International Spillovers of Unconventional Monetary Policy*

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We study and measure how forward guidance of the United States Federal Funds rate at its zero lower bound affects monetary policy and the dynamics of a small open economy. To do so, we set up and estimate a two-country open economy model on US and Canadian data from 1984Q1 to 2014Q4. Using piecewise linear solution methods, we jointly estimate the model's structural parameters and the expected durations of the fixed nominal interest rate regimes in the US and Canada at each quarter from 2009Q1. We decompose the estimated durations into a component implied by the constraint itself and an additional one which we interpret as calendar-based forward guidance. The results strongly support the existence of international spillovers of US forward guidance. Using the estimated model, we conduct counterfactuals and find that without forward guidance in the US, monetary policy in Canada would not have been as constrained by its effective lower bound because its exchange rate would have appreciated significantly less.

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1 Introduction

In response to the financial crisis which originated in the United States, the Federal Reserve cut its policy interest rate to near zero. As the financial crisis spread, a number of small open economies followed suit, also cutting their policy rates to near zero. The Bank of Canada, for example, reduced its policy rate to 0.25 per cent in April of 2009 and the Bank of England set its official Bank Rate to 0.5 per cent in March of 2009. Since 2009, Denmark, Sweden and Switzerland have also lowered their policy interest rates to zero and, more recently, to just below zero.[1]

Soon after running up against their effective lower bounds, central banks turned to unconventional policies. One unconventional policy which has been used and has received considerable attention is forward guidance. Forward guidance refers to announcements made by the central bank about the future path of its policy rate with the explicit aim of influencing expectations so as to increase the degree of policy accommodation.[2]

There is a good argument in theory for why forward guidance can alleviate the contractionary impact of the zero lower bound (ZLB). In forward-looking *closed economy* models the current stance of monetary policy depends on the expected path of the nominal interest rate. Therefore forward guidance can, in principle, stimulate aggregate demand to the extent it lowers private agents' forecasts of future nominal interest rates. So, a credible commitment to maintain interest rates at zero for longer than would have otherwise been implied by the zero bound itself represents an additional channel of monetary stimulus. Eggertsson and Woodford (2003), Jung et al. (2005) and more recently Werning (2012) all make this point: monetary policy can stimulate an economy by creating the right kind of expectations about the way the policy rate will be used once the constraint ceases to bind.[3]

Forward guidance in a *small open economy* also works to provide monetary stimulus. Its impact, however, is not independent of the stance of monetary policy abroad. It therefore matters when calibrating unconventional policies in an open economy to be able to measure if foreign forces as opposed to domestic ones are responsible for driving the open economy to, or keeping it at, its ZLB. For a small open economy there are two relevant bounds highlighted in the literature: the bound that applies to the foreign economy's policy rate and the one that applies to its own policy rate.

[1] It does not matter for our purposes if this bound is zero or some other number. In any case it is clear that with physical cash whose rate or return is zero, there must exist some lower bound on the nominal interest rate that a central bank can set. No equilibria exist for nominal interest rates beyond that bound given predominant current legal arrangements that establish that balances at the central bank may be converted into notes at an exchange rate of unity. In the paper, we use the more familiar term 'zero lower bound' instead of the more precise term 'effective lower bound'.

[2] Following the classification of Campbell et al. (2012), forward guidance that commits the central bank to a particular path of its policy rate beyond that implied by a policy interest rate rule is *Odyssean* guidance. This is different than *Delphic* forward guidance, which is an announcement that is informative about the state of the economy. Odyssean announcements can be calendar-based, with the announcement ending at a specified time period, or threshold-based, with the policy rate held at zero so long as specified thresholds are not breached. Our focus is on calendar-based Odyssean forward guidance policies near the zero lower bound.

[3] Krugman (1998) was the first to recast the liquidity trap as an expectations-driven phenomenon.

