

Equalization and Spatial Location Efficiency: The Example of Australia

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Abstract

This paper shows that regional economies, such as federations or unitary countries with sub national governments, may need a system of optimal inter regional transfers to correct for various types of externalities related to factor mobility and location decisions. It is then argued that equalization schemes which take account of the differing expenditure and revenue needs of regions, create a pattern of inter regional transfers of income, but that they are inconsistent with what is required for spatial efficiency. Therefore, equalization is incompatible with the efficient spatial allocation of mobile factors of production. It is also shown that regions have an incentive to act strategically over equalization and distort their provision of local public goods.

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1. The Issue

Decentralized economies with states, provinces or local governments frequently use equalization models when distributing grants from higher level governments. Many schemes, particularly the Canadian one, are purely revenue (or tax) capacity equalization models. Others are more comprehensive in the sense that they also equalize for differences in the costs of providing sub-national government services across regions. Schemes that take account of regional cost differences include those used in Japan, Switzerland and the UK for distributing grants to local governments. These schemes calculate the impact of factors such as geography, industrial structure (Japan) and population dispersion on the cost structure of sub-national regions. The implication is that regions with relatively higher costs than the average receive more than otherwise. In Australia, the equalization scheme used to distribute grant funds to the states also takes account of regional cost differences.

Economists have shown concern about the economic efficiency implications of equalization. The work to date has concentrated on schemes of tax capacity equalization and the distortions that they create for tax and public spending decisions at the recipient government level. This literature starts with Courchene and Beavis (1973) who argue that states may act strategically if equalization formulas are dependent on the use of national average tax rates which are in turn functions of each state's tax policy. Other papers have also discussed these possibilities including Bird and Slack (1990) and Usher (1995). Courchene (1994) also proposes that equalization might deter states from exploiting new tax bases. Smart (1998) adopts a partial equilibrium model of a single jurisdiction and a central authority which mandates a Canadian-style tax base equalization scheme, together with a model of optimal taxation at the sub-national level. He shows that revenue equalization distorts the region's perceived marginal cost of public funds and hence tax policy. Similar theoretical results are obtained by Swan and Garvey (1995) and Dahlby and Warren (2003) for the Australian equalization model, though the latter paper models only the tax capacity component of the scheme.

The basic idea to emerge from these papers is that by lowering a region's perceived marginal cost for public funds tax capacity equalization encourages over

provision relative to Samuelson efficient levels, and hence over-taxation. In contrast, Köthenburger (2002) develops a model with identical multiple regions and a common capital market in which regional governments engage in tax competition over mobile capital. It is well known that tax competition creates positive fiscal externalities and induces under taxation and under provision of local public goods². Köthenburger incorporates a tax capacity equalization scheme within his model of tax competition and shows that the over taxation and over provision encouraged by equalization offsets the under taxation and under provision caused by tax competition. The result is an efficient outcome. Bucovetsky and Smart (2004) confirm this result, again with a purely tax capacity equalization scheme, but generalize it to allow for elastic tax bases, regional inequality and differences in preferences and regional populations. The implication of both papers is that a higher level government can use equalization to achieve its preferred allocation of resources among regions whilst also correcting for any inefficiencies associated with tax competition at the sub-national level. Rather than being a source of inefficiency equalization ensures efficient outcomes at the sub-national government level.

Two observations about this literature stand out. One is that it deals exclusively with tax capacity equalization schemes of the type in operation in countries like Canada. Yet as noted above equalization for costs is incorporated into many equalization schemes used in practice. Second, the models developed do not focus on efficiency in the spatial allocation of mobile factors of production; rather, they concentrate on efficiency in the rates of tax set, or alternatively, levels of public expenditure. But we know that equalization schemes result in substantial transfers of income between regions - indeed that is their primary motivation - so one would expect them to have consequences for the spatial allocation of mobile resources³.

The aim of this paper is to focus on this question, namely, what is the link between equalization, the spatial allocation of mobile factors of production, and economic efficiency? This is done by constructing a model of a regional union of states, which might be thought of as a federation, a confederation such as the European Union, or even as a unitary state where there is some devolution of tax and

² See Wilson (1991).

spending powers to local jurisdictions (e.g. the UK). The model employed is one commonly found in the fiscal federalism literature. I then embed within the model an equalization scheme which takes account of both revenue (tax capacities) and expenditure needs (based on differences in regional cost of producing government services). By studying spatial efficiency within the context of an equalization scheme that allows for revenue and expenditure needs, this paper departs from the existing literature in two significant ways.

The scheme used is the Australian one because this model incorporates tax capacity and cost equalization and is, arguably, the most comprehensive equalization system in use in the world today. The regional model has two states, each with a government that receives an equalization grant and chooses public expenditures to maximize state welfare whilst taking account of the impact of policy choices on the grant received and on the location decisions of a mobile factor. Hence, the model allows for policy competition between regions. Because of factor mobility and the link between states induced by the presence of equalization there is interdependence between state policy decisions. Adopting Nash conjectures the model is therefore characterized as a two stage *equalization game*. In stage 1 the states make their choices whilst taking account of the impact of their policy choices on their net transfer through the equalization system, as well as the spatial location effects of their decisions. The equalization model is part of the environment of the game and the states take the structure of the model, but not various variables within the model, as given. In stage 2 the mobile factor makes its location choices conditional on state policies chosen in stage 1. An optimization problem is characterized and the necessary conditions that must be satisfied in a Nash equilibrium to the game are discussed.

The main finding of the paper is that the inter-state transfer that occurs with equalization is inconsistent with an efficient spatial allocation of mobile factors of production within a regional economy. The problem is that the equalization induced inter-state transfer is a function of equity motivated cost disabilities (which are used to estimate expenditure needs) and differences in tax capacities. On the other hand,

³ The size of these transfers can be seen from Annex 1 which presents equalization data for Australia.

the inter-state transfer needed for efficient spatial location decisions is a function of the fiscal and economic rent externalities associated with mobile factor location decisions. Therefore, under equalization one gets inter state transfers that are not equal to the efficient transfers. Though not the emphasis of the paper I also find that public good provision is distorted by equalization, consistent with the results of Dalhby and Warren, as well as Smart. However, I do not find evidence for the results of Köthenburger, and Bucovetsky and Smart, that the under provision caused by tax competition may offset the distortions of equalization. Some possible reasons for this are offered in the paper. They include the different way that I have incorporated factor mobility and allowed private location decisions, absent any government intervention, to be potentially inefficient, and the relative complexity of my equalization which allows for expenditure needs.

