aggregated budget constraints in countries H and F as follows:

$$R_{t-1} \left[D_{t-1}^n + B_{t-1}^n \Gamma\left(-sp_{t-1}\right)(1-\delta_t) \right] + W_t N_t + PR_t \geq P_t C_t + D_t^n + B_t^n, \tag{5}$$

where $R_t \equiv 1 + r_t$ denotes the gross (risk-free) nominal interest rate, r_t is the net interest rate , D_t^n denotes the nominal state contingent claim and B_t^n are the nominal government debts, respectively, , W_t is the nominal wage, PR_t denotes profits from the ownership of the firms, δ_t is the default rate and $sp_t \equiv \frac{SP_t}{SP} - 1$ is the percentage deviation of (real) real fiscal surplus from its steady state value, $SP_t \equiv \tau_t Y_t - G_t$ denotes the real fiscal surplus, τ_t denotes the tax rate, $Y_t \equiv \left[\int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}$ denotes (aggregated) output and $G_t \equiv \left(\int_0^1 G_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}$ denotes (aggregated) government expenditure. Further, we define V as the steady state value of any variables V_t. Thus, SP is the steady state value of fiscal surplus.

Now we discuss the term $R_t \Gamma(-sp_t)$ where $\Gamma'(-sp_t) > 0$ by assumption.[3] Our assumption implies that government decides to government debt coupon rate depending on its fiscal situation. If the fiscal situation becomes worse, the government dicides to hike the coupon rate. If the fiscal

[3] As long as government debt in the steady state is positive, fiscal surplus in the steady state is positive in our model. Even if the percentage deviation of fiscal surplus turns to negative, it cannot be said fiscal balance turns negative at onece because fiscal surplus is not zero but positive in the steady state. To make the fiscal balance negative, the fiscal surplus must be below its steady state value at least.

situation becomes bettwe, the government dicides to lower the coupon rate. Note that the government debt coupon rate $R_t\Gamma(-sp_t)$ is not government debt yield which is decided fully endogenized. In our setting, the government debt yied is decided by households' itertemporal optimal condition, namely, Euler equation. Thus, the government debt yied is decided dndogenously although the government debt coupon rate depends on our assumption.

As mentioned, the function $\Gamma(-sp_t)$ is hinted by Benigno[3] although the details are somewhat different from Benigno[3]. Benigno[3] assumes that households in home country face a burden in international financial markets. As borrowers, households in home country will be charged a premium on the foreign interest rate; as lenders, they will receive remuneration lower than the foreign interest rate. Following his setting, Benigno[3] assumes $\Gamma'(\cdot) < 0$, which implies that the higher country F's government debt, the lower the remuneration for holding country F's government debt.[4] However, on the contrary, our setting implies that the lower the fiscal surplus, the lower the remuneration for holding country F's government debt because of default. Thus, the government has to pay additional remuneration for holding government debt, which provides households with a motivation for holding government debt. Thus, we assume that $\Gamma'(\cdot) > 0$. That is, the lower the fiscal surplus, the higher the interest rate multiplier. Another assumption different from Benigno[3] is that $\Gamma(\cdot)$ is a function of the fiscal surplus while Benigno[3] assumes that it is a function of current government debt with interest payment, namely R_tB_t. Our setting on $\Gamma(\cdot)$ is following Corsetti, Kuester, Meier and Muller[9] indirectly. Corsetti, Kuester, Meier and Muller[9] assumes that the higher the fiscal deficit, the higher the probability of default and visa versa. If we are given that the higher the probability of default, the higher the nominal ineterst rate for risky assets, our assumption that $\Gamma(\cdot)$ is an decreasing function of fiscal surplus is consistent. Further, our setting on $\Gamma(\cdot)$ is supported by some empirical evidence.[5] Thus, our assumption on $\Gamma(\cdot)$ is consistent with some previous work and data.

The representative household in a currency union maximizes Eq.(1) subject to Eq.(5). The optimality conditions are given by:

$$\beta \mathrm{E}_t\left(\frac{P_tC_t}{P_{t+1}C_{t+1}}\right) = \frac{1}{R_t}, \tag{6}$$

which is intertemporal optimality condition and:

$$C_tN_t^{\varphi} = \frac{W_t}{P_t}, \tag{7}$$

which are standard intratemporal optimality conditions.

Because we introduce default risk for government debt, households are not indifferent between holding the state contingent claims and holding government debt. Even if holding government debt, Eq.(6) is still applied, however, it requires an additional condition, which depicts households' motivation to hold government debt, stemming from one of the FONCs for households, which can be obtained by differentiating the Lagrangean by government nominal debt and is given by:

$$\beta \mathrm{E}_t\left(\frac{P_tC_t}{P_{t+1}C_{t+1}}\right) = \frac{1}{R_t\left\{\Gamma(-sp_t) + B_t\Gamma'(-sp_t)\left[B(R-1)\right]^{-1}\right\}\mathrm{E}_t(1-\delta_{t+1})}. \tag{8}$$

[4]Benigno[3] mentions that this function, which depends only on the level of real government bonds in his setting, captures the costs of undertaking positions in the international asset market or the existence of intermediaries in the foreign asset market.

[5]See Appendix C for details.

Combining Eqs.(8) and (9), we have:

$$B_t = \frac{\left[\frac{1}{\mathrm{E}_t(1-\delta_{t+1})} - \Gamma\left(-sp_t\right)\right] SP}{\Gamma'\left(-sp_t\right)}, \tag{9}$$

which is demand schedule on the government debt. Log-linearizing Eq.(9), we have:

$$b_t = \frac{1-\beta}{\beta} sp_t + \frac{1-\beta}{\phi\beta} \mathrm{E}_t\left(\delta_{t+1}\right), \tag{10}$$

where $\phi \equiv \Gamma'(0)$ denotes the interest rate spread in the steady state. Following Benigno[3], we define the interest rate spread for government debtϕ and assume $\Gamma(0) = 1$. Eq.(10) is log-linearized demand schedule on the government debt Eq.(9). The coefficient of the first term in the RHS is positive and implies that an increase in the fiscal surplus increases demand for government debt, and vice versa. An increase in fiscal surplus increases government's solvency and house holds prefere to purchase government debt. This is not true. The reason is that an increase in fiscal surplus decreases returns on holding government debt through decreasing interest rate multiplier. Eq.(9) also can be rewritten as:

$$R_t = R_t \mathrm{E}_t\left(1-\delta_{t+1}\right)\left\{\Gamma\left(-sp_t\right) + B_t\Gamma'\left(-sp_t\right)\left[B\left(R-1\right)\right]^{-1}\right\},$$

which shows that the marginal rate of substitution for consumption is same either households hold state contingent claim D_t or government debt B_t. That is schedule of consumption is same either households hold state contingent claim D_t or government debt B_t. An increase in fiscal surplus in period t decreases payoff on government debt held by households in $t+1$ and this decrease brings less consumption in period $t+1$ compared with the case of holding state contingent claim. To keep same consumption in period $t+1$, in that case, households have to purchase more government debt in period t which brings more consumption in period $t+1$ Because of holding more government debt in period t, consumption in period t decreases. As a result, the marginal rate of substitution for consumption is not changed. On contrary, a decrease in fiscal surplus in period t decreases demand for government debt in period t through an increase in interest rate multiplier. Because an increase in interest rate multiplier brings more consumption in period $t+1$, households slush holding government debt and keep the same consumption schedule. Thus, demand for government debt is an increasing function of fiscal surplus. The same can be said of the sign of the second term in RHS. Positive sign of the second term in RHS shows that an increase in the expected default rate increases demand for government debt and vice versa. Although this relationship is not intuitive, however, the fundamental mechanism is similar to the relationship between demand for government debt and the fiscal surplus. An increase in the expected default rate decreases payoff on debt held by households in period $t+1$ and this decrease brings less consumption in period $t+1$ compared with the case of holding state contingent claim. Thus, households have to purchase more government debt in period t to keep same consumption in period $t+1$ As a result, the marginal rate of substitution for consumption is not changed because consumption in period t decreases via purchasing more government debt in period t. On contrary, a decrease in the expected default rate increases the payoff in period $t+1$ and households have to slush holding government debt in period t. Thus, demand for government debt is an increasing function of the expected default rate. This phenomenon is regarded as one of the wealth effect.

Another feature on Eq.(10) is that the equality shows that the default risk disappears when there are no interest rate spreads. In fact, by plugging $\phi = 0$ into Eq.(10), we have:

$$\mathrm{E}_t\left(\delta_{t+1}\right) = 0.$$

For convenience, we define $R_t^G \equiv R_t \Gamma\left(-sp_t\right)$ and $R_t^H \equiv R_t \left\{\Gamma\left(-sp_t\right) + B_t \Gamma'\left(-sp_t\right)\left[B\left(R-1\right)\right]^{-1}\right\}$, and dub R_t^G and R_t^H are government debt's counpon rate and the government bond yield, respectively. Log-linearizing the definition of the nominal interest rate for the government debt and the government bond yield are given by:

$$\hat{r}_t^G = \hat{r}_t - \phi sp_t, \tag{11}$$

$$\hat{r}_t^H = \frac{\omega_\phi}{1-\beta}\hat{r}_t - \frac{\phi\omega_\gamma}{1-\beta}sp_t + \frac{\phi\beta}{1-\beta}b_t, \tag{12}$$

with $\hat{r}_t^G \equiv \frac{dR_t^G}{R^G}$ and $\hat{r}_t^H \equiv \frac{dR_t^H}{R^H}$, with $\omega_\gamma \equiv 1 + \beta\left(\gamma - 1\right)$ and $\omega_\phi \equiv 1 - \beta\left(1 - \phi\right)$ where $\gamma \equiv \frac{\Gamma''(0)}{\Gamma'(0)}$ denotes the elasticity of the interest rate spread to a one percent change in the fiscal deficit in the steady state. The elasticity γ is an unfamiliar parameter and we assume $\mid \Gamma'\left(\cdot\right) \mid < \mid \Gamma''\left(\cdot\right) \mid$ thus $\gamma > 1$. Our assumption implies that a decrease in fiscal surplus increases interest rate for government debt via an increase in interest rate multiplier and vice versa and one parcent change in fiscal surplus increases causes further changes in interest rate for government debt. Note that our assumption is supported by data and we discuss in section 5.1.2 where we try to estimate the elasticity of the interest rate spread to a one percent change in the fiscal deficit γ. Given our assumption, Eq.(10) implies that an increase in fiscal surplus decreases expected default rate and vice versa. As we mention in section 2.3, fiscal surplus is decreasing function of government debt. Thus, Eq.(10) also implies that an increase in government debt increases expected default rate, as long as we assume $\mid \Gamma'\left(\cdot\right) \mid < \mid \Gamma''\left(\cdot\right) \mid$.