Forward guidance abroad at the foreign ZLB lowers the expected future path of foreign policy rates inducing an appreciation of the home currency. Because this appreciation reduces imported goods inflation it could drive an inflation targeting central bank to its lower bound. But forward guidance spillovers also operate through a foreign demand channel for the home economy's output, a channel emphasized by Bodenstein et al. (2009), Haberis and Lipinska (2012) and Fujiwara et al. (2013). As a result of a forward guidance announcement in the foreign economy, the home exchange rate appreciates. The resulting effect on the demand for domestic output is determined by the *trade elasticity* of substitution. When domestic goods easily substitute for foreign goods, the decline in foreign demand for domestic goods caused by the exchange rate appreciation outweighs the effect of stronger demand for domestic goods from a stimulated foreign economy–a 'begger-thy-neighbor' effect. But when domestic and foreign goods are complements, the higher demand from the foreign economy can outweigh the substitution effect caused by the exchange rate appreciation, so that the home economy can benefit from forward guidance abroad. Because of this uncertainty about the effect of the foreign demand channel, it is an empirical issue to establish the sign and magnitudes of the responses of domestic variables to foreign forward guidance.

Cook and Devereux (2014) highlight another reason why the ZLB and forward guidance may work differently in open economies. In an open economy at the ZLB, the exchange rate can exacerbate the adverse effects of domestic shocks. In normal times, i.e. away from the lower bound, floating exchange rates adjust to buffer the effects of these shocks. At the ZLB, however, domestic monetary policy cannot respond to adverse shocks so that the exchange rate is lower than it would otherwise have been. The lack of a domestic monetary policy response reinforces the contraction.[4]

The papers highlighted are part of a literature that studies *in theory* the ZLB and forward guidance in open economies. Taken together, they illustrate that there are possibly competing mechanisms which govern the strength and direction of cross-country spillovers of monetary policy at the ZLB. To the best of our knowledge there are no empirical studies that include the period of unconventional monetary policy and attempt to quantify these mechanisms in the context of a structural open economy model. This paper fills this gap.

Our objective is to measure the spillovers of forward guidance from large to small economies as well as assessing the impact of domestic forward guidance itself.[5] To do so, we set up a two-country small open economy model based on Galí and Monacelli (2005) and estimate jointly the parameters of the model and the expected durations of the fixed interest rate regimes of the US and Canada for each quarter since 2009Q1.[6] Estimating forward-looking structural models at the ZLB requires accounting for a regime shift in policy. In estimation we follow Kulish et al. (2014) and allow, but

[4]More generally, the unresponsiveness of the policy interest rate when combined with unanticipated shocks can change not only the magnitude of the response of the system, as pointed out by (Christiano et al., 2011), but also the sign of as shown by (Wieland, 2014; Eggertsson, 2010).

[5]We capture other unconventional policies such as quantitative easing but only to the extent suggested by Bernanke and Reinhart (2004) that they may influence expectations of the future path of policy rates.

[6]In the case of Canada we estimate forward guidance at one per cent for the period 2010Q3 onwards.

do not require, central banks to engage in forward guidance that extends the expected duration of the interest rate policy beyond that implied by the bound itself.

A methodological innovation of our analysis is to combine estimation at the ZLB with the piecewise linear method to handle occasionally binding constraints proposed by Jones (2016). This allows us to decompose the estimated durations of the fixed interest rate regime into a component due to structural shocks and one which has the interpretation of an intended extension of the fixed interest rate duration, a calendar-based commitment policy. We use this decomposition to study the contribution of US forward guidance to fluctuations in output, inflation and the exchange rate in Canada. We account for two occasionally binding interest rate constraints. In particular, we exploit the block-exogenous structure of the two-country small open economy model and identify, first, foreign ZLB durations, and with those expected durations in hand, we then identify domestic ZLB durations.[7]

As the theory emphasizes that the impact of foreign forward guidance on a small open economy depends on the trade elasticity of substitution, we set a wide prior over this parameter. We find a posterior mode of the trade elasticity of substitution in line with the estimates of Justiniano and Preston (2010). We then assess the impact of foreign forward guidance across the entire posterior distribution and find that it exerts a contractionay effect on Canada. So while it is possible in theory for foreign forward guidance to be expansionary for the domestic economy, this does not seem to be an empirically relevant case for Canada. We estimate that forward guidance by the US and Canada during our ZLB sample raised the rate of inflation and the level of output of both economies. In a counterfactual exercise, we remove Canadian forward guidance and find that it would have taken 2 years longer for Canadian output to reach its pre-crisis level and that annualized Canadian inflation would have been on average 1.5 percentage points lower.