The paper follows this outline. Section 2 develops the basic model of a regional economy and derives conditions that must be satisfied for global efficiency. Section 3 develops the equalization model. Section 4 embeds this model within a two-stage equalization game and Section 5 examines efficiency. Section 6 concludes with lessons for other countries that have schemes of equalization.

2. A Regional Economy

Consider a regional economy such as a federation of states/provinces, a regional union of semi-autonomous states, or a unitary country with local jurisdictions. Suppose that the regional union has $i = 1, 2$ ‘regions’ each with its own government. From now on we think of the regions as states. Let us also assume that the regional union has a fixed supply N of residents each with identical preferences. We also abstract from leisure/work decisions by supposing that each resident supplies one unit of labor and that n_i is the population and hence labor supply of state i . The given population of the regional economy (labor supply) is

$$N = n_1 + n_2. \quad (1)$$

The implication is that we have a regional union with a labor market that is closed to outward and inward migration, though as will be seen later, we do allow for free

migration between the two states: thus, the regional union has a closed but ‘common’ labor market.

2.1 *Production Technology*

The production process in each state is simple. There are two inputs, the first, immobile and in fixed supply, can be thought of as land, fixed physical capital (public or private), or natural resources. The supply of the immobile factor in state i is denoted as k_i . The second factor is the labor supplied by the residents of the regional economy. Labor is perfectly mobile between states so its supply can vary from the perspective of each state. The two factors are combined within a state using a CRS production technology so factor payments exactly exhaust state output. A numeraire output is produced by this activity; its price is equal to its marginal cost which is assumed to be one. The value y_i of a state’s production of the numeraire is represented by the production function⁴ $y_i = f_i(n_i, k_i)$ but since the immobile factor is in fixed supply in each state this can be expressed as

$$y_i = f_i(n_i) \quad (2)$$

where $f_i'(n_i) > 0$ and $f_i''(n_i) < 0$. Factor markets are competitive and $w_i = f_i'(n_i)$.

Each resident of state i receives the same wage but because k_i may differ across states, and states may have different production technologies, wage rates may differ between states. Residents are also assumed to have full ownership of the fixed factor and hence receive the economic residual $R_i = y_i - w_i n_i$ as income. Assuming that its distribution within a state is on an equal per capita basis the income of a representative resident of state i is the average product of the region,

$$\frac{y_i}{n_i} = w_i + \frac{R_i}{n_i} \quad (i = 1, 2). \quad (3)$$

⁴ The aggregate state production function can be rationalised by assuming that there is a large number, for example, H_i , of perfectly competitive profit maximising firms in state i . All firms, denoted by the index $h_i = 1, \dots, H_i$ are the same and hence have identical production functions, $h_i^j = h_i \left(\frac{n_i}{H_i}, \frac{k_i}{H_i} \right)$. Aggregate production in state i is simply $H_i y_i^j$ and can be represented as in (2).

2.2 *Inter-Regional Cost Differences*

Let us suppose that the numeraire output of state i , y_i , is transformed by the state's government, to be modeled in section 4, into a pure⁵ local public good q_i whilst the rest is consumed as a private good x_i . It is common, at least in models of this nature applied in a federalism setting, to assume that the prices of the public and private goods are one. However, this assumption does not serve well here since we wish to allow for differences in the cost of providing public services in different regions due to so-called 'cost disability factors' which include the effects of population dispersion, geographic features, age/sex distribution of the population, the proportion of indigenous people in the population and other socio economic variables⁶. It is argued that variations in these factors across states imply that some states must incur higher (lower) expenditure per capita to provide one unit of public service relative to the per capita average expenditure of all the states. This is so even if all states face a common marginal cost of public good provision. For example, a state with a highly dispersed population may need to spend more to provide one unit of education service in a remote location relative to a state providing the same unit of education in a city.

So we need to have some way of capturing this idea within our regional model since, as will be seen later, it has a significant impact on the efficiency of equalization. The way I do this is to define the public good price in state i as consisting of two parts, the underlying marginal cost of production, which is a function of the production technology employed by state i , and a variable constructed to capture the expenditure related to the cost disability factors discussed above. Thus, I define the public good price in state i as

$$p_i = mc + \gamma_i \quad (4)$$

⁵ Services provided by Australian states (mainly, public health, education and transport) have a substantial public good component to them. Nevertheless, this is an abstraction from reality, made in order to keep the model tractable. Where appropriate its potential impact on the results is explained.

⁶ Many federations or decentralized economies incorporate cost disabilities into grant schemes but Australia's equalization for cost disabilities is more comprehensive. The most significant disability in an Australian context is the indigenous one: it is the major determinant of the pattern of cost disabilities across the Australian states.

where mc is the marginal cost of producing the local public good, assumed to be the same across all states - hence there is no i subscript for the marginal cost⁷, and $0 \leq \gamma_i$ is a variable which captures the aggregate impact of cost disabilities on the price of the public good. This variable is constructed by the CGC using accounting methods which are not amenable to modeling. However, in a paper which develops a cost function methodology for estimating cost inter-regional cost disabilities econometrically, Petchey, Shapiro, MacDonald and Koshy (2000) provide a model which explains how one can construct the disability variable in a way that can be integrated into a regional model such as the one here. The way that Petchey et al do this is discussed in Annex B where it is noted that they defined the cost disability as

$$\gamma_i(q_1, q_2) = e^{\phi_i(q_1, q_2)} \quad (i = 1, 2) \quad (5)$$

where $\phi_i(q_1, q_2)$ is a function which captures, in aggregate, how a state's cost disabilities deviate from the average cost disabilities suffered by all states. This function also shows us that this deviation is dependent on the joint policies selected by the states; hence, the cost disability variable γ_i is also a function of joint policies. That is why I have expressed the cost disability as a function of state policies in (4).