2.2 Government

2.2.1 Government Budget Constraint and the FTPL

Fiscal policy consists of choosing the mix between taxes and one period nominal debt with sovereign risk to finance exogenous process of government expenditure. The flow government budget constraint is given by:

$$B_t^n = R_{t-1}^G\left(1 - \delta_t\right)B_{t-1}^n - \int_0^1 P_t\left(i\right)\left[\tau_t Y_t - \left(h\right)G_t\right]di.$$

Because optimal allocation of generic goods are given by $Y_t\left(i\right) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} Y_t$ and $G_t\left(i\right) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} G_t$, similar to Eq.(4), this equality can be rewritten as:

$$B_t^n = R_{t-1}^G\left(1 - \delta_t\right)B_{t-1}^n - P_t SP_t.$$

Dividing both sides of this equality yields:

$$B_t = R_{t-1}\Gamma\left(-sp_{t-1}\right)\left(1 - \delta_t\right)B_{t-1}\Pi_t^{-1} - SP_t. \tag{13}$$

with $\Pi_t \equiv \frac{P_t}{P_{t-1}}$ being gross inflation rate. The first term on the RHS correspond to the amount of redemption with the nominal interest payment and shows that the lower the fiscal surplus, the higher the interest payments.

Log-linearizing Eq.(13) yields:

$$b_t = \frac{1}{\beta}\hat{r}_{t-1} - \frac{1}{\beta}\delta_t - \frac{1}{\beta}\pi_t + \frac{1}{\beta}b_{t-1} - \frac{1-\beta}{\beta}sp_t - \frac{\phi}{\beta}sp_{t-1}, \tag{14}$$

where we use Eq.(11). Eq.(14) implies that not only the higher the current fiscal surplus but also the higher the past fiscal surplus the lower the current government debt because an increase in fiscal surplus decreases the interest payment via a decrease in interest rate multiplier.

The appropriate transversality condition for the government debt is given by:

$$\lim_{j\to\infty} \beta^{t+j+1}\mathrm{E}_t\left[R^G_{t+j}\left(1-\delta_{t+j+1}\right)\frac{P_{t+j}B_{t+j}}{P_{t+j+1}}\right] = 0.$$

Iterating the second equality in Eq.(13) forward, plugging Eq.(6) into this iterated equality and imposing the appropriate transversality condition for the government debt, we have:

$$C_t^{-1}R^G_{t-1}B_{t-1}\Pi_t^{-1}\left(1-\delta_t\right) = C_t^{-1}SP_t + \beta\frac{R^H_t}{R^G_t}\mathrm{E}_t\left(C^{-1}_{t+1}SP_{t+1}\right) + \beta^2\mathrm{E}_t\left(\frac{R^H_t}{R^G_t}\frac{R^H_{t+1}}{R^G_{t+1}}C^{-1}_{t+1}SP_{t+1}\right) + \cdots. \tag{15}$$

This equality, shows that the government debt with interest payment in terms of marginal utility of consumption, the LHS corresponds to the expected sum of discounted value of fiscal surplus in terms of marginal utility of consumption, the RHS because of the transversality condition. Here, $\beta\frac{R^H_t}{R^G_t}$ and so forth can be regarded as some discount factor. As the Euler equation, Eqs.(6) and (8) shows that the households' intertemporal marginal rate of substitution corresponds to the nominal interest rate $R_t = R^H_t\mathrm{E}_t\left(1-\delta_{t+1}\right)$, fraction $\frac{R^H_t}{R^G_t}$ and so forth appears. If $R^G_t = R^H_t$ is applied for all t, which implies that the nominal interest rate for the government debt corresponds to the government bond yield, this equality boils down to:

$$C_t^{-1}R^G_{t-1}B_{t-1}\Pi_t^{-1}\left(1-\delta_t\right) = C_t^{-1}SP_t + \beta\mathrm{E}_t\left(C^{-1}_{t+1}SP_{t+1}\right) + \beta^2\mathrm{E}_t\left(C^{-1}_{t+1}SP_{t+1}\right) + \cdots.$$

Eq.(15) can be rewritten as:

$$\delta_t = 1 - \frac{\frac{R^G_{t-1}}{R^H_{t-1}}\sum_{k=0}^{\infty}\prod_{h=0}^{k}\beta^k\mathrm{E}_t\left(\frac{R^H_{t+h-1}}{R^G_{t+h-1}}C^{-1}_{t+k}SP_{t+k}\right)}{C_t^{-1}R^G_{t-1}B_{t-1}\Pi_t^{-1}}. \tag{16}$$

Eq.(A) is not Uribe[18]'s but our FTSR and implies that an increase in inflation, does not necessarily occur if solvency is less than the government's debt. Not only inflation, but also default, mitigates the government's liability. Suppose that price level is constant and there is no inflation. In this situation, if the solvency is about equal to the government's liability, the RHS is less than unity. On the other hand, the LHS is definitely less than unity through an increase in the default rate. In other words, if the government falls insolvent while price level is strictly stable, default is inevitable. Uribe[18] pointed out that there is a trade-off between inflation stabilization and suppressing default by introducing default, namely sovereign risk, into the central equation of the FTPL. Similar to Uribe[18], at first glance Eq.(A) also implies that there is such a trade-off. Furthermore, he calibrates his model and compares it with the Taylor rule, which stabilizes inflation and the interest rate peg. Under the interest rate peg, the interest rate on risky assets corresponds to the risk-free asset interest rate pegged to the steady-state rate. This calibration further shows that default ceases just one period after the shock decreased the fiscal surplus, although default continues under the Taylor rule. This result implies that a Taylor rule to stabilize inflation includes the unwelcome possibility of magnifying sovereign risk, and this calls for an interest rate peg to counter default. Although Uribe[18] ignores the welfare perspective of these actions, his policy implications are persuasive. Paying attention to just Eq.(A), which is similar to Uribe's[18] model, we seem to obtain policy implications quite similar to those in Uribe[18].

We now show on the relationship between Eq.(A) and the Fiscal Theory of the Price Level (FTPL). If there are neither the default risk nor interest rate multiplier in Eq.(A), Eq.(A) boils down to as follows because of $R_t^G = R_t^H = R_t$:

$$1 = \frac{\sum_{k=0}^{\infty} \beta^k \mathrm{E}_t \left(C_{t+k}^{-1} SP_{t+k} \right)}{C_t^{-1} R_{t-1} B_{t-1} \Pi_t^{-1}},$$

which is the central equation of the FTPL. On the RHS in this equality, the numerator is the net present value of the sum of fiscal surplus in terms of the marginal utility of consumption, namely solvency, and the denominator is the past nominal government debt with interest payments divided by current price level, namely the government's liability. The LHS is unity. Thus, this equality implies that solvency definitely corresponds to the government's liability. If solvency is less than the government's liability, price level increases, that is, inflation occurs so that the government's liability is mitigated. For now, we introduce sovereign risk and this mechanism is no longer applicable, as Eq.(A) implies.

2.2.2 Default Rule

Eq.(A) and its above equality are also the default rule which is analogous to one shown by Uribe[18] who considers a policy rule the government does not default unless the tax-to-debt-ratio below a certain threshold. Let define $\Psi \equiv \frac{\frac{R_{t-1}^G}{R_{t-1}^H} \sum_{k=0}^{\infty} \prod_{h=0}^{k} \frac{R_{t+h-1}^H}{R_{t+h-1}^G} \beta^k \mathrm{E}_t \left(C_{t+k}^{-1} SP_{t+k} \right)}{C_t^{-1} R_{t-1}^G B_{t-1} \Pi_t^{-1}}$ where Ψ denotes the threshold chosen arbitrary by government. Around the steady state, our threshold Ψ is 1. If the solvency over the government's liability corresponds to Ψ, that is, $\Psi = 1$, the government has no motivation to default on and chooses $\delta_t = 0$.[6] If the solvency is smaller than the government's liability, namely $\Psi < 1$, the government has an incentive to default go on and chooses $\delta_t > 0$. If the solvency exceeds the government's liability, namely $\Psi > 1$, the government subsidizes debt holders and choose $\delta_t < 0$. In the steady state, $\Psi = 1$ is applied and we assume that government chooses $\delta_t = 0$.

2.2.3 Relationship between Default Rate and Fiscal Surplus

By leading Eq.(A) one period and plugging this Eq.(A) itself, we can rewritten Eq.(A) to a second-order differential equation as follows:

$$\delta_t = 1 - \frac{1}{R_{t-1}^G \Pi_t^{-1} B_{t-1}} \left\{ SP_t + \beta \mathrm{E}_t \left[\left(\frac{C_t}{C_{t+1}} \Pi_{t+1}^{-1} \right) R_t^H \left(1 - \delta_{t+1} \right) B_t \right] \right\}. \tag{17}$$

In Eq.(17), still current government debt B_t appears in the second term in the RHS. That is, a decrease in current government debt increases default rate, and visa versa. Why does current government debt B_t appear in the second term in the RHS ? This stems from the transversality condition for the government debt. Because of the transversality condition, Eq.(A) and its second

[6]To be exact, the solvency and the government's liability are evaluated by the marginal utility of consumption and the redeeming cost is affected by inflation.

order differential version Eq.(17) are strictly applicable. That is, once government debt is issued, this is redeemed. Otherwise, the burden of redemption is mitigated by default or inflation. To keep Eq.(A), once government debt is issued, fiscal surplus must be improved while newly issued government debt is about to slush fiscal surplus. Because fiscal surplus must be improved to redeem, default rate declines when government debt increases. This is the reason why current government debt B_t appears in the second term in the RHS.

In addition, we can imagine fiscal surplus is a function of output gap easilly. In fact, log-linearized fiscal surplus is given by:

$$sp_t = \frac{\beta\tau}{(1-\beta)\,\varsigma_B}\hat{r}_t + \frac{\beta\tau}{(1-\beta)\,\varsigma_B}y_t - \frac{\beta\varsigma_G}{(1-\beta)\,\varsigma_B}g_t. \tag{18}$$

with $\varsigma_B \equiv \frac{B}{Y}$ and $\varsigma_G \equiv \frac{G}{Y}$ being the steady state share of government debt to output and the steady state share of government expenditure, respectively. By using Gali and Monacelli's[13] definition of output gap, namely, $\tilde{y}_t \equiv y_t - \bar{y}_t$, where $\tilde{y}_t$ and $\bar{y}_t$ denotes output gap and the natural rate of output, respectively, stabilizing fiscal surplus leads to stabilize output gap.[7] As mentioned, stabilizing default rate needs to stabilizing fiscal surplus. Thus, it can be said that there is not necessarily a trade-off between stabilizing inflation and default rate.