We estimate that the expected durations of the fixed interest rate policy were longer for the US than for Canada. But decomposing these durations reveals that the fraction of the duration that is due to forward guidance – the duration which is not explained by shocks – is relatively larger in Canada. In a counterfactual exercise, we remove from the estimated durations the component due to forward guidance and find that the Canadian exchange rate depreciates. This suggests forward guidance policy in the US has been relatively more expansionary than that of Canada.

We make the following additional contributions to the literature. For samples that go beyond 2009, we show how solution and estimation methods may be extended and applied efficiently when it comes to understanding monetary policy and the impact of foreign and domestic shocks in small open economies. The key methodological contribution of the paper is to decompose the expected durations of the ZLB into an endogenous component and a policy component. This decomposition therefore addresses the issues highlighted by Fratto and Uhlig (2014) regarding the importance of measuring expectations of future policy at the ZLB. As we mentioned before, to do so one must

[7]The method for handling two constraints can be generalized further to models that lack block exogeneity as well as to models like that of Philippon and Midrigan (2011) that have more than two occasionally binding constraints.

account for an additional occassionally binding constraint. Forward guidance may well become a part of the monetary policy toolkit, in which case these methods are useful. But even if monetary policy were not to run up against its lower bound ever again, these methods are useful because future samples of data will still contain a long spell of low and fixed interest rates.

The rest of the paper is structured as follows. Section 2 outlines the model. Section 3 discusses the solution and estimation strategy used. Section 4 presents the cross-country effects of forward guidance in our estimated model. Section 5 discusses through the estimation results, while section 6 presents the counterfactual results. Section 7 concludes.

2 The model

We conduct our analysis in a New Keynesian small open economy model along the lines of Galí and Monacelli (2005). The model features two economies: a large (foreign) economy and a small (domestic) economy. Economic developments in the large economy affect the small economy, but the reverse is not true. As in De Paoli (2009), the model can be thought of as the limiting case of a two-country model as the relative size of one of the economies goes to zero. We extend this otherwise standard framework in four dimensions: (i) we include imperfect exchange rate pass-through; (ii) we allow for trend inflation; (iii) we incorporate interest rates of longer maturities; and (iv) we include habits in the utility function. The final set of log-linear equations used in estimation is given in the appendix.

2.1 The Large Economy

Variables with a star superscript ($*$) correspond to the large economy. This economy is populated by a large number of households who maximize:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\xi_t^* \left(\log(C_t^* - hC_{t-1}^*) - \frac{{N_t^*}^{1+\varphi}}{1+\varphi} \right) \right],$$

where N_t^* is labor supply and ξ_t^* denotes intertemporal preference shocks that follow an AR(1) process in logs. C_t^* is a composite consumption index given by $C_t^* = \left[\int_0^1 C_t^*(i)^{\frac{\chi-1}{\chi}} di \right]^{\frac{\chi}{\chi-1}}$, where χ is the elasticity of substitution between types of differentiated goods. Households face the budget constraint:

$$P_t^* C_t^* + \mathbb{E}_t \left\{ Q_{t,t+1}^* D_{t+1}^* \right\} \leq D_t^* + W_t^* N_t^* + T_t^*,$$

for all $t > 0$, where P_t^* is the large economy's CPI, W_t^* is the nominal wage rate and T_t^* denotes taxes and transfers. D_{t+1}^* is the nominal payoff in period $t+1$ of the portfolio held at the end of period t and $Q_{t,t+1}^*$ is the stochastic discount factor for one-period ahead nominal payoffs. We assume that households have access to a complete set of contingent claims, traded internationally.