The construction of (5) implies that: (i) if $\phi_i = 0$ state i has average cost disabilities, $\gamma_i = 1$ and $p_i = mc + 1$; (ii) if $\phi_i < 0$ state i has lower than average cost disabilities, $0 \leq \gamma_i < 1$ and $p_i = mc + (0 \leq \gamma_i < 1)$; and (iii) if $\phi_i > 0$ state i has higher than average cost disabilities, $\gamma_i > 1$ and $p_i = mc + (\gamma_i > 1)$. Therefore, a state with $\phi_i = 0$ will face a public good price equal to its given marginal cost, plus one. Alternatively, a state whose disabilities interact such that $\phi_i < 0$ faces a public good price equal to its marginal cost plus something less than one (possibly zero). A state with $\phi_i > 0$ has a price equal to its marginal cost plus something greater than one. If all states have the same marginal cost, a reasonable assumption in Australia, the only

⁷ Marginal cost can be allowed to vary across states. For example, states might have identical outputs of the public good, but different production technologies, and hence marginal costs. Alternatively, they may have the same production technologies but different outputs (the more likely scenario in a homogeneous economy such as Australia) and hence marginal costs. These possibilities are not considered here since the intent is to focus on the cost disabilities as a source of price differences across states, and strategic behavior over these disabilities.

reason public good prices will differ is because of inter-state variations in the cost disabilities. Thus, γ_i is the only source of variation in the price of the public good across states; states with relatively high cost disabilities will have a relatively higher public good price, while states with relatively low cost disabilities will have relatively lower public good prices – even though all states are assumed to face the same underlying marginal cost of production. Finally, assuming that the price of the private good is one the marginal rate of transformation between the public and private good is $MRT_{xq} = 1/p_i$.

2.3 *Regional Budget Constraint and Mobility*

The aggregate budget constraint in state i can now be defined as $n_i x_i + p_i q_i = y_i$ where x_i is equal per capita consumption of the private good. The constraint says that total consumption of the private good and total consumption of the public good must equal output of the numeraire in state i . In per capita terms, the budget constraint is

$$x_i + \frac{p_i q_i}{n_i} = \frac{y_i}{n_i}. \quad (6)$$

Note that $(p_i q_i)/n_i$ is the per capita contribution by a representative resident of state i to the provision of the public good. This can be thought of as a per capita lump sum tax t_i paid by a representative resident in state i so that $t_i = (p_i q_i)/n_i$. The per capital budget constraint can also be written as $x_i + t_i = y_i/n_i$.

Each identical resident of state i is assumed to have a quasi-concave, continuous and differentiable utility function, $u(x_i, q_i)$. Since I have supposed that residents are freely mobile across states, in other words, the regional union has a common (though closed) labor market, we must make some assumption about how these residents migrate. The simplest free migration assumption is adopted, namely, that labor migrates between states until the following equal return condition is satisfied,

$$u(x_1, q_1) = u(x_2, q_2). \quad (7)$$

One can see from (9) that the public good choices made by each state government will have an impact on the distribution of mobile labour across states within the regional economy.

2.4 Global Efficiency Conditions

What conditions must be satisfied for global efficiency in this regional economy? One approach to answering this is to suppose there is a (mythical) and benevolent central planner who chooses local public good provision and private consumption in each state, and the distribution of mobile citizens across states, to maximize per capita utility in state 1 (or 2) while holding per capita utility in state 2 (or 1) fixed at some given level. This problem and its solution are standard and not reproduced here but it should be noted that two conditions are necessary for global efficiency⁸.

Condition 1: Efficient provision of the public good: The first is that the Samuelson condition must be satisfied in each state:

$$n_i MRS_{xq}^i = p_i, \quad (8)$$

where MRS_{xq}^i is the marginal rate of substitution between the private and public good in state i and $p_i = mc + \gamma_i$ as previously defined. Thus, the public good should be provided in each state to the point where the sum of the marginal benefit to state residents is equal to the price of the public good. But note that the price consists of the underlying marginal cost of providing the public good and the effect of the cost disability variable γ_i .

Condition 2: Spatial Efficiency: The second condition relates to the efficiency with which the mobile factor, labor, is allocated across regions. The first step in explaining this is to consider: what is the net benefit of adding an additional resident (unit of labor) to region i? One way to think of this is that an additional worker

⁸ See the papers by Flatters, Henderson, Vernon and Mieszkowski (1974), Boadway and Flatters (1982), Myers (1990) and Boadway (2004) for analysis of the central planner problem and the conditions necessary for a global optimum. Only a brief summary is presented here.

consumes x_i of resources but adds their marginal product (wage) to the state's output. Hence, the net benefit to region i can be thought of as

$$nb_i = (w_i - x_i). \quad (9)$$

This can be expressed in another way by noting that from equation (6) per capita consumption is $x_i = (f_i(n_i) - p_i q_i) / n_i$ whilst from equation (3) the wage for the additional worker is $w_i = f'_i(n_i) / n_i - R_i / n_i$. Substitution of these terms for per capita consumption and the wage into (9) yields:

$$nb_i = \frac{p_i q_i}{n_i} - \frac{R_i}{n_i}, \quad i = 1, 2 \quad (10)$$

We know that $p_i q_i / n_i = t_i$ is the tax contribution of the new resident. Since we have a pure local public good this is a 'fiscal externality' generated by the extra worker since their tax contribution benefits all the existing residents of state i. On the other hand, R_i / n_i is the new resident's share of state i's economic rent. This is a negative 'rent externality' created by the additional worker since the share of existing residents in the state's rent must decrease by this amount.

Thus, one can characterize the net benefit of adding a person to state i in terms of marginal product relative to consumption, or in terms of the fiscal (tax) and rent externalities they generate. If nb_i is positive then welfare in region i increases as we add a worker; if nb_i is negative then welfare falls with the additional worker. Alternatively, if nb_i is zero then state i has its 'optimal population' in the sense that per capita consumption is equal to marginal product for the last worker added to the population, or alternatively, the (positive) fiscal externality created is exactly offset by the negative rent externality (see Hartwick (1980)).

This is all well and good but we must remember that we have a regional union with a common, but closed, labor market. Thus, the extra worker we add to state i has to come from one of the other states within the regional union. In this two state model, if i is state 1, then a worker added to state 1 must come from state 2, and vice versa. So what we must really think about is the optimal allocation of a given supply of mobile workers between the two states. It seems obvious that the condition that will ensure this is that the net benefit of adding a worker in either state

should be the same. If it is not then clearly total welfare in the regional union can be raised by reallocating workers. To think about what condition will ensure that workers are allocated within the regional union in this way let us define $nb = nb_1 - nb_2$. This just says that the net benefit to the regional union of shifting a worker from state 2 to state 1, or nb , is equal to the net benefit for state 1 less the net benefit for state 2. We want workers to be allocated across states so these net benefits are equal which implies that $nb = 0$. Using (10) this condition can alternatively be written as

$$\left(\frac{p_1 q_1}{n_1} - \frac{R_1}{n_1} \right) - \left(\frac{p_2 q_2}{n_2} - \frac{R_2}{n_2} \right) = 0. \quad (11)$$

Expression (11) must be satisfied if mobile labour is to be allocated across the two states of the regional union in a spatially efficient manner. However, as shown in the fiscal externalities literature (see papers listed in footnote 7) in a union of states in which workers migrate freely to equate utility (see (7)) this spatial efficiency condition is not satisfied. As can be seen from (11) spatial efficiency requires that the *net* externality created in state 1 be equal to the *net* externality created in state 2. But mobile labourers will locate to satisfy the equal utility condition (7), and in general there is no reason why this will equate the net externalities created by location decisions, as is required for spatial efficiency.