To make sure, we review the relationship between default rate and fiscal surplus under Uribe's[18]. In his setting, that is, fiscal surplus is exogenous and follows AR(1) process, Eq.(A) can be rewritten as:

$$\delta_t = 1 - \frac{SP_t}{R_{t-1}\Gamma\left(-sp_{t-1}\right)B_{t-1}\Pi_t^{-1}\left(1-\beta\varrho\right)},$$

with ϱ being AR(1) coefficient of the exogenous fiscal surplus. Note that consumption is constant in Uribe[18] and consumption disappears in this equality. Different from Eq.(15), the default rate is clearly a decreasing function of fiscal surplus. The higher the fiscal surplus, the lower the default rate, and visa versa. In addition, this equality shows that an increase in inflation decreases the default rate, and visa versa. Because of endowment economy in Uribe[18], there is no supply curve such as the NKPC, inflation is decided by this kind of equality in Uribe[18]. Thus, there is the SI-SD trade-off rate in Uribe[18]. However, this trade-off is not necessarily applicable if the fiscal surplus is endogenized.

2.2.4 Log-linearizing the Government Budget Constraint

Log-linearizing Eq.(17) yields:

$$\begin{aligned}\delta_t \quad = \quad & \hat{r}_{t-1} - \phi sp_{t-1} + b_{t-1} - \pi_t + \beta \mathrm{E}_t\left(c_{t+1}\right) - \beta c_t - \frac{\beta\omega_\phi}{1-\beta}\hat{r}_t - \frac{\omega_{sp}}{1-\beta}sp_t - \frac{\beta\omega_\phi}{1-\beta}b_t + \beta\mathrm{E}_t\left(\pi_{t+1}\right) \\ & + \beta\mathrm{E}_t\left(\delta_{t+1}\right).\end{aligned} \tag{19}$$

with $\omega_\phi \equiv 1-\beta\left(1-\phi\right)$ and $\omega_{sp} \equiv \left(1-\beta\right)^2 - \phi\omega_\gamma\beta$. Because Eq.(17) is not only the Euler equation, but also iterated government budget constraint with default risk, Eq.(19) is somewhat complicated. In the RHS, the expected default rate $\mathrm{E}_t\left(\delta_{t+1}\right)$ and the current default rate appears in the fourth and fifth term. An increase in the expected default rate mitigates a burden of redeeming

[7] In our model, steady state is not efficient because friction stemming from monopolistical competitive market cannot be dissolved by taxation. Thus, the target level of output gap or efficient output gap is not zero although the target level is zero in Gali and Monacelli[13] because the steady state is efficient.

government debt and the solvency shall decline. A decrease in the solvency lowers marginal utility of consumption, if the fiscal surplus does not change. The Consumption which corresponds to the inverse of marginal utility of consumption in our model increases. Thus, the sign of the expected default rate is positive. On contrary, an increase in the current default implies relatively increase in a burden of redeeming government debt in terms of current marginal utility of consumption. If inflation does not change, consumption must decrease via an increase in marginal utility of consumption, as shown in Eq.(A). Thus, the sign of the current default rate is positive.

The eighth term in the RHS in Eq.(19) is fiscal surplus and this term shows that an increase in fiscal surplus decreases the default rate, at glance. However, this is not necessarily applied. Indeed, our fiscal theory of sovereign risk Eq.(A) as if implies that negative correlation. However, we have to pay attention that an increase in fiscal surplus decreases the percentage deviation of government bond yield $\hat{r}_t^H$ and vice versa, as shown in Eq.(14). That is, an increase in fiscal surplus decreases interest rate multiplier which decreases government debt's yield. As shown in Euler equation Eq.(8), a decrease in the government bond yield R_t^H increases present consumption if other variables do not change. As shown in Eq.(15) or our theory of sovereign risk, government's solvency is measured by not the fiscal surplus but the fiscal surplus interms of marginal utility of consumption which corresponds to inverse of consumption in our setting, an increase in consumption decreases government's solvency which applies pressure to increase default rate. In our parameterization which is mentioned in section 5.1.1, latter pressure is higher than direct pressure to decrease default rate via an increase in fiscal surplus, an increase in fiscal surplus increases the default rate. This means that $\omega_{sp} < 0$.

The second term in the RHS is (logarithmic) previous fiscal surplus sp_{t-1} . The lower the this term, the lower the burden of redeeming government debt via decrease in interest payment for holding government debt, and visa versa. Thus, the lower the previous fiscal surplus, the higher the current consumption because if the burden of redeeming government debt decreases, there is no need to high solvency and rather the solvency must be lowering. As shown in Eq.(A), solvency is measured by marginal utility of consumption, which definitely corresponds to inverse of consumption in our model. Considering this fact, the reason of negative sign of sp_{t-1} because the lower the previous fiscal surplus, the higher the current consumption via decreases in the solvency and the marginal utility of consumption.

Note that Eq.(19) is also log-linearized Euler equation. Eq.(19) can be rewritten as:

$$\begin{aligned} c_t &= \mathrm{E}_t\left(c_{t+1}\right) - \frac{\beta\omega_\phi}{1-\beta}\hat{r}_t + \mathrm{E}_t\left(\pi_{t+1}\right) - \frac{\beta\omega_\phi}{1-\beta}b_t + \mathrm{E}_t\left(\delta_{t+1}\right) - \frac{\omega_{sp}}{\beta\left(1-\beta\right)}sp_t + \frac{1}{\beta}\hat{r}_{t-1} - \frac{1}{\beta}\pi_t \\ &\quad + \frac{1}{\beta}b_{t-1} - \frac{\phi}{\beta}sp_{t-1}. \end{aligned} \tag{20}$$

When we assume $\delta_t = 0$ for all t and $\phi = 0$, that is, there is neither sovereign risk nor an interest rate spread, Eq.(19) boils down to:

$$c_t = \mathrm{E}_t\left(c_{t+1}\right) - \hat{r}_t + \mathrm{E}_t\left(\pi_{t+1}\right) - b_t + \frac{1}{\beta}\hat{r}_{t-1} - \frac{1}{\beta}\pi_t + \frac{1}{\beta}b_{t-1} - \frac{1-\beta}{\beta}sp_t,$$

which can be viewed as log-linearized central equation of the FTPL and implies that a decrease in fiscal surplus needs a decrease in consumption via an increase in the marginal utility of consumption to keep solvency. Otherwise, the price level increases via an increase in the inflation.

2.3 Firms

This subsection depicting the production, price setting and marginal cost and features of the firms is quite similar to Gali and Monacelli[13], although here tax is levied on firm sales and is not constant.[8]

A typical firm in each country produces a differentiated good with a linear technology represented by the production function:

$$Y_t(i) = A_t N_t(i),$$

where $Y_t(i)$ denotes the output of a generic good and A_t denotes the productivity.

Analogous to consumption indexes, we define $Y_t \equiv \left[\int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}}$. Combining these definitions and the PPI indexes, we have:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} Y_t. \tag{21}$$

By combining the production technology in the currency union and Eq.(21), we have an aggregate production function relating to aggregate employment as follows:

$$N_t = \frac{Y_t Z_t}{A_t}, \tag{22}$$

where $Z_t \equiv \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} di$ denotes the price dispersions.

Log-linearizing Eq.(22) yields:

$$n_t = y_t - a_t. \tag{23}$$

Notice that Z_t disappears in Eq.(A) because of $o\left(\|\xi\|^2\right)$.

Each firm is a monopolistic producer of one of the differentiated goods. Each firm sets their prices $P_t(i)$ taking as given P_t and C_t. We assume that firms set prices in a staggered fashion in the Calvo–Yun style, according to which each seller has the opportunity to change its price with a given probability $1-\theta$, where an individual firm's probability of reoptimizing in any given period is independent of the time elapsed since it last reset its price. When a firm has the opportunity to set a new price in period t, it does so in order to maximize the expected discounted value of its net profits. The FONCs for firms are given by:

$$\tilde{P}_t = \frac{\mathrm{E}_t\left(\sum_{k=0}^{\infty} \theta^k \beta^k \tilde{Y}_{t+k} \frac{\varepsilon}{\varepsilon-1} P_{t+k} MC_{t+k}\right)}{\mathrm{E}_t\left(\sum_{k=0}^{\infty} \theta^k \beta^k \tilde{Y}_{t+k}\right)}, \tag{24}$$

where $MC_t \equiv \frac{W_t}{(1-\tau_t) P_t A_t}$ denotes the real marginal cost, $\tilde{Y}_{t+k} \equiv \left(\frac{\tilde{P}_t}{P_{t+k}}\right)^{-\varepsilon} Y_{t+k}$ denotes the demand for goods when firms choose a new price, $\tilde{P}_t$ denotes the newly set prices. Note that we assume government levies tax on firms sales.

By log-linearizing Eq.(24), we have:

$$\pi_t = \beta \mathrm{E}_t(\pi_{t+1}) + \kappa mc_t, \tag{25}$$

[8]Unlike our setting, Gali and Monacelli[13] assume that constant employment subsidies and monopolistic power completely disappear.

with $\kappa \equiv \frac{(1-\theta)(1-\theta\beta)}{\theta}$ being the slop of New Keynesian Philips curve (NKPC). Eq.(25) is fundamental equality of our NKPC.

Substituting Eq.(7) into the definition of the real marginal cost yields:

$$MC_t = \frac{C_t N_t^{\varphi}}{(1-\tau_t) A_t}. \tag{26}$$

Note that the marginal cost in the steady state, which is the inverse of a constant markup, is smaller than one, while the gross tax rate is definitely smaller than one. In such a case, the steady-state wedge between the marginal product of labor and the marginal utility of consumption is not unity. That is, monopolistic power remains because it is unable to be completely absorbed through taxation. As we discuss later, we need to derive our welfare criteria following Benigno and Woodford[5] because monopolistic power is no longer removed completely and the steady state is distorted.

Log-linearizing Eq.(26) yields:

$$mc_t = c_t + \varphi y_t + \frac{\tau}{1-\tau}\hat{\tau}_t - (1+\varphi) a_t, \tag{27}$$

where $\hat{\tau}_t \equiv \frac{d\tau_t}{\tau}$ denotes the percentage deviation of the tax rate from its steady-state value. We simply refer to the percentage deviation of the tax rate from its steady-state value $\hat{\tau}_t$ as the tax gap.

2.4 Equilibrium

2.4.1 Market-Clearing Conditions

Up to now, our model's features do not include market-clearing conditions, which in our analysis are quite similar to that in Gali and Monacelli[13] and Ferrero[11].