Firms produce differentiated goods with the technology:

$$Y_t^*(i) = Z_t N_t^*(i),$$

where $Y_t^*(i)$ is the production and $N_t^*(i)$ the labor input of firm i. Z_t is total factor productivity, which follows a random walk with drift, μ, in logs. Real marginal costs are common across firms and given by:

$$MC_t^* = \frac{W_t^*}{P_t^* A_t^*}.$$

Firms face Calvo-style pricing frictions. Each quarter, a fraction, $1-\theta^*$, set prices optimally, while the remainder adjust their prices by the steady-state inflation rate, Π^*. The pricing problem for a representative firm i is:

$$\max_{P_t^*(i)} \sum_{k=0}^{\infty} (\beta\theta^*)^k \mathbb{E}_t \left\{ \frac{\Lambda_{t+k}^* P_t^*}{\Lambda_t^*} \left[\frac{P_t^*(i)\,(\Pi^*)^k}{P_{t+k}^*} Y_{t+k}^*(i) - MC_{t+k}^* Y_{t+k}^*(i) \right] \right\},$$

subject to the demand constraint $Y_{t+k}^*(i) = \left[\frac{P_t^*(i)(\Pi^*)^k}{P_{t+k}^*}\right]^{-\chi} Y_{t+k}^*$, where Y_{t+k}^* is aggregate output. In the expression above Λ_t^* is the shadow value of an additional unit of income in period t. Goods market clearing in the large economy requires that all production is consumed, that is $Y_t^* = C_t^*$.

Monetary policy follows an interest rate rate rule that responds to inflation, output growth and the deviation of the level of output from trend, subject to a lower bound:

$$\frac{R_t^*}{R^*} = \max\left\{ \bar{R}^*, \left[\frac{R_{t-1}^*}{R^*}\right]^{\rho_R^*} \left[\left(\frac{\Pi_t^*}{\Pi^*}\right)^{\phi_\pi^*} \left(\frac{Y_t^*}{Y_{t-1}^*\mu}\right)^{\phi_g^*} \left(\frac{Y_t^*}{Z_t}\right)^{\phi_y^*} \right]^{1-\rho_R^*} \exp(\varepsilon_{R,t}^*) \right\}, \qquad (1)$$

where R_t^* is the policy rate in the large economy, $R^* = \mu\Pi^*/\beta$ is the steady-state policy rate and $\varepsilon_{R,t}^*$ is a monetary policy disturbance. Notice that the specification of (1) allows for the lower bound of the nominal interest rate, $\bar{R}^*$, to be different from zero. This is useful to handle fixed interest rates regimes and situations where the effective lower bound on policy may be different from zero.

Longer-term interest rates are determined via the expectations hypothesis. We link model longer-term interest rates to observed longer-term interest rates following Graeve et al. (2009). For any maturity $m > 1$:

$$R_{m,t}^{*,\text{obs}} = R_{m,t}^* \exp\left(c_m^* \eta_t^* \varepsilon_{m,t}^*\right),$$

where $R_{m,t}^*$ is the interest rate on a bond that pays one unit of the large economy's currency in m quarters as determined by the expectations hypothesis, c_m^* is a constant risk premia on the m quarter interest rate, η_t^* is shock, common to all interest rates in the large economy, that follows exogenous autoregressive process and $\varepsilon_{m,t}^*$ is an idiosyncratic shock to the m quarter interest rate in the large economy. Because the expectations hypothesis holds, longer-term nominal interest rates in the model, $R_{m,t}$, are also subject to the lower bound of the short-rate R_t^*.

2.2 The Small Economy

The structure of the small economy is similar to the large economy, except that we must take account of the fact that its households can consume goods and services produced abroad and its firms can sell their output overseas as well as domestically. Unless stated otherwise, variables have the same interpretation as in the large economy.