2.5 *The Optimal Inter-Regional Transfer*

In the fiscal externality literature the potential inefficiency of decentralized mobile factor location decisions described above has created an ‘efficiency case’ for an inter-state income transfer designed to create an allocation of the mobile factor that is consistent with the equal utility condition (7) and the spatial efficiency condition (11). Letting T be the lump sum transfer of numeraire from state 1 to 2, and substituting for x_1 and x_2 using $x_i = (f_i(n_i) - p_i q_i) / n_i$, one can solve for the value of the transfer which ensures that $nb = 0$. Thus, we solve for T from

$$f_1'(n_1) - \frac{f_1(n_1) - T - q_1}{n_1} - f_2'(n_2) + \frac{f_2(n_2) + T - q_2}{n_2} = 0 \quad (12)$$

and find the equation for the optimal transfer to be

$$T^* = \frac{n_1 n_2}{N} \left(\left(\frac{p_1 q_1}{n_1} - \frac{p_2 q_2}{n_2} \right) - \left(\frac{R_1}{n_1} - \frac{R_2}{n_2} \right) \right). \quad (13)$$

This is the transfer (from state 1 to 2) which ensures that any free migration between states to satisfy the equal utility condition is also consistent with spatial efficiency. The transfer corrects for the distorting effects of the fiscal (tax) and rent externalities on location decisions. The transfer is positive if the externalities are such that output must be transferred from state 1 to 2 and negative if the transfer needs to go the other way on efficiency grounds. Notice also that if the externalities balance one another and sum to zero, the optimal transfer is zero (no transfer is required on efficiency grounds and free migration is efficient without intervention).

There are various complexities one can add to this story. For example, if q_i is allowed to be an impure public good using a congestion parameter α , as is sometimes done in the fiscal federalism literature, then $p_1 q_1$ and $p_2 q_2$ in (13) are each pre-multiplied by $(1 - \alpha)$. Therefore, q_i will be a pure private good if α is equal to one. In this case the fiscal externality terms disappear from (13). This implies that the optimal transfer is only a function of differences in per capita rents, that is, $T^* = (n_1 n_2 / N) (R_1 / n_1 - R_2 / n_2)$. Further, if rents accrue to the national government and are distributed regardless of location, there are no rent externalities to distort migration decisions, and $T^* = 0$; private sector migration decisions are fully optimal. Since I have assumed that q_i is a pure local public good, and that state specific rents are allocated only to state residents, in general there will be an efficiency case for a non-zero transfer.

The optimal transfer can also be expressed in terms of the net benefits;

$$T^* = \frac{n_1 n_2}{N} (nb_1 - nb_2). \quad (14)$$

As will be shown later, the Australian equalization scheme creates substantial transfers of income across states. The question I am interested in is whether these transfers are consistent with the optimal transfer described above. This is the issue that no one else has confronted in the literature on fiscal federalism or equalization and is a key focus of the paper. Moreover, whether the optimal transfer is non-zero

or zero makes no difference to my fundamental result, namely, that equalization results in inter-state transfers that are inconsistent with the spatially efficient transfer. For example, equalization will make a transfer even when none is called for on spatial efficiency grounds, and even when a transfer is needed to correct for migration externalities, the transfer that equalization makes will be incorrect.

3. Equalization

An equalization model for the simple regional economy constructed in section 2 can now be developed. The first step is to note that the pool of revenue to be allocated to the states is equal to central revenue collections less central expenditures. The difference, a ‘fiscal gap’, is a function of the particular tax and expenditure assignment. As noted in the Introduction, in Australia the fiscal gap is broadly equal to the GST revenue and health care grants, and because taxation powers are highly centralized, the pool is relatively larger than in other more decentralized economies (e.g. Canada). Since I wish to concentrate on the efficiency with which a given pool is allocated amongst regions (states) I will abstract from the issue of how the pool is created. This is done, somewhat artificially, by assuming that the national government generates some revenue pool G using a lump sum tax s levied on each of the N citizens in the federation; hence $G = sN$. States are assumed to treat N and G as exogenous. Hence, s is exogenous from their perspective.

3.1 The Grant Formula

In Australia G is allocated using a fiscal capacity equalization model described in Commonwealth Grants Commission (2005). This model uses the following equation to determine the per capita grant g_i to state i :

$$g_i = \frac{G}{N} + \frac{T+G}{N}(\gamma_i - 1) + \frac{T}{N}(1 - \rho_i) \quad i = 1, 2. \quad (15)$$

Equal per capita Expenditure need Revenue need

The right hand side has three parts⁹. The first, G/N , is what everyone in the country would get if the allocation was made purely on an equal per capita basis. If the other components of the formula did not exist, the Australian model would be a simple equal per capita one. The second and third parts measure expenditure and revenue needs, as explained below.

3.2 *Expenditure and Revenue Needs*

The term $((T + G)/N)(\gamma_1 - 1)$, is the ‘expenditure need’ of state i , where T is the tax revenue of all states. Since G is the grant pool allocated to the states, $T + G$ is the total expenditure of all states. In the two-state regional union modeled here, where each state provides one service, we know that total state spending is $T + G = p_1q_1 + p_2q_2$ and hence the total tax revenue raised by all states is just

$$T = p_1q_1 + p_2q_2 - G. \quad (16)$$

The expenditure need for state i is positive if state i has relative high costs ($\gamma_1 > 1$), thus tending to push g_i above its equal per capita share, G/N , and negative if state i has relatively low costs ($\gamma_1 < 1$), tending to pull g_i below its equal per capita share, G/N . Thus, the expenditure need is positive or negative depending on whether state i has relatively high or low costs of providing the local public good.