The market-clearing condition is given by:

$$Y_t(i) = C_t(i) + G_t(i).$$

By plugging optimal allocation for generic goods including Eq.(4) into the market-clearing condition, we have:

$$Y_t = C_t + G_t. \tag{28}$$

By log-linearizing Eq.(28), we obtain:

$$y_t = \varsigma_C c_t + \varsigma_G g_t. \tag{29}$$

2.4.2 Output, Nominal Interest Rate and Inflation Dynamics

Plugging Eq.(29) into Eq.(20) yields:

$$\begin{aligned} y_t \;=\;& \mathrm{E}_t(y_{t+1}) - \frac{\varsigma_C \omega_\phi}{1-\beta}\hat{r}_t + \varsigma_C \mathrm{E}_t(\pi_{t+1}) - \frac{\varsigma_C \omega_\phi}{1-\beta} b_t + \varsigma_C \mathrm{E}_t(\delta_{t+1}) + \frac{\varsigma_C}{\beta}\hat{r}_{t-1} - \frac{\varsigma_C}{\beta}\pi_t + \frac{\varsigma_C}{\beta} b_{t-1} \\ &+ \frac{\varsigma_C}{\beta}\delta_t - \frac{\varsigma_C \omega_{sp}}{\beta(1-\beta)} sp_t - \frac{\phi \varsigma_C}{\beta} sp_{t-1} + \varsigma_G (1-\rho_G) g_t, \end{aligned} \tag{30}$$

where we assume that the government expenditure follows AR(1) process and $\mathrm{E}_t(g_{t+1}) = \rho_G g_t$. As we discussed in Section 2.2.4, the expected default rate raise consumption while the current default rate lowers the consumption. Because an increase in consumption raises output and vice versa, as shown in Eq.(29), the signs of the expected default rate and the current default rate in the fourth and the fifth term in the RHS are positive and negative, respectively. In the seventh and eighth terms in the RHS in Eq.(30), there are the current and past tax gap. As we discussed in Section 2.2.4, an increase in the current and past fiscal surplus decreases consumption. As shown in Eq.(29), an increase in consumption raise output and an increase in tax gap raises fiscal surplus. Thus, the signs of the current and past tax gap are negative.

Plugging Eq.(10) into Eq.(30) yields:

$$\begin{aligned} y_t &= \mathrm{E}_t\left(y_{t+1}\right) - \frac{\varsigma_C \omega_\phi}{1-\beta}\hat{r}_t + \varsigma_C \mathrm{E}_t\left(\pi_{t+1}\right) - \frac{\varsigma_C\left(1-\beta\right)}{\beta\phi}\mathrm{E}_t\left(\delta_{t+1}\right) + \frac{\varsigma_C}{\beta}\hat{r}_{t-1} - \frac{\varsigma_C}{\beta}\pi_t + \frac{\varsigma_C \omega_o}{\beta^2\phi}\delta_t \\ &- \frac{\varsigma_C \omega_{\varsigma\pi}}{\beta\left(1-\beta\right)}sp_t + \frac{\varsigma_C \omega_o}{\beta^2}sp_{t-1} + \varsigma_G\left(1-\rho_G\right)g_t, \qquad (31) \end{aligned}$$

with $\omega_{\varsigma\pi} \equiv \left(1-\beta\right)\omega_\phi + \omega_{sp}$ and $\omega_o \equiv \beta\left(1+\phi\right) - 1$.

Plugging Eqs.(27) and (29) into Eq.(25), we have:

$$\pi_t = \beta \mathrm{E}_t\left(\pi_{t+1}\right) + \frac{\kappa\left[1+\varphi\varsigma_C\right]}{1-\varsigma_G}y_t + \frac{\kappa\tau}{1-\tau}\hat{\tau}_t - \frac{\kappa\varsigma_G}{1-\varsigma_G}g_t - \kappa\left(1+\varphi\right)a_t. \qquad (32)$$

We do not introduce novel feature into firms, Eq.(32) mainly stemming from firms' FONC Eq.(17) does not have notable feature. However, in our model tax is levied on firms' sales and the tax rate is not constant in our model, changes in tax gap changes marginal cost, as shown in Eqs.(18) and (19). Thus, the tax gap appears in the third term in the RHS. Eq.(32) implies that an increase in the tax gap lowers the inflation, and vice versa. As we will discuss, the tax gap is strong fiscal policy tool to stabilize inflation because of distorted steady state where the monopolistical competitive power is not dissolved by taxation.

3 Policy Target

We analyze three policies, the OM policy, the OMF policy, the minimizing interest rate difference policy to contrast with Uribe[18] who analyzes inflation stabilization plocy such as Taylor rule and the price level targeting, and the interest rate peg which pegs both the nominal interest rate for safty assets and the nominal interest rate for risky assets. Because the OM and the OMF policies are de facto inflation stabilization policy, thease are clearly correspond to Taylor rule and the price level targeting in Uribe[18]. At glance, there is some difference between the interest rate peg in Uribe[18] and the minimizing interest rate difference policy which minimizes difference between the nominal interest rate $\hat{r}_t$ and the government bond yield $\hat{r}_t^H$. However, as mentioned by Uribe[18], expected default rate converges zero under the interest rate peg in Uribe[18] and the minimizing interest rate difference policy makes the expected default rate close to zero. In fact, as shown in the definition of the government bond yield and Euler equations Eqs.(6) and (9), the expected default rate shal be zero if the nominal interest rate corresponds to the government bond yield completely, that is, $R_t = R_t^H$. Thus, the minimizing interest rate difference policy imitaes the interest rate peg in Uribe[18] in that point. Also, regarding with that the nominal interest rate for safty assets is constant overtime because of constant consumption in Uribe[**?**], we can be convinced that our minimizing interest rate difference policy is definitely corresponds to the interest rate peg in Uribe[18] because there is no difference between the nominal interest rate for safty assets and it for risky asstes in Uribe[18].[9]

Now we discuss on details on each policy. Under the MIS policy, the policy authorities minimize the interest rate spread $\hat{r}_t^S \equiv \hat{r}_t^H - \hat{r}_t$ risk over time. That is, they minimize following:

$$\mathcal{L}^R \equiv \sum_{t=0}^{\infty} \beta^t \mathrm{E}_0 \left(L_t^R \right) \tag{33}$$

with:

$$L_t^R \equiv \frac{1}{2} \left(\hat{r}_t^S \right)^2 .$$

Because the gross nominal interest rate corresponds to the expected government debt yield, namely, $R_t = R_t^H \mathrm{E}_t \left(1 - \delta_{t+1}\right)$, the MIS policy minimizes the expected default rate. As mentioned, from the view point of minimizing the expected default rate, this policy corresponds to the interest rate peg in Uribe[18]. Note that Uribe[18] shows that the default is settled just one period after exogenous negative fiscal surplus shock under the interest rate peg. Because of zero expected default rate, the default no longer occurs after the second period. In our setting, however, even if zero expected default rate, the default may occur after the second period because the fiscal surplus is not a shock but endoginized in our model. By paying attention to Eq.(17), we can be convinced that changes in the inflation cannot absorb the effects of changes in the fiscal surplus completely even if the expected default rate is zero. Thus, the MIS policy cannot necessarily stabilize the default rate. Rather, as we repeatedly mention, the stabilized fiscal surplus contributes to stabilizing the inflation and surpressing the default and there is a possibility that the default rate under the OMF policy is more stabilized than one under the MIS policy.

Under the he the OM policy and the OMF policy, the policy authorities such as the central bank and the government minimize the welfare cost function over time. the period welfare cost function is derived from the welfare criterion. Basically, we derive the welfare criterion following Gali[12]. However, because of distorted steady state, where a linear term which generates welfare reversal, we have to eliminate such a linear term.[10] To eliminate such a linear term, we need to derive the second-order approximated aggregate supply equation which corresponds to Eq.(17) and the second-order approximated inter temporal government solvency condition, which corresponds to Eq.(A). Thus, we follow not only Gali[12] but also Benigno and Woodford[5] and Benigno and Woodford[6] to derive the welfare criterion similar to Ferrero[11]. Note that we impose $R_t^G = R_t^H$ when we derive the second-order approximated inter temporal government solvency condition because of limits of our abilities. However, this restriction has no problem on analyzing a trade-off between inflation stabilization and default risk because our welfare cost function implies that stabilizing inflation is almost only one policy target as shown in later.

Following Gali[12], second ordered approximated utility function is given by:

$$\begin{aligned}\sum_{t=0}^{\infty} \beta^t \mathrm{E}_0 \left(\frac{U_t - U}{U_C C} \right) &= \sum_{t=0}^{\infty} \beta^t \mathrm{E}_0 \left[\frac{\Phi}{1-\varsigma_G} y_t - \frac{(1-\Phi)(1+\varphi)}{\varsigma_C 2} y_t^2 + \frac{(1-\Phi)(1+\varphi)}{1-\varsigma_G} y_t a_t \right. \\ &\quad \left. - \frac{(1-\Phi)\varepsilon}{\varsigma_C 2\kappa} \pi_t^2 \right] + \text{t.i.p.} + o\left(\|\xi\|^3 \right), \end{aligned} \tag{34}$$

[9]Different from Uribe[18], we do not give policy objective exogenously because of determinacy. See Gali and Monacelli[13].

[10]The presence of linear terms generally leads to the incorrect evaluation of welfare simple example of this result is proposed by Kim and Kim[**?**].

where t.i.p. denotes terms independent of policy, $o\left(\|\xi\|^3\right)$ are terms of order three or higher and $\Phi \equiv 1 - \frac{1-\tau}{\frac{\varepsilon}{\varepsilon-1}}$ denotes the steady-state wedge between the marginal rate of substitution between consumption and leisure and the marginal product of labor. In the RHS, there is a linear term generating "Welfere Reversal[11]" To avoid welfare reversal, we have to eliminate the linear term in the RHS in Eq.(34). Following Benigno and Woodford[5] and Benigno and Woodford[6], that linear term can be rewritten as follows:

$$\begin{aligned}\sum_{t=0}^{\infty}\beta^t \mathrm{E}_0\left(\frac{\Phi}{1-\varsigma_G}y_t\right) &= -\sum_{t=0}^{\infty}\beta^t \mathrm{E}_0\left[\frac{\Phi\left[(1-\tau)(1+\omega_g)\omega_{\nu 1} - \tau\omega_{\omega 1}\right]}{2\Gamma\varsigma_C^2}y_t^2\right. \\ &\quad -\frac{\Phi\left[\omega_{\omega 2}\tau - (1-\tau)(1+\omega_g)\omega_{\nu 3}\right]}{\Gamma\varsigma_C^2}y_t g_t - \frac{\Phi(1-\tau)(1+\omega_g)\omega_{\nu 4}}{\Theta\varsigma_C}y_t a_t \\ &\quad \left.+\frac{\Phi(1-\tau)(1+\omega_g)\varepsilon(1+\varphi)}{2\Theta\kappa}\pi_t^2\right] + \Upsilon_0 + o\left(\|\xi\|^3\right),\end{aligned}$$