2.2.1 Households

The small economy is populated by a representative household that maximizes the expected present value of lifetime utility, given by:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\xi_t \left(\log(C_t - hC_{t-1}) - \frac{N_t^{1+\varphi}}{1+\varphi} \right) \right],$$

subject to the budget constraint:

$$P_t C_t + \mathbb{E}_t \left\{ Q_{t,t+1} D_{t+1} \right\} \leq D_t + W_t N_t + T_t.$$

As in the large economy, we assume that households have access to a complete set of contingent claims, traded internationally. The assumption of complete markets and optimization by households implies the risk-sharing condition linking the marginal utility of consumption in the large and small economies, $\Lambda_t = \frac{\Lambda_t^*}{Q_t}$ where Λ_t is the marginal value of an additional unit of domestic income to households in the small economy, $Q_t = S_t P_t^* / P_t$ is the real exchange rate between the small and large economy and S_t is the nominal exchange rate between the small and large economy. We define the exchange rate as the number of units of the small economy's currency required to purchase one unit of the large economy's currency. In estimation, we add a risk premium shock to this expression that follows an AR(1) process in logs to account for deviations from uncovered interest rate parity.

2.2.2 Consumption Retailers

The final consumption good is assembled by perfectly competitive retailers using the technology:

$$C_t = \left[(1-\alpha)^{\frac{1}{\tau}} (C_{H,t})^{\frac{\tau-1}{\tau}} + \alpha^{\frac{1}{\tau}} (C_{F,t})^{\frac{\tau-1}{\tau}} \right]^{\frac{\tau}{\tau-1}},$$

where $C_{H,t}$ and $C_{F,t}$ are composite consumption indices of dometically- and foreign- produced final goods. The parameter τ is the elasticity of substitution between domestic- and foreign-produced goods. The price index corresponding to this bundle is:

$$P_t = \left[(1-\alpha) \left(P_{H,t}\right)^{1-\tau} + \alpha \left(P_{F,t}\right)^{1-\tau} \right]^{\frac{1}{1-\tau}},$$

where $P_{H,t}$ is the price of the domestic composite good and $P_{F,t}$ is the price of the imported composite good, both expressed in domestic currency.

2.2.3 Domestic Final Goods Retailers

The domestically-produced final good, $Y_{H,t}$ is assembled by a perfectly competitive final good retailer that combines domestically-produced intermediate goods using the technology:

$$Y_{H,t} = \left[\int_0^1 Y_{H,t}(i)^{\frac{\chi-1}{\chi}} di \right]^{\frac{\chi}{\chi-1}},$$

where χ is the elasticity of substitution between varieties of domestic intermediate goods. The price of the domestic final good is:

$$P_{H,t} = \left[\int_0^1 P_{H,t}(i)^{1-\chi} di \right]^{\frac{1}{1-\chi}}.$$

It follows that the final goods firm's demand for each variety is given by:

$$Y_{H,t}(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\chi} Y_{H,t}.$$

2.2.4 Domestic Intermediate Goods Producers

Domestic intermediate goods producers manufacture heterogeneous goods using the technology:

$$Y_t(i) = Z_t A_t N_t(i),$$

where A_t is a stationary technology process that follows an AR(1) process in logs. Real marginal costs are equal across firms and given by:

$$MC_t = \frac{W_t}{P_t A_t}.$$

Firms face Calvo-style pricing frictions. Each quarter a fraction, $1-\theta$, of firms are able to adjust their prices freely. The remaining firms index their prices to the steady-state inflation rate, Π. The resulting pricing problem for firm i is

$$\max_{P_t(i)} \sum_{k=0}^{\infty} (\beta\theta)^k \mathbb{E}_t \left\{ \frac{\Lambda_{t+k} P_t}{\Lambda_t} \left[\frac{P_{H,t}(i) Y_{t+k}(i) \Pi^k}{P_{t+k}} - MC_{t+k} Y_{t+k}(i) \right] \right\},$$

subject to the domestic final goods demand condition given above.