The last part of the grant formula, $(T/N)(1 - \rho_i)$, is the ‘revenue need’ where ρ_i is a ‘revenue disability’ which measures the relative strength of the state’s tax base. The CGC constructs a disability for each state tax base using various approaches which cannot be modeled here. However, the measures adopted by the CGC are highly correlated with a state’s per capita output, as one might expect (e.g. states with richer tax bases are likely to have higher per capita output). Therefore, in the model here I use a proxy for the revenue disability, namely, per capita output in state i divided by per capita output across all states:

$$\rho_i = \frac{N \cdot y_i}{n_i(y_1 + y_2)} \quad i = 1, 2 \quad (17)$$

⁹ In the CGC model equation (15) also includes a term which reflects the budgetary status of state i (see CGC (2005)). This is inconsequential to the analysis and is excluded. Also, I apply the formula

If $\rho_i > 1$ (state i is relatively rich) the revenue need is negative and tends to pull g_i below the per capita share, G/N . If $\rho_i < 1$ (state i is relatively poor) the revenue need is positive tending to push g_i above state i's per capita share, G/N .

In summary, each state receives its equal per capita share of the revenue pool G adjusted for its revenue and expenditure needs, each of which can be positive or negative, depending on the state's cost and revenue disabilities. For example, a state with relatively high cost disabilities and low per capita output would be assessed as having positive expenditure and revenue needs and receive a per capita grant which exceeds G/N .

3.3 *The Inter-Regional Transfer with Equalization*

Though not explicitly stated in CGC papers there is also a 'balanced grant condition' which ensures that the sum of the grants paid to states exactly exhausts the revenue pool, G . Thus,

$$n_1 g_1 + n_2 g_2 = G = sN. \quad (18)$$

To see the implications of this note that the total grant to state i is $n_i g_i$ and the state's contribution to G is $n_i s$. Therefore, the net transfer to state i, denoted as θ_i , is the difference between the two; namely, $\theta_i = n_i g_i - n_i s$. This is zero if $g_i = s$ (the state's per capita grant is equal to its per capita contribution), positive when $g_i > s$ (the state's per capita grant exceeds its per capita contribution to the pool) and negative if $g_i < s$ (the state's per capita grant is less than its per capita contribution to the pool). But (18) implies that the net transfer to state 1 must also be the negative of the net transfer to state 2 so the following must also be true:

$$\theta_1 = n_1 (g_1 - s) = -\theta_2. \quad (19)$$

Say, for example, that θ_1 is \$100; (19) means that state 2 must be transferring \$100 to state 1, hence the transfer to state 2 is -\$100. Alternatively, suppose that θ_1 is

to a two state regional union whereas Australia has $i = 1, \dots, 8$ states and territories.

-\$100; (19) implies that state 2 has a net transfer of \$100 in its favour. Equation (19) is, therefore, a formalization of the transfers between the Australian states that appear in the Table in Annex A.

The following Lemma will prove useful in the later discussion on the efficiency of equalization:

Lemma 1: The inter-state transfer with fiscal equalization is not the one required for an efficient spatial allocation of the mobile factor.

Proof: The inter-state transfer with equalization is given by (19) and is a function of the per capita grant to state 1 which is, in turn, a function of state expenditures, taxes, as well as cost and revenue disabilities. However, the optimal inter-state transfer (13) is a function of fiscal and rent externalities. There is no reason why the inter-state transfer (19) will be the same as the one implied by (13), except fortuitously. Thus, the inter-regional transfer with equalization is not the one required to achieve spatial efficiency//.

4. Equalization Game

This section integrates the equalization model developed in section 3 into the regional economy of section 2. The analysis begins by assuming that each state within the regional union has a benevolent government. Each chooses its public good provision to maximize within-state per capita utility taking account of production feasibility, the equalization scheme and labor location decisions. The policies of the states are interdependent for two reasons; first, the policies of one state have an impact on welfare in the other state by affecting certain variables within the equalization formula which in turn influences the pattern of grants and inter-state transfers, and second, the policies of one state affect the other by changing the distribution of the mobile factor (labor) within the regional economy because labor location decisions are sensitive to state policies via the equal utility condition. For these reasons states can be thought of as playing an *equalization game* where q_1 and q_2 are the (continuous) strategy choices of the states. Though I

shall think of public good outputs as the choice variables from now on, it should be noted that the states can also be thought of as playing the game in taxes if one recalls that $t_i = p_i q_i / n_i$. As will be seen in Lemma 2, n_i and p_i are functions of q_1 and q_2 . Therefore, once a state has chosen its public good output, for a given public good output in the other state, it has also chosen its per capita tax on mobile labor. Thus, one can think of this as a game with public good (policy) competition over mobile labor, or tax competition over mobile labor. Moreover, as the discussion on global optimality in section 2.2 has shown this competition leads to mobile labor location choices that are sub-optimal because of the presence of fiscal and rent externalities. Migrating labor makes its location choices conditional on state policies to satisfy the equal utility condition. Hence states and mobile labor are rational agents making optimal choices to maximize their respective objectives.

It is impossible to characterize the CGC model depicted in section 3 as the outcome of any optimization process. This is not surprising given that the equalization scheme is not a normatively ideal construct. Therefore, the CGC is not included explicitly as a player in the game. Rather, the equalization model is taken to be a part of the environment in which states and labor make their decisions. More formally the equalization game is as follows:

- *Stage 1 (states' move)*: States 1 and 2 choose q_1 and q_2 simultaneously, adopting Nash conjectures. States take account of the effect of their policy choices on their net equalization transfer¹⁰ and correctly anticipate labor location decisions made in stage 2.
- *Stage 2 (mobile factor moves)*: Conditional on the state policy choices in stage 1 citizens make their location choices to equate per capita utility.

4.1 State Optimization

Now let us consider public good provision in state 1 in the equalization game. Taking into account the net equalization transfer given by equation (19) the

¹⁰ This is a reasonable assumption in Australia where the states invest considerable resources into understanding the CGC model and simulating how their own decisions affect their grants.

aggregate budget constraint for state 1 can be expressed as $n_1 x_1 + p_1 q_1 + \theta_1 = f_1(n_1)$. Recalling that $p_1 = mc + \gamma_1$ per capita consumption in state 1 inclusive of the transfer is

$$x_1 = \frac{f_1(n_1) - (mc + \gamma_1) \cdot q_1 + \theta_1}{n_1}. \quad (20)$$

The objective of state 1 in stage 1 is to choose q_1 (given q_2) to maximize¹¹

$$u\left(\frac{f_1(n_1) - (mc + \gamma_1) \cdot q_1 + \theta_1}{n_1}, q_1\right) \quad (21)$$