with $\omega_g \equiv \frac{G}{SP} = \frac{\beta\varsigma_G}{(1-\beta)\varsigma_B}$, $\Theta \equiv (1+\omega_g)(1-\tau)\left[1+\varsigma_C\varphi\right]+\tau\left[1-\varsigma_C(1+\omega_g)\right]$, $\omega_{\nu 1} \equiv \varsigma_C\varphi\left[\varsigma_C(1+2\varphi)+2(2-\varsigma_G)\right]$, $\omega_{\omega 1} \equiv (1+\varsigma_G)\left[1-\varsigma_C(1+\omega_g)\right]$, $\omega_{\omega 2} \equiv \varsigma_C\left[\varsigma_G(1+\omega_g)+\omega_g\right]-2\varsigma_G$, $\omega_{\nu 3} \equiv 1-\varsigma_C\left\{\varsigma_G(1-2\varsigma_G)-\varphi\left[\varsigma_G(2-\varsigma_G)-2\right]\right\}$ and $\omega_{\nu 4} \equiv \varphi\varsigma_C\left[1+2(1+\varphi)\right]+(1+\varphi)(2-\varsigma_G)$ where $\Upsilon_0 \equiv -\frac{\tau\Phi}{\Gamma(1-\beta)}\omega + \frac{(1-\tau)(1+\omega_g)\Phi}{\Theta\kappa}\nu$ denotes a transitory component and ω and ν are the second-order approximated FONC for firms and the second-order approximated solvency condition for government. Plugging that equality into Eq.(30) yields:

$$\sum_{t=0}^{\infty}\beta^t \mathrm{E}_0\left(\frac{U_t - U}{U_C C}\right) \simeq -\mathcal{L} + \Upsilon_0 + \text{t.i.p.} + o\left(\|\xi\|^3\right),$$

where:

$$\mathcal{L} \equiv \sum_{t=0}^{\infty}\beta^t \mathrm{E}_0(L_t) \tag{35}$$

denotes the expected welfare costs,

$$L_t \equiv \frac{\Lambda_x}{2}x_t^2 + \frac{\Lambda_\pi}{2}\pi_t^2,$$

with $\Lambda_x \equiv \frac{\omega_{u1}}{\Theta\varsigma_C^2}$ and $\Lambda_\pi \equiv \frac{\varepsilon\left[\Phi(1-\tau)(1+\omega_g)(1+\varphi)\varsigma_C + \Theta(1-\Phi)\right]}{\Theta\kappa\varsigma_C}$, L_t being the period welfare costs, $x_t \equiv y_t - y_t^*$ being output gap from the target level (OGTL), $y_t^* \equiv \frac{\omega_{u2}}{\omega_{u1}}a_t + \frac{\omega_{u3}}{\omega_{u1}}g_t$ being the target level of output, $\omega_{u1} \equiv \Phi\left[(1-\tau)(1+\omega_g)\omega_{\nu 1} - \tau\omega_{\omega 1}\right]+(1-\Phi)(1+\varphi)\varsigma_C\Theta$, $\omega_{u2} \equiv \varsigma_C\left[\Phi(1-\tau)(1+\omega_g)\omega_{\nu 4} + (1-\Phi)(1+\varphi)\Theta\right.$ and $\omega_{u3} \equiv \Phi\left[\omega_{\omega 2}\tau - (1-\tau)(1+\omega_g)\omega_{\nu 3}\right]$.

4 The LQ Problem

4.1 NKIS, NKPC, Government Budget Constraint and the Fiscal Surplus

Now we arrange our model. Plugging the definition of the OGTL into Eq.(30) yields:

$$\begin{aligned}x_t &= \mathrm{E}_t(x_{t+1}) - \frac{\varsigma_C\omega_\phi}{1-\beta}\hat{r}_t + \varsigma_C\mathrm{E}_t(\pi_{t+1}) - \frac{\varsigma_C(1-\beta)}{\beta\phi}\mathrm{E}_t(\delta_{t+1}) + \frac{\varsigma_C}{\beta}\hat{r}_{t-1} - \frac{\varsigma_C}{\beta}\pi_t + \frac{\varsigma_C\omega_o}{\beta^2\phi}\delta_t \\ &\quad - \frac{\varsigma_C\omega_{\varsigma\pi}}{\beta(1-\beta)}sp_t + \frac{\varsigma_C\omega_o}{\beta^2}sp_{t-1} + \epsilon_{x,t}\end{aligned} \tag{36}$$

[11] Tesar[?] and Kim[?] have used the log-linearization method and derived a paradoxical result that the incomplete-markets economy produces a higher level of welfare than the complete-markets economy. Kim and Kim[?] point out that a reversal of welfare ordering implies that approximation errors due to the linearization.

where $\epsilon_{x,t} \equiv -\frac{\omega_{u2}(1-\rho_A)}{\omega_{u1}} a_t - \frac{\omega_{u3}-\varsigma_G\omega_{u1}(1-\rho_A)}{\omega_{u1}} g_t$ denotes the demand shock and we assume that the productivity follows AR(1) processes and $\mathrm{E}_t(a_{t+1}) = \rho_A a_t$.

Eq.(36) is our version of NKIS. Because of Eq.(10), the terms of the default rate replace the terms of the government debt in Eq.(36).

Plugging the definition of the OGTL into Eq.(32) yields:

$$\pi_t = \beta \mathrm{E}_t(\pi_{t+1}) + \frac{\kappa(1+\varphi\varsigma_C)}{\varsigma_C} x_t + \frac{\kappa\tau}{1-\tau}\hat{\tau}_t + \epsilon_{\pi,t}, \tag{37}$$

where $\epsilon_{\pi,t} \equiv \frac{\kappa[(1+\varphi\varsigma_C)\omega_{u3}-\varsigma_G\omega_{u1}]}{\varsigma_C\omega_{u1}} g_t - \frac{\kappa[(1+\varphi\varsigma_C)\omega_{u2}-(1+\varphi)\omega_{u1}]}{\varsigma_C\omega_{u1}} a_t$ denotes the cost push sock. Eq.(37) is our version of NKPC.

Plugging Eq.(10) into Eq.(14) yields:

$$sp_t = \frac{1}{2(1-\beta)}\hat{r}_{t-1} - \frac{1}{2(1-\beta)}\delta_t - \frac{1}{2(1-\beta)}\pi_t + \frac{1}{2\beta} sp_{t-1} + \frac{1-\beta}{2\phi\beta}\mathrm{E}_t(\delta_{t+1}), \tag{38}$$

which is still government budget constraint although government debt dissapears because of demand schedule for the government debt Eq.(10).

Plugging the definition of the OGTL into Eq.(18) yields:

$$sp_t = \frac{\beta\tau}{(1-\beta)\varsigma_B}\hat{\tau}_t + \frac{\beta\tau}{(1-\beta)\varsigma_B} x_t + \epsilon_{sp,t}, \tag{39}$$

where $\epsilon_{sp,t} \equiv \frac{\beta\tau}{(1-\beta)\varsigma_B} a_t - \frac{\beta(\varsigma_G\omega_{u1}-\tau\omega_{u3})}{(1-\beta)\varsigma_B\omega_{u1}} g_t$ is the fiscal surplus shock.

4.2 FONCs for the Policy Authorities

The policy authorities minimize Eq.(35) under the OM and OMF policies while they minimize Eq.(33) under the MIS, subject to Eqs.(36)–(39). We assume that there are two policy instruments, the tax gap and the nominal interest rate. Under the OM and the MIS policies, the policy instrument is just the nominal interest rate $\hat{r}_t$ and the tax gap $\hat{\tau}_t$ is zero over the time, that is, the tax rate is fixed at its steady state level. The policy authorities chooses the sequence $\{x_t, \pi_t, \hat{r}_t, \delta_t, sp_t\}_{t=0}^{\infty}$.[12] Under the OMF policy, the policy instruments are not only the nominal interest rate $\hat{r}_t$ but also the tax gap $\hat{\tau}_t$. The policy authorities select the sequence $\{x_t, \pi_t, \hat{r}_t, \hat{\tau}_t, \delta_t, sp_t\}_{t=0}^{\infty}$.

The OM and the OMF policy is a synonym of inflation stabilization policy because the weight on the quadratic term of inflation is extremely high. Thus, analyzing effects on the default rate under the OM or the OMF policy is analogous to analyzing effects on the default rate under inflation stabilization policy. Further, there is one policy instrument under the OM policy while there are two policy instruments under the OMF policy. This means that the OM policy regime lacks one of the available policy instruments to conduct the OMF policy or to stabilize inflation and the taxation regime is more aggressive in stabilizing inflation than the OMF policy regime. Thus, we can find how stabilizing inflation affects the default rate through comparing dynamics both on the inflation and the default rate under both policies.

[12] Because Eq.(10) is used to eliminate government debt in our model, the policy authorities' instrument is just the nominal interest rate at glance. However, government debt is indirectly chosen by choosing the fiscal surplus and the default rate. Follwoing Ferreo[11] who assumes the policy authorities choosing the nominal interest rate, the government debt and the tax gap, the OM policy in this paper is not an optimal monetary policy but an optimal monetary and fiscal policy to be exact because the policy authorities choose the government debt indirectly. However, the OMF policy in our paper is definitely an optimal monetary and fiscal policy and we have to distinguish these two policies. Thus, we dub this policy the OMP policy.

4.2.1 FONCs under the MIS Policy

The FONCs for the OGTL and for the inflation are given by:

$$\mu_{1,t} = \frac{\kappa(1+\varphi\varsigma_C)}{\varsigma_C}\mu_{2,t} + \frac{\beta\tau}{(1-\beta)\varsigma_B}\mu_{4,t}, \tag{40}$$

$$\mu_{2,t} = -\frac{\varsigma_C}{\beta}\mu_{1,t} - \frac{1}{2(1-\beta)}\mu_{3,t}. \tag{41}$$

Because the period loss function L_t^R does not include quadratic terms of the inflation and the OGTL, these itself dissapear from the FONCs.

The FONCs for the nominal interest rate, the default rate and the fiscal surplus are given by:

$$\left(\frac{\phi\beta}{1-\beta}\right)^2\hat{r}_t = \frac{(\phi\beta)^2\gamma}{(1-\beta)^2}sp_t - \frac{\phi\beta}{1-\beta}\mathrm{E}_t(\delta_{t+1}) - \frac{\varsigma_C\omega_\phi}{1-\beta}\mu_{1,t} + \varsigma_C\mathrm{E}_t(\mu_{1,t+1}) + \frac{\beta}{2(1-\beta)}\mathrm{E}_t(\mu_{3,t+1}), \tag{42}$$

$$\frac{\varsigma_C\omega_o}{\phi\beta^2}\mu_{1,t} = -\frac{\phi-(1-\beta)}{2\phi(1-\beta)}\mu_{3,t} \tag{43}$$

$$\left(\frac{\phi\beta\gamma}{1-\beta}\right)^2 sp_t = \frac{(\phi\beta)^2\gamma}{(1-\beta)^2}\hat{r}_t + \frac{\phi\beta\gamma}{1-\beta}\mathrm{E}_t(\delta_{t+1}) - \frac{\varsigma_C\omega_{\varsigma\pi}}{\beta(1-\beta)}\mu_{1,t} - \mu_{3,t} - \mu_{4,t} - \frac{\varsigma_C\omega_o}{\beta}\mathrm{E}_t(\mu_{1,t+1}) - \frac{\omega_o}{2(1-\beta)}\mathrm{E}_t(\mu_{3,t+1}). \tag{44}$$

Here, Eqs.(42) and (44) show that to minimize the interest rate difference, the policy authorities have to hike the nominal interest rate facing an increase in the fiscal surplus, and vice versa. Suppose that an increase in the fiscal surplus. An increase in the fiscal surplus to applies pressure to lower the government debt's yield excluding the default risk, as shown in Eq.(12). To cancel this pressure, the nominal interest rate has to be hiked. As shown in Eq.(12), an increase in the (percentage deviation of) gross nominal interest rate increases the (percentage deviation of) gross government debt's yield excluding the default risk amount of $\frac{\omega_\phi}{1-\beta} > 1$. As a result, the interest rate difference is minimized.