2.2.5 Exporters

Exporters purchase the domestic final good at price $P_{H,t}$ and differentiate it through branding for sale in the foreign economy. The exporters are owned by domestic households. However, all export contracts and prices are specified in the currency of the large economy. An export retailer bundles these goods before selling them overseas according to the technology:

$$X_t = \left[\int_0^1 X_t(i)^{\frac{\chi_X - 1}{\chi_X}} (i) \right]^{\frac{\chi_X}{\chi_X - 1}},$$

where χ_X is the elasticity of substitution between different varieties for export. The corresponding price index, in foreign currency terms, is:

$$P^*_{X,t} = \left[\int_0^1 P^*_{X,t}(i)^{1-\chi_X} di \right]^{\frac{1}{1-\chi_X}}.$$

The export retailer faces the demand function:

$$X_t = \left(\frac{P^*_{X,t}}{P^*_t} \right)^{-\tau} Y^*_t,$$

As in Justiniano and Preston (2010) the elasticity of export demand, τ, is the same as the elasticity of substitution between domestic and foreign goods in the consumption basket. This assumes that household's preferences in the small and large economies are the same, which for the US and Canada is a reasonable assumption. It follows that the demand for each exporter's goods are given by:

$$X_t(i) = \left(\frac{P^*_{X,t}(i)}{P^*_{X,t}} \right)^{-\chi_X} X_t.$$

Exporters face Calvo-style pricing frictions, with the parameter θ_x governing the share of firms that are able to adjust their prices each quarter. The resulting pricing problem for firm i is:

$$\max_{P^*_{X,t}(i)} \sum_{k=0}^{\infty} (\beta\theta_x)^k \mathbb{E}_t \left\{ \frac{\Lambda_{t+1} P_t}{\Lambda_t} \left[\frac{P^*_{X,t}(i) \Pi^k X_{t+k}(i)}{S_{t+k} P_{t+k}} - \frac{P_{H,t+k} X_{t+k}(i)}{P_{t+k}} \right] \right\},$$

subject to the demand constraint given above.

Finally, market clearing for the domestic final good requires that all production is sold, either domestically or overseas. That is:

$$Y_{H,t} = C_{H,t} + X_t.$$

2.2.6 Importers

Importers bring in homogeneous products from abroad at price $S_t P_t^*$ and differentiate them through branding. Importers then sell the differentiated goods to a retailer that combines them into the final imported good using the technology:

$$C_{F,t} = \left[\int_0^1 C_{F,t}(i)^{\frac{\chi_F - 1}{\chi_F}} di \right]^{\frac{\chi_F}{\chi_F - 1}},$$

where $C_{F,t}(i)$ is the quantity of the imported good of variety i used in the production of the final imported good and χ_F is the elasticity of substitution between different imported good varieties. The price index corresponding to the imported final good is:

$$P_{F,t} = \left[\int_0^1 P_{F,t}(i)^{1-\chi_F} di \right]^{\frac{1}{1-\chi_F}}.$$

Consequently, each importer faces the demand curve:

$$C_{F,t}(i) = \left(\frac{P_{F,t}(i)}{P_{F,t}} \right)^{-\chi_F} C_{F,t}.$$

Importers face Calvo-style pricing frictions. Each quarter, a fraction, $1 - \theta_m$ set prices optimally, while the remainder adjust their prices by the small economy's steady-state inflation rate, Π. The pricing problem for a representative firm i is

$$\max_{P_{F,t}(i)} \sum_{k=0}^{\infty} (\beta \theta_m)^k \mathbb{E}_t \left\{ \frac{\widetilde{\Lambda}_{t+1} P_t}{\widetilde{\Lambda}_t} \left[\frac{P_{F,t}(i) C_{F,t}(i) \Pi^k}{P_{t+k}} - \frac{S_{t+k} P_{t+k}^* C_{F,t+k}(i)}{P_{t+k}} \right] \right\},$$

subject to the demand constraint above.

2.2.7 Monetary Policy

The domestic central bank follows a Taylor Rule and is subject to a lower bound parameterized by $\bar{R}$:

$$\frac{R_t}{R} = \max \left\{ \bar{R}, \left[\frac{R_{t-1}}{R} \right]^{\rho_R} \left[\left(\frac{\Pi_t}{\Pi} \right)^{\phi_\pi} \left(\frac{Y_t}{Y_{t-1}\mu} \right)^{\phi_g} \left(\frac{Y_t}{Z_t} \right)^{\phi_y} \right] e^{\varepsilon_{R,t}} \right\}. \tag{2}$$

As in (1) we allow for the lower bound of the nominal interest rate, $\bar{