S to: (A) *Migration Constraints*:

$$(i) u\left(\frac{f_1(n_1) - (mc + \gamma_1) \cdot q_1 + \theta_1}{n_1}, q_1\right) = u\left(\frac{f_2(n_2) - (mc + \gamma_2) \cdot q_2 - \theta_1}{n_2}, q_2\right)$$

$$(ii) n_1 + n_2 = N$$

(B) *Equalization Constraints*

$$(iii) g_1 = \frac{G}{N} + \frac{T+G}{N}(\gamma_1 - 1) + \frac{T}{N}(1 - \rho_1) \quad (vi) \gamma_1 = e^{\delta}$$

$$(iv) \theta_1 = n_1(g_1 - s) \quad (vii) \gamma_2 = e^{\delta}$$

$$(v) T = (mc + \gamma_1) \cdot q_1 + (mc + \gamma_2) \cdot q_2 - G \quad (viii) \rho_1 = \frac{N \cdot f_1(n_1)}{n_1(f_1(n_1) + f_2(n_2))}$$

In the set up of (21) I have substituted into the objective function and the equal utility condition for x_1 and x_2 - the latter using the regional budget constraint for state 2 (analogous to (20)). Thus, feasibility does not appear as a separate constraint. Also, using (19), and the balanced grant condition built into (19), the net transfer to state 2 in the equal utility condition is incorporated as the negative of the net transfer to state 1. This eliminates g_2 and s from the right side of the equal utility condition. The balanced grant condition is not therefore included explicitly as a separate constraint nor is it necessary to incorporate the equation for g_2 into the constraint set.

With Nash conjectures nine variables are perceived by the state to be endogenous; $q_1, n_1, n_2, \theta_1, g_1, \gamma_1, \gamma_2, \rho_1$ and T . Since there are eight constraints there

¹¹ State 2 solves an analogous problem.

is one free dimension for optimization. The parameters are G, N, s and mc . The following Lemma is useful for the later analysis of the efficiency of any Nash equilibrium:

Lemma 2: Each of the endogenous variables in the equalization game is a function of collective state policies, q_1 and q_2 .

Proof: See Annex B.

Interestingly, Lemma 2 shows that the grant allocations, and the inter-state transfer that occurs with equalization are, for a given equalization formula, actually determined by the states' policy choices. This idea is novel since if one asked CGC and state officials in Australia who determines the equalization grant and the resulting inter-state redistribution, the answer would be that it is the CGC. It is true that the national government and the CGC choose the formula, but once this is given, the states choose the grants and the degree of inter-regional redistribution.

4.2 *Equilibrium*

The optimization problem above can be solved to find the necessary conditions that must be satisfied in any Nash equilibrium. The first step is to differentiate the objective function with respect to q_1 , for given q_2 (with Nash conjectures), allowing state 1 to perceive (correctly) the effects of its choice on its net equalization transfer and (through migration) population. This yields

$$n_1 MRS_{xq} = MCPF_1 \quad (22)$$

where $MRS_{xq} = u_{q_1} / u_{x_1}$ is the marginal rate of substitution between the local public good and the private good (marginal benefit of the public good) and $MCPF_1$ is the perceived marginal cost of public funds defined as

$$MCPF_1 = p_1 + \frac{\partial \gamma_1}{\partial q_1} q_1 - \frac{\partial \theta_1}{\partial q_1} - nb_1 \frac{\partial n_1}{\partial q_1}. \quad (23)$$

Constraint (ii) from the state optimisation problem can be written as $n_2 = N - n_1$ and substituted into the equal utility condition, constraint (i). The constraint set then consists of seven equations. One can then differentiate each constraint with respect to q_1 , for given q_2 . The result in $Ax=d$ form is¹²:

$$\begin{pmatrix} \left(\frac{u_{x_1}}{n_1} nb_1 + \frac{u_{x_2}}{n_2} nb_2 \right) & 0 & \left(\frac{u_{x_1}}{n_1} + \frac{u_{x_2}}{n_2} \right) & -\frac{u_{x_1}}{n_1} q_1 & 0 & 0 & 0 & 0 \\ 0 & N & 0 & -(T+G) & 0 & 0 & 0 & T \\ -(g_1 - s) & -n_1 & 1 & 0 & 0 & 0 & 0 & 0 \\ (g_1 - s) & n_1 & 0 & 0 & \frac{1}{2} & -\frac{1}{2} & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ (nb_1 - w_1 n_1) & 0 & 0 & -q_1 & n_1 & 0 & 0 & 0 \\ -(nb_2 - w_2 n_2) & 0 & 0 & 0 & 0 & n_2 & 0 & 0 \\ -B & 0 & 0 & 0 & 0 & 0 & 0 & n_1 y \end{pmatrix} \begin{pmatrix} \frac{\partial n_1}{\partial q_1} \\ \frac{\partial g_1}{\partial q_1} \\ \frac{\partial \theta_1}{\partial q_1} \\ \frac{\partial \gamma_1}{\partial q_1} \\ \frac{\partial nb_1}{\partial q_1} \\ \frac{\partial nb_2}{\partial q_1} \\ \frac{\partial c_1}{\partial q_1} \\ \frac{\partial \rho_1}{\partial q_1} \end{pmatrix} = \begin{pmatrix} \frac{u_{x_1}}{n_1} p_{q_1} - u_{q_1} \\ 0 \\ 0 \\ 0 \\ \phi_1 \gamma_1 \\ p_{q_1} \\ 0 \\ 0 \end{pmatrix} \quad (24)$$

There are seven simultaneous equations in (24), which I think of from now on as the *strategic behaviour matrix*, with seven unknowns, $\frac{\partial n_1}{\partial q_1}$, $\frac{\partial g_1}{\partial q_1}$, $\frac{\partial \theta_1}{\partial q_1}$, $\frac{\partial T}{\partial q_1}$, $\frac{\partial \gamma_1}{\partial q_1}$, $\frac{\partial \gamma_2}{\partial q_1}$ and

$\frac{\partial \rho_1}{\partial q_1}$. The parameters are as given before. Three of unknowns, the migration

response, $\frac{\partial n_1}{\partial q_1}$, the net equalization transfer response, $\frac{\partial \theta_1}{\partial q_1}$, and the net benefit

response, $\frac{\partial nb_1}{\partial q_1}$, appear in the expression for the marginal cost of public funds

above. What the strategic behaviour matrix tells us is that their values are interdependent and determined within a larger system of equations and unknowns arising from the constraint set that state 1 faces; including feasibility, equalization and migration. Equilibrium to the equalization game can now be defined as:

¹² Where in the A matrix $B = Nw_1 - \rho_1(y + n_1 w_1 - n_1 w_2)$.

Definition of Equilibrium: A sub-game perfect Nash equilibrium (SPNE) is a policy pair for the equalization game, (q_1^*, q_2^*) , that solves (22), (23) and (24), and analogous expressions for state 2, simultaneously¹³.