4.2.2 FONCs under the OM and OMF Policies

Now, we show the FONCs under the OM policy. The FONCs for the OGTL and for the inflation are given by:

$$\Lambda_x x_t = -\frac{1}{\beta}\mu_{1,t} + \frac{\kappa(1+\varphi\varsigma_C)}{\varsigma_C}\mu_{2,t} + \frac{\beta\tau}{(1-\beta)\varsigma_B}\mu_{4,t}, \tag{45}$$

$$\Lambda_\pi \pi_t = -\frac{\varsigma_C}{\beta}\mu_{1,t} - \mu_{2,t} + \frac{1}{2(1-\beta)}\mu_{3,t}, \tag{46}$$

where $\mu_{1,t}$, $\mu_{2,t}$, $\mu_{3,t}$ and $\mu_{4,t}$ are the Lagrange multipliers on Eqs.(36), (37), (38) and (39), respectively. Because of the default risk, these FONCs are somewhat different from familiar one. However, by ignoring Lagrange multipliers $\mu_{3,t}$ and $\mu_{4,t}$, we can understand that inflation is stabilized via stabilizing OGTL because Lagrange multipliers $\mu_{1,t}$ and $\mu_{2,t}$ are multiplied on NKIS Eq.(36) and NKPC Eq.(37). Basically, mechanism on stabilizing inflation is similar to New Keynesian literature including Benigno and Benigno[4].

The FONCs for the nominal interest rate and the default rate are given by:

$$\frac{\varsigma_C \omega_\phi}{1-\beta}\mu_{1,t} = \varsigma_C \mathrm{E}_t\left(\mu_{1,t+1}\right) + \frac{\beta}{2\left(1-\beta\right)}\mathrm{E}_t\left(\mu_{3,t+1}\right), \tag{47}$$

$$\frac{\varsigma_C \omega_o}{\phi\beta^2}\mu_{1,t} = -\frac{1}{2\left(1-\beta\right)}\mu_{3,t}, \tag{48}$$

which show that there is a close relationship between NKIS Eq.(36) and the government budget constraint Eq.(38).

The FONCs for the fiscal surplus is given by:

$$\frac{\varsigma_C \omega_{\varsigma\pi}}{\beta\left(1-\beta\right)}\mu_{1,t} = -\mu_{4,t} - \frac{\varsigma_C \omega_o}{\beta}\mathrm{E}_t\left(\mu_{1,t+1}\right), \tag{49}$$

which shows that changes in fiscal surplus affects the NKIS Eq.(36) and the government budget constraint Eq.(38). In addition, Eqs.(48) and (49) implies that changes in fiscal surplus affects default rate because of Laglange multipliers on the government budget constraint and the definition of fiscal surplus, $\mu_{3,t}$ and $\mu_{4,t}$.

Under the OMF policy, the FONCs are given by not only Eqs.(45)–(49), but also given by the FONC for the tax gap as follows:

$$\mu_{2,t} = -\frac{\left(1-\tau\right)\beta}{\left(1-\beta\right)\varsigma_B\kappa}\mu_{4,t},$$

which shows that changes in the fiscal surplus affects the NKPC via changes in the tax gap. Although the OM policy does not has this mechanism, changes in tax gap affects not only the default rate via changes in fiscal surplus, as shown in Eqs.(48) and (49), but also the inflation because of Laglange multiplier on the NKPC Eq.(37) $\mu_{2,t}$. That is, the tax gap not only works to stabilize inflation but also affects the default rate via the fiscal surplus. Thus, it cannot be necessarily said that there is the SI-SD trade-off.

5 Numerical Analysis

5.1 Parameterization

We run a series of dynamic simulations and adopt the following benchmark parameterization. Calibrated parameters mainly follows Ferrero[11] analyzing the OMF policy while unfamiliar parameters, the interest rate spread for risky assets ϕ and the elasticity of the interest rate spread to a one percent change in the fiscal deficit γ based on emprical evidence. In addition, we assume the productivity and government expenditure follow an AR(1) processes and we estimate persistence and standard errors of the innovations from data.

Familiar parameters, the values for the subjective discount factor β, the elasticity of substitution across goods ε, price stickiness θ, the inverse of the labor supply elasticity φ, the steady-state share of government bonds to output σ_B and the steady-state share of government expenditure to output σ_G, the steady-state tax rate τ, are set to 0.99, 11, 0.75, 0.47, 2.4, 0.276, 0.3, respectively, following Ferrero[11].[13] Following our results on empirical analysis, unfamiliar parameters, the spread of the nominal interest rate ϕ and the elasticity of the interest rate spread to the fiscal deficit γ are

[13] $\tilde{\sigma}_B = 2.4$ is consistent with quarterly time periods in the model and implies that the annual steady-state debt–output ratio is 0.6.

set to 0.138 and 1.145, respectively. In addition, following our results on empirical analysis, we set persistences on the productivity and on the government expenditure, ρ_A and ρ_G are set to 0.976 and 0.927, respectively while standard errors of the innovations on the productivity and on the government expenditure are set to 0.0316 and 0.0728, respectively. Appendix A mentions our empirical analysis.

5.2 Macroeconomic Dynamics

5.2.1 Macroeconomic Volatility and Correlation

We discuss macroeconomic volatility at first (Tab. 1). The volatility on the inflation under the MIS policy which is 0.7769 which is higher than one under the OM which is 0.0044 although the volatility on the default rate under the OM policy which is 2.5590 is higer than one under the MIS policy which is 0.8076. This implies that there is a trade-off between stabilizing inflation and surpressing default. If policy authorities choose stbilizing inflation, they have to give up surpressing default and vice versa. This result is consistent with Uribe[18]. However, by comparing the OM policy with the OMF policy, we have to recognize that there is not necessarily such a trade-off. Note that both the OM and the OMF policies focusing on stabilizing inflation. While the OM policy has just one policy instrument, the OMF policy has two policy instruments. Thus, volatilities on OGTL and inflation are definitely zero, which means that inflation–output gap trade-off is completely dissolved and these are smaller than volatilities on OGTL and inflation under the OM policy, 0.0508 and 0.0044, respectively. Notable results are volatilities on default rate. The volatility under the OMF policy is 0.6271 which is 75% smaller than one under the OM policy. In addition, it is 22% smaller than one under the MIS policy. Because volatility on the inflation under the OMF policy is definitely zero and it is smaller than one under both the OM and the MIS policies, it can be said that there is not necessarily trade-off between stabilizing inflation and suppressing default. This result is quite different from Uribe[18].

As mentioned by Uribe[18], there is the SI-SD trade-off at glance. In fact, there is a negative correlation between the inflation and the default rate under the MIS policies and the correlation is -0.9954 (Tab. 2). however, as mentioned, the volatilities on both the inflation and the default rate under the OMF policy is the lowest. As we mentioned, fiscal surplus is endogenized following DSGE literature in our model, our result is not necesarily the SI-SD trade-off. Now we find the mechanism mitigating a trade-off by focusing on correlation among variables. The correlation between default rate and fiscal surplus under the OM policy and the OMF policy is 0.9395 and 0.9751, and is very strong. That is the higher the fiscal surplus, the higher the default rate and vice versa. In addition, correlation between the inflation and the fiscal surplus under the OM policy is 0.3067, which is higher than correlation between the inflation and the default rate. Thus, stabilizing fiscal surplus not only stabilizes the default rate but also stabilizes the inflation. The reason why stabilizing the fiscal surplus stabilizes the inflation is NKIS Eq.(36) and the FONC for the OGTL and the inflation Eqs.(45) and (46). That is, stabilizing fiscal surplus stabilizes the OGTL via stabilizing consumption as shown in Eq.(20) and stabilizing OGTL brings stabilized inflation as shown in Eqs.(45) and (46). Anyway, the fiscal surplus strongly affects not only the default rate but also the inflation and stabilizing fiscal surplus is consistent not only with stabilizing the default rate but also with the inflation. Thus, there is not necessarily trade-off between stabilizing inflation and suppressing default.

Although it is taking a side trip, it is noteworthy to mention on positive correlation between the fiscal surplus and the default rate. This positive correlation is not marvelous. An increase in the fiscal surplus decreases the government bond yield. This decraese increases marginal rate of substition of consumption through Euler equation Eq.(8). Because of adopting de facto inflation stabilization policy which makes price level stable, expected price level shall decrease. A decrease in expected price increases a burden of government debt and government solvency relatively lowers. This relative decrease in government solvency increases the default rate. Thus, there is positive correlation between the fiscal surplus and the default rate.

5.2.2 Impulse Response Functions

Now we discuss on the impulse response functions (IRFs) . To examine IRFs, we consider one standard deviation negative change in productivity and one standard deviation positive change in government expenditure.

Now, we focus on negative change in productivity (Fig. 1). A decrease in productivity applies pressure to decrease output while the OGTL rises to cancel a decrease in the income, as shown in NKIS Eq.(36) (Panel 1). Under the MIS policy, the nominal interest rate is hiked to minimize the interest rate spread. That is, a decrease in the fiscal surplus stemming from a decrease in output applies pressure to increase the government bond yield excluding the default risk. To minimize the interest rate spread, the nominal interest rate increase. As mentioned and as shown in Eq.(12), an increase in the nominal interest rate increases the government bond yield excluding the default risk beyond. As a result, there is no interest spread and policy object is achieved (Panel 8). The inflation rises through NKPC Eq.(37) because of an increase in the OGTL and a decrease in the productivity. The default rate is constant and is zero over time because of no interest rate spread (Panels 5 and 8). Indeed, Uribe[18] shows that the default occurs but it ends after just one period because the expected default rate is difinitely zero under the interets rate peg which corresponds to the MIS policy in our paper. As shown in the definition of the the government bond yield excluding the default risk $R_t \equiv R_t^H \mathrm{E}_t\left(1-\delta_{t+1}\right)$, No interest spread implies that $\mathrm{E}_t\left(\delta_{t+1}\right)=0$, that is, the expected default rate is zero. Because the expected default rate is zero which implies that the default rate is an i.i.d. random variable with mean zero in Uribe[18] who assumes the fiscal surplus is exogenous shock follwoing AR(1) process, interest rate peg which is conssitent with the MIS policy in our paper stabilizes the default rate immediately after the negative fiscal surplus shock. However, we now analyze commitment policy, through controling households expectation, the default rate is difinitely zero. While the default is completely suppressed, there is inflation. Thus, there is clearly a trade-off between stabilizing inflation and surpressing default.

Next, we discuss on the IRFs under OM and OMF policies. A decrease in productivity increases output under the OM policy. Under the OM policy, just central bank copes with this change. A decrease in productivity works as cost-push shock under the our assuption, namely, distorted steady state and inflation rises. To stabilize the inflation, the nominal interest rate is hiked to stablize inflation through applying pressure to decrease the OGTL, although the OGTL increases because an increase in the inflation which increases expected inflation. Thus, inflation-output gap trade-off is not dissolved. A decrease in productivity decreases the fiscal surplus through a decrease in the taxable, namely output. A decrease in the fiscal surplus increases the default rate via a decrease in government's solvency. Under the OMF policy, however, the tax gap is avaiaable as a fiscal policy tool and an increase in this mitigates a decrease in the fiscal surplus (Panel 9). Thus, an increase in the default rate is not so severe than the increase under the OMP. Needless to say, an increase in the tax gap is not intended to stabilize the default rate but is intended to stabilize inflation and the inflation is completely stabilized (Panel 3). Thus, there is not necesarrily the SI-SD trade-off.

An increase in government expenditure shows similar IRFs (Fig. **??**). An increase in government expenditure applies pressure to decrease the fiscal surplus and to increase the OGTL. Under the MIS policy, the default rate is completely stabilized while the inflation severely rises (Panels 2 and 7). That is, there is clearly the SI-SD trade-off. Similar to the MIS policy, the OM policy generates the SI-SD trade-off. While the inflation is more stable than one under the MIS policy, the default rate severly rises (Panels 2 and 7). However, while the default rate is not comletely stabilized, it is more stable than one under the OM policy, under the OMF policy (Panel 7). In addition, the inflation is completely stabilized under the OMF policy (Panels 2 and 7). Thus, there is not necessarily the SI-SD trade-off.

6 The Trade-off between Stabilizing Inflation and Suppressing default Rate

Is the SI-SD trade-off is so severe as highlited by Uribe[18]? We calculate both volatilities on the inflation and the default rate under various price stickiness θ from 0.6 to 0.95 every 0.05 (Fig. **??**). Note that we just focus on an incraese in the government expenditure. Under the OM policy, there is the SI-SD trade-off clearly (Panel 1). The higher the price stickiness, the higher the volatility on the default rate and the lower the volatility on the default rate, and vice varsa. Because higher the price stickiness, the higher the weight on inflation in the period welfare costs Λ_π, The higher the price stickiness, the lower the volatility on the inflation. That is, when price stickiness is high, Λ_π is also high. In that case, inflation is well stabilized. However, as mentioned, Stabilized inflation agressively under the OM policy induces high volatility on the default rate. Thus, there is the SI-SD trade-off clearly. Volatility on the inflation is depeds on the price stickiness under the MIS policy, similar to the OM policy (Panel 2). The higher the price stickiness, the higher the Λ_π, as mentioned. Thus, higher the price stickiness, the lower the volatility on the inflation. However, different from the OM policy, the volatility on the default is not depending on the price stickiness and is definitly zero. In addition, the standard deviation on the inflation is just 0.008 when the price stickiness is 0.95. While the standard deviation on the inflation is nearly zero $(3.4 \times 10^(-4))$, the standard deviation on the default rate is 0.97, under the default rate. Policy authorities may choose the MIS policy because the volatility on the default rate is quite high, under the OM policy. Uribe[18] shows not only the SI-SD trade-off but also subscription of suppressing the default and the subscription is giving up stabilizing inflation. It seems that Uribe[18]'s subscription is not so irrelevant but may be realistic, if the price stickiness is high. How about the SI-SD trade-off under the OMF policy? There is no the SI-SD trade-off and standard deviation on both the inflation and the default rate are definitely zero and 0.008, respectively (Panel 2). In addition, the standard deviation on the defalt rate under the OMF policy is almost corresponding to the standard deviation on the inflation under the MIS. Which policy should be adopted? Further, when price stickiness is 0.6, the standard deviation of the inflation is 0.62 under the MIS policy while the standard deviation of the default rate is still 0.008 under the OMF policy. Our emprical result on price stickiness is 0.705. In that case, the standard deviation on the inflation is 0.31 under the MIS policy, while the standard deviation of the default rate is still 0.008 under the OMF policy. While the statement is too sweeping, there is a possibility that choosing the OMF policy as the first option. Further, what we can say surely is that the SI-SD trade-off is not so severe as what mentioned by Uribe[18].

7 Conclusion

We develop a class of DSGE model with nominal rigidities and find that stabilizing inflation is not inconsistent with suppressing default. Further, we have a policy implication that by stabilizing inflation, default risk shall be stabilized and this policy implication is quite different from Uribe[18]'s. While the ECB seems to reluctant to stabilize inflation because of smoldering sovereign risk, our results imply that there is the another choice for policy authorities without concernig about the SI-SD trade-off. That is, the OMF policy may be the first option.

In this paper, welfare criteria thus welfare cost function is not completely consistent with households' utility function Deriving welfare criteria which is completely consistent with households' utility function is one of future work.

Appendices

A Non Stochastic Steady State

We focus on equilibria where the state variables follow paths that are close to a deterministic stationary equilibrium, in which $\Pi_t = 1$. Note that $\tilde{\mathsf{X}} = 1$ is applied in this steady state with $\tilde{\mathsf{X}}_t \equiv \frac{\tilde{P}_t}{P_t}$. Because this steady state is nonstochastic, the productivity is unit values, i.e., $A = 1$. We assume the default rate is zero, $\delta = 0$ is applied.

In this steady state, the gross nominal interest rate is equal to the inverse of the subjective discount factor, as follows:

$$R = \beta^{-1}.$$

Eq.(24) can be rewritten as:

$$\tilde{P}_t = \mathrm{E}_t \left(\frac{K_t}{P^{-1} F_t} \right) \tag{A.1}$$

with:

$$K_t \equiv \frac{\varepsilon}{\varepsilon - 1} \sum_{k=0}^{\infty} \left(P_{t+k} C_{t+k} \right)^{-1} \tilde{Y}_{t+k} MC^n_{t+k} \quad ; \quad F_t \equiv P_t \sum_{k=0}^{\infty} \left(P_{t+k} C_{t+k} \right)^{-1} \tilde{Y}_{t+k}$$

which implies that:

$$K = \frac{\frac{\varepsilon}{\varepsilon-1} Y MC^n}{(1 - \alpha\beta)(PC)} \quad ; \quad F = \frac{PY}{(1 - \alpha\beta)(PC)}.$$

These equalities imply that:

$$P = \frac{\varepsilon}{\varepsilon - 1} MC^n$$

Thus, we have:

$$MC = \frac{\varepsilon}{\varepsilon - 1}^{-1}. \tag{A.2}$$

Furthermore, Eqs.(26) and (**??**) imply the following:

$$CN^{\varphi} = \frac{1-\tau}{\frac{\varepsilon}{\varepsilon-1}}. \tag{A.3}$$

Eq.(A.3) implies the familiar expression:

$$(1-\tau)\,U_C \quad = \quad \frac{\varepsilon}{\varepsilon-1}U_N.$$

Note that because $\tau \in (0,1)$ and $\varepsilon > 1$, this steady state is distorted.

Eq.(13) yields the following:

$$B\left(\frac{1-\beta}{\beta}\right) = SP, \tag{A.4}$$

with $B \equiv \frac{B^n}{P}$.

Note that $R = R^H$ because of $\delta = 0$ and $R^G = R\Gamma(0)$. Plugging this into Eq.(15) yields:

$$C^{-1}R\Gamma(0)\,B = C^{-1}SP + \frac{\beta}{\Gamma(0)}C^{-1}SP + \left(\frac{\beta}{\Gamma(0)}\right)^2 C^{-1}SP + \cdots. \quad = \quad \frac{1}{1-\beta\left[\Gamma(0)\right]^{-1}}C^{-1}SP,$$

which implies:

$$\Gamma(0)\,B\beta^{-1} = \frac{1}{1-\beta\left[\Gamma(0)\right]^{-1}}SP \tag{A.5}$$

Plugging Eq.(4) into this equality yields:

$$\Gamma(0) = \frac{1-\beta}{1-\beta\left[\Gamma(0)\right]^{-1}},$$

which implies that $\Gamma(0) = 1$. Thus, our assumption that $\delta = 0$ is consistent with $\Gamma(0) = 1$.

Because of $\Gamma(0) = 1$, $R^G = R$. Thus,

$$R^G = R^H \tag{A.6}$$

In the steady state, boild down to:

$$1 = \frac{\frac{1}{\left(1-\beta\frac{R^H}{R^G}\right)}\left(C^{-1}SP\right)}{C^{-1}RB}. \tag{A.7}$$

note that the RHS in Eq.(A.7) corresponds to the steady state value of Ψ. That is, $\Psi = 1$ is applied in the steady state. This implies that the default rate is zero in the steady state.

B Empirical Evidences on Calibrated Unfamiliar Parameters and AR(1) Processes

One of our calibrated parameters, the elasticity of the interest rate spread to the fiscal deficit γ is based on following regression:

$$\ln\left(R_t^{risky} - R_t\right) = \alpha_0 + \alpha_1 df_t + \alpha_2 DUM_t + \alpha_3 df_t DUM_t, \tag{B.1}$$

where R_t^{risky} corresponding to $R_t\Gamma(-sp_t)$ denotes the nominal interest rate for risky assets where DUM_t is a dummy. α_1 and α_3 measure how changes in the percentage deviation of fiscal deficit $df_t \equiv -sp_t$ widen or narrow interest rate spread $R_t^{risky} - R_t$. Thus, α_1 and α_3 correspond to γ. Data is monthly and is retrieved from Datastream and we use yield on government bonds with maturities of 10 years in Greece and yield on government bonds with maturities of 9–10 years and real government budget balance in Greece.[14] Sample period is from January, 2005 to April, 2015. Note that Athens Olympic was held in January, 2005, beginning of the period when unhealthy fiscal deficit started to continue. Real government budget balance is seasonally adjusted and HP filtered. Data frequency is monthly. We assign $DUM_t = 1$ during May, 2010 to June. 2012 otherwise $DUM_t = 0$. Note that Greece asked fiscal support to both the IMF and ECB on April, 2015 and May, 2015 is its next month and Greece decided to adopt a reduced budget following results on the poll. That is, $DUM_t = 1$ is assigned during severe debt crisis in Greece. The estimators on α_0, α_1, α_2 and α_3 are -4.302, 0.170, 2.196 and 1.145, respectively. Those of standard errors are 0.896, 0.099, 0.852 and 0.077, respectively. Results of estimators on α_0 and α_3 are significant in 1% level and on α_2 is significant in 5% level. The results that α_1 is significant and α_3 is not significant implies that the elasticity of the elasticity of the interest rate spread to the fiscal deficit γ is significant during severe debt crisis when the nominal interest rate roses rapidly and its elasticity is 1.145. Thus, we set γ to 1.145. Because γ is significant during May, 2010 to June, 2012, we regard average of the interest rate spread $R_t^{risky} - R_t$ as risk premium and we set the interest rate spread for risky assets ϕ to 0.138 following our interpretation.

AR(1) processes are also estimated from data on domestic productivity and government expenditure in Greece retrieved from Datastream and the sample period is from January, 2005 to April, 2015. The data is HP filtered. Our results on the persistence on the productivity ρ_A and the persistence on the government expenditure are 0.976 and 0.927, respectively and the innovations on the productivity and on them are 0.0316 and 0.0728, respectively, as mentioned in section 5.1.1.

As we mentioned in section 2.1, our assumption on the elasticity of the interest rate spread to the fiscal deficit $\gamma > 1$ is supported by data. It's true because t-statics on the null hypothesis $\alpha_3 = 1$ and the alternative hypothesis $\alpha_3 > 1$ is 1.88, which is larger than its 5% critical value 1.7 and $\alpha_3 > 1$ is supported statistically. Note that α_3 corresponds to γ as mentioned.

C Empirical Evidences on Government Debt with Interest Payment as an Argument of $\Gamma(\cdot)$

Similar to Eq.(B.1), we estimate following:

$$\ln\left(R_t^{risky} - R_t\right) = \tilde{\alpha}_0 + \tilde{\alpha}_1 rb_t + \tilde{\alpha}_2 DUM_t + \tilde{\alpha}_3 rb_t DUM_t,$$

where $\tilde{\alpha}_1$ and $\tilde{\alpha}_3$ measure how changes in the percentage deviation of government debt with interets payment from its steady sate value $rb_t \equiv \frac{R_t B_t}{RB} - 1$ widen or narrow interest rate spread $R_t^{risky} - R_t$. Thus, $\tilde{\alpha}_1$ and $\tilde{\alpha}_3$ correspond to γ. Data is qurterly and is retrieved from Datastream and we use government debt in Greece and government interest payment in Greece. We sum up overnment debt in Greece and government interest payment in Greece and divide it by CPI in Greece. That generated data is HP filetred. Sample period runs from Q1, 2005 to Q1, 2015. We assign $DUM_t = 1$ during Q2, 2010 to Q2, 2012 otherwise $DUM_t = 0$. The estimators on $\tilde{\alpha}_0$, $\tilde{\alpha}_1$, $\tilde{\alpha}_2$ and $\tilde{\alpha}_3$ are -4.316,

[14]Original data is nominal government budget balance and we deflate it by the CPI.

0.385, 2.366 and -4.120, respectively. Those of standard errors are 0.890, 10.307, 0.841 and 10.367, respectively. Results of estimators on $\tilde{\alpha}_0$ and $\tilde{\alpha}_2$ are significant in 1% level. The fact that $\tilde{\alpha}_0$ is not significant means that γ cannot be estimated if we assume that the argument of $\Gamma(\cdot)$ is the government debt in Greece. This estimation result and the result on Appendix B imply that the fiscal surplus as an argument of $\Gamma(\cdot)$ is plausible although government debt with interest payment as an argument of $\Gamma(\cdot)$ is not plausible.

D Empirical Evidences on Price Stickiness

To be added.

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Table 1: Macroeconomic Volatility

Variable	OMP	OMFP	MIS
x_t	0.0508	0.0000	0.2558
π_t	0.0044	0.0000	0.7769
$\hat{r}_t$	2.7636	0.6765	0.0876
$\hat{\tau}_t$	NA	0.2269	NA
δ_t	2.5590	0.6271	0.8076
sp_t	2.6020	0.6373	0.0819
$\hat{r}_t^S$	0.3872	0.1002	0.0000

Table 2: Correlation of Selected Variables

Variable	Policy	x_t	π_t	$\hat{r}_t$	$\hat{\tau}_t$	δ_t	sp_t	$\hat{r}_t^S$
x_t	OMP	1.0000						
	OMFP	1.0000						
	MIS	1.0000						
π_t	OMP	-0.1098	1.0000					
	OMFP	NA	1.0000					
	MIS	-0.0577	1.0000					
$\hat{r}_t$	OMP	0.0835	-0.7653	1.0000				
	OMFP	NA	NA	1.0000				
	MIS	-0.1318	0.9972	1.0000				
$\hat{\tau}_t$	OMP	NA	NA	NA	1.0000			
	OMFP	NA	NA	0.9164	1.0000			
	MIS	NA	NA	NA	1.0000			
δ_t	OMP	0.2879	-0.8770	0.8405	NA	1.0000		
	OMFP	NA	NA	-0.4621	-0.5566	1.0000		
	MIS	NA	NA	-0.2082	NA	1.0000		
sp_t	OMP	-0.9102	0.2644	-0.4343	NA	-0.4754	1.0000	
	OMFP	NA	NA	0.4104	0.7191	-0.4537	1.0000	
	MIS	0.9360	-0.0541	-0.1241	NA	NA	1.0000	
$\hat{r}_t^S$	OMP	-0.3444	0.2108	0.3720	NA	-0.1270	-0.0022	1.0000
	OMFP	NA	NA	-0.3571	-0.6769	0.4331	-0.9982	1.0000
	MIS	NA	NA	NA	NA	NA	NA	1.0000

Figure 1: IRFs to Negative Productivity

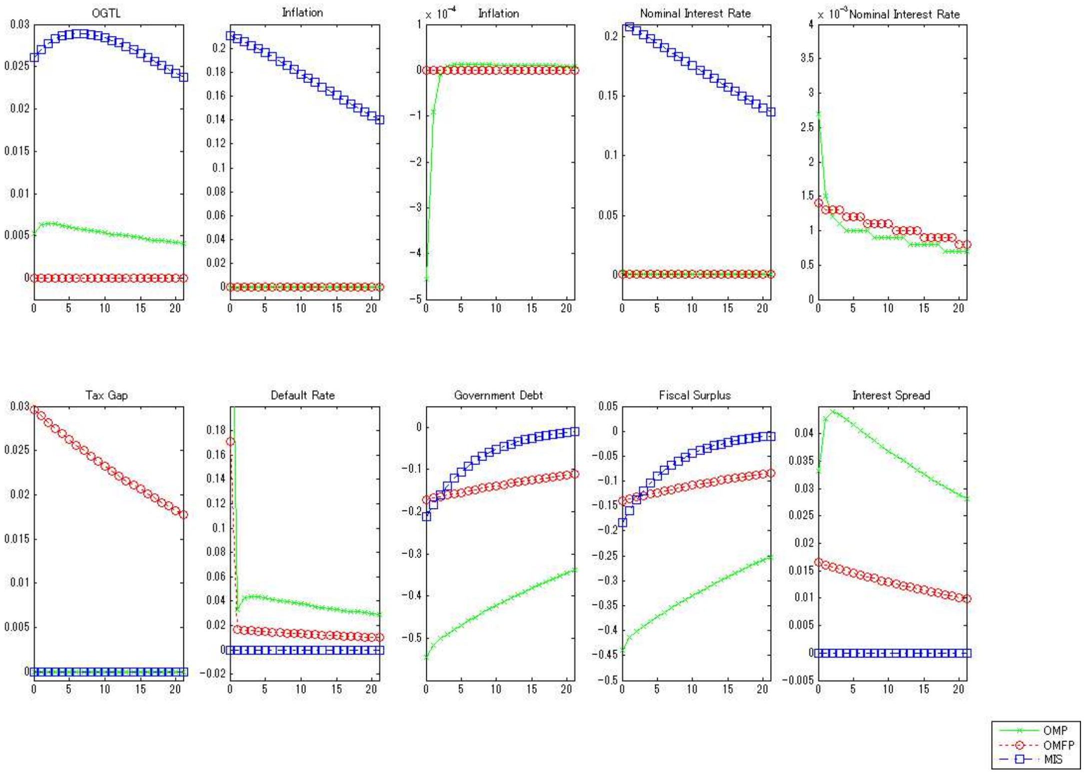

Figure 2: IRFs to Government Expenditure

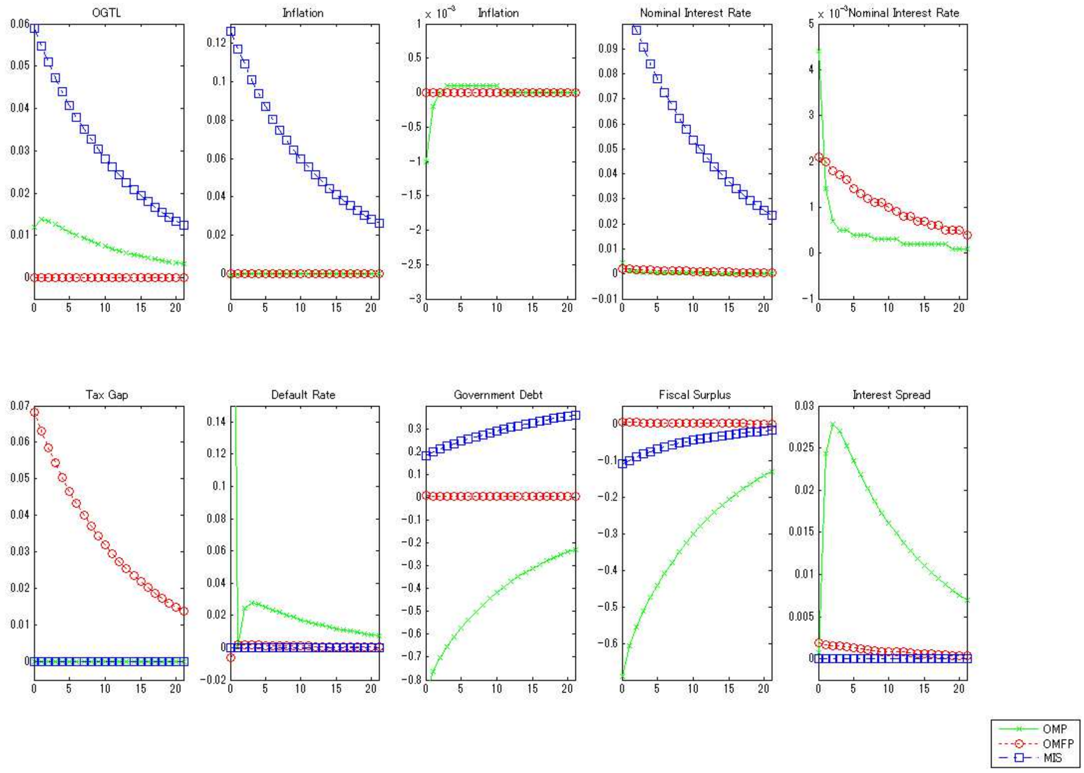

Figure 3: The Trade-off between Stabilizing Inflation and Default Rate Volatilities