Given the complexity of (24) I have not solved analytically for an equilibrium policy pair though a simulation model has been constructed based on specific functional forms for production technology and preferences (this model is available from the author on request)¹⁴. Further, the issue of existence of equilibrium is not examined here. It is possible that an equilibrium may not exist, or if it does, that there is more than one.

5. Efficiency

The main result on the spatial efficiency of a SPNE to the game is now presented:

Theorem - Spatial Location: In a SPNE of the equalization game the mobile factor, labour, is allocated inefficiently across states.

Proof: A SPNE, if one exists, yields a policy pair q_1^*, q_2^* . From Lemma 2 we know that all endogenous variables in the equalization game are functions of state policies. Therefore, a Nash equilibrium yields equilibrium values for state specific wage rates, w_1^*, w_2^* , the net benefits, nb_1^*, nb_2^* , the net equalization transfer to state 1 (which is the negative of the transfer to state 2), $\theta_1^* = -\theta_2^*$, and the populations, n_1^* and n_2^* . Furthermore, in any equilibrium mobile labor will be allocated across states to satisfy the equal utility condition,

$$u_1^*(w_1^* - nb_1^* - \theta_2^* / n_1^*, q_1^*) = u_2^*(w_2^* - nb_2^* + \theta_2^* / n_2^*, q_2^*). \quad (24)$$

¹³ An equilibrium is sub game perfect since in each stage of the game players use best responses.

¹⁴ If impurity is allowed with respect to the local public good the left side of the necessary condition becomes $(n_1 / n_1^\alpha) MRS_{xq}$. The A matrix is also slightly modified since when one differentiates the equal utility condition the resulting migration response changes to reflect the presence of the alpha parameter. Other than this the remaining strategic behavior responses in $Ax = d$ are unaffected. This is also the case in the alternative grant models considered in Section 6 below. Thus, the nature of public expenditures is immaterial to the general results.

From section 2.2, we know that the satisfaction of this equal utility condition will also yield spatial efficiency only if the inter-state transfer from state 1 to 2 is equal to the optimal transfer. However, we know from Lemma 1 that this is, in general, not the case. Therefore, a SPNE to the equalization game yields an allocation of the mobile factor across regions that is inefficient.//

The spatial inefficiency arises because the inter-state transfer consistent with an equilibrium to the equalization game is not consistent with the optimal transfer, as given by expression (13). This is the case even when the optimal transfer is zero, for example, because there externalities to distort private location decisions. In this case, equalization will still make some inter-state transfer when none is called for on efficiency grounds. The problem is that as shown in section 2.2 the optimal inter-state transfer is a function of externalities, while the actual inter-state transfer under equalization is a function of cost and revenue disabilities which are unrelated to these externalities. This, in turn, reflects the fact that Australian equalization, in common with most other schemes of equalization, is motivated by equity.

Though not the main focus of the paper, it is also possible to say something about the efficiency of local public good provision. In particular, we know from the discussion in section 2.2. that public good provision should satisfy the Samuelson condition. However, this will only be the case in a SPNE to the equalization game if the migration, cost disability and net benefit responses on the right hand side of the expression for the marginal cost of public funds, (23), are zero. Though I have not solved the game analytically because of its complexity, in simulations undertaken with the numerical version of the model mentioned above, it is clear that these terms are not zero, and that therefore, public good provision is not consistent with the Samuelson condition. Note that these terms have an influence on the marginal cost of public funds, distorting it away from the true opportunity cost, p_1 ; they are present because of strategic behaviour over the equalization formula and migration. This result supports the findings in Dahlby and Warren, and also in the paper by Smart, who found that equalization distorts the marginal cost of public funds.

However, I cannot replicate the results of Köthenburger, or Bucovetsky and Smart with respect to public good provision. Recall from the Introduction they

found that tax competition creates under provision whilst equalization induces under provision with the two effects canceling each other out resulting in efficient provision of local public goods. Admittedly, I have not solved my model analytically for a SPNE, but I have developed a numerical simulation model, and in simulations with that model equilibrium local public good provision is consistently inefficient (the response terms affecting the marginal cost of public funds are non zero). I am unable to explain this difference of results. It is true that I do not have tax competition over mobile capital, as in the Köthenburger and Bucovetsky and Smart papers, but I do have tax competition over mobile labor. Important differences lie in what is assumed about mobility and the type of equalization scheme employed. In my model labour migration can be inefficient (distorted by fiscal and rent externalities) whereas in the Köthenburger and Bucovetsky and Smart papers this potential for private location choices absent any state intervention is not incorporated. Also, in my model the equalization scheme is a real world one which includes expenditure needs and the strategic behavior that might occur over such needs whereas Köthenburger and Bucovetsky and Smart build an equalization scheme which includes revenue needs only. These differences are likely to account for the differences in results over public good provision.

To summarize, a decentralized equilibrium in which states choose levels of provision of local public goods (or taxes) but act strategically with respect to an equity-motivated, centrally mandated, fiscal capacity equalization scheme, is inefficient for two reasons; the mobile factor of production is allocated inefficiently across regions because the inter-regional transfer that occurs with equalization is not the one required for spatially optimal location decisions and public good provision is inefficient. One can think of a SPNE to this game as ‘constrained efficient’; it is the best that states can do given the national equalization scheme and the incentives that it creates for strategic behavior.

6. Conclusion

The new result in this paper is that fiscal equalization for revenue and expenditure needs results in strategic behavior and a pattern of inter regional transfers of income

that are inconsistent with the efficient spatial location of mobile factors of production. This is essentially because the inter regional transfer under equalization is not the one required to correct for any of the externalities that might be associated with location choices. It was also found that local public goods would be provided inefficiently because of strategic behavior over the equalization formula which distorts the marginal cost of providing public goods.

It should be remembered of course that equalization is undertaken for mostly equity reasons; indeed, inter state equity is the stated aim of Australian equalization¹⁵. These equity goals must be weighed against any possible efficiency consequences of the sort found in this paper. The empirical significance of the efficiency effects identified above has also not been established. Though I have shown all the conceptually plausible routes by which states may be induced to adopt strategic behavior over equalization, and choose the ‘wrong’ inter-state transfer, the magnitude of the welfare effects may, or may not, be small. We simply do not know the answer to this question.

Finally, regions/states would require full information about the grant model and migration, as well as how grants, location decisions and inter-state transfers respond to their policy choices (i.e (24)), if the distortions identified here were to be of concern. One might argue instead that states are myopic with respect to the equalization formula adopted, and migration. If this is so, then all the terms in the strategic behavior matrix become zero and public good provision at the regional level satisfies the Samuelson condition. However, the inter-state transfer that results from equalization will still be inefficient.

¹⁵ See Morris (2003).

Annex A

Regional Redistribution Caused by Equalization, Australia, 2006-07

	Equalization Distribution¹⁶	Equal Per Capita	Difference	Pop.	Per Capita Redistribution
	A\$m (1)	A\$m (2)	A\$m (1)-(2)	(m)	A\$m
New South Wales	13,728.8	15,726.7	-1,997.9	6.9	-291.0
Victoria	10,477.3	11,703.6	-1,226.3	5.1	-240.0
Queensland	9,573.9	9,354.6	219.4	4.1	53.7
Western Australia	4,745.1	4,724.3	20.7	2.1	10.1
South Australia	4,231.6	3,561.6	670.0	1.6	430.9
Tasmania	1,735.4	1,120.5	614.8	0.5	1,256.8
ACT	863.3	753.8	109.5	0.3	332.8
NT	2,067.7	478.0	1,589.7	0.2	7,617.9
Total (Pool)	47,423.0	47,423.0	na	20.7	na

Source: Commonwealth Budget Paper No. 3, Federal Financial Relations, 2006-07.

¹⁶ The revenue pool allocated to the states during 2006-07 consists of revenue from the Goods and Services Tax (GST) and health care grants. The GST is levied by the Commonwealth as a 10% value added tax and all of the revenue is passed back to the states via the equalization model.

Annex B

1. Construction of the Variable Capturing Inter-Regional Cost Differences

The Australian equalization model estimates values for γ_i using an accounting approach (Commonwealth Grants Commission (2005)) which is impossible to capture in a model. However, Petchey et al (2000) develop a cost function model to estimate γ_i for the Australian states using an approach that captures the intent of the CGC methodology but is amenable to inclusion in the model here. They define the cost disability as

$$\gamma_i = e^{\phi_i} \quad (i = 1, 2) \quad (\text{B1})$$

where

$$\phi_i = \sum_{j=1}^J \beta_{i,j} D_{i,j} \quad (i = 1, 2) \quad (\text{B2})$$

is a ‘disability function’ for state i where: (i) $D_{i,j}$ is the percentage deviation of the j^{th} disability measure (e.g. population dispersion) from the mean value of that measure for all states; (ii) $0 \leq \beta_{i,j}$ captures the impact of the percentage deviation of the j^{th} disability from its mean value on the value of the disability function in state i ; and¹⁷ (iii) $D_{i,j} = (X_{i,j} - \bar{X}_j) / \bar{X}_j$ where $X_{i,j}$ is the value of the j^{th} disability measure for state i and \bar{X}_j is the mean value of the disability measure for all states.

The construction of (1) is neat since it implies that the cost disability is normalized around one. If $\phi_i = 0$ then $\gamma_i = 1$ and $p_{qi} = mc_{qi} + 1$ whereas $\phi_i < 0$ implies $0 \leq \gamma_i < 1$ and hence $p_{qi} = mc_{qi} + (0 \leq \gamma_i < 1)$. When $\phi_i > 0$ we have $\gamma_i > 1$ and $p_{qi} = mc_{qi} + (\gamma_i > 1)$. Therefore, a state whose disabilities offset one another within (2) such that $\phi_i = 0$ will face a public good price equal to its given marginal cost, plus one. Alternatively, a state whose disabilities interact such that $\phi_i < 0$ faces a public good price equal to its marginal cost plus something less than one

¹⁷ Petchey et al estimate $\beta_{i,j}$ for each of the $i=1, \dots, 8$ Australian states and $j=1, \dots, 4$ disabilities based on their cost function model.

(possibly zero). A state with $\phi_i > 0$ has a price equal to its marginal cost plus something greater than one. If all states have the same marginal cost, a reasonable assumption in Australia, the only reason public good prices will differ is because of inter-state variations in the cost disabilities.

State policies will influence the disabilities captured within (2). This can be seen by considering state 1. The value of the j^{th} disability measure in the state, $X_{1,j}$, is a function of q_1 . For example, suppose $X_{1,j}$ is a measure of population dispersion or decentralization in state 1. The state's policy choices will in general have an influence on the degree of decentralization. This implies that $D_{1,j}$, the percentage deviation (for state 1) of its decentralization disability measure from the mean value for all states, is also a function of state 1 policy choices. In addition state 1's policy choices will affect \bar{X}_j and hence $D_{2,j}$, the percentage deviation of state 2's dispersion disability from the mean.

Sorting out the precise nature of the interaction between state policies and the disabilities is a major project in its own right and beyond the scope of this paper. But I wish to capture in general terms the idea that state policies have an impact on cost disabilities and hence, as will be seen later, on a state's grant, creating potential for strategic behaviour. Thus, define the general function $\phi_i(q_1, q_2)$ and hence

$$\gamma_i(q_1, q_2) \quad i = 1, 2, \quad (\text{B3})$$

where no specific restrictions are placed on the precise relationship between state policies and the cost disability of state i .

2. Proof of Lemma 2

From constraint (viii) the revenue disability is a function of state populations, $\rho_1(n_1, n_2)$, and constraint (iii) implies that the per capita grant is a function of parameters, the cost disability and the revenue disability, $g_1(G, N, T, \gamma_1, \rho_1)$. Furthermore, from equation (15) we know that total state tax collected is a function of joint state policy choices, $T(q_1, q_2)$, and we also know that the cost disability is a function of joint policies, $\gamma_1(q_1, q_2)$. Therefore, one can define the per capita grant

simply as a function of state policies and populations, $g_1(q_1, q_2, n_1, n_2)$. Meanwhile, constraint (iv) implies that the net transfer to state 1 is a function of state policies and the population of state 1, $\theta_1(n_1, q_1, q_2)$. Thus, for given state policies, constraints (i) and (ii) imply that the population of state 1 is a function of joint state policies, $n_1(q_1, q_2)$, and analogously for state 2's population, $n_2(q_1, q_2)$. The revenue disability, per capita grant and net transfer can, therefore, all be defined as functions only of collective state policies, namely, $\rho_1(q_1, q_2)$, $g_1(q_1, q_2)$ and $\theta_1(q_1, q_2)$. By implication we also know that the net benefit of adding an extra worker to state i is a function of state policies, $nb_i(q_1, q_2)$, as is the wage in state i , $w_i(q_1, q_2)$ // .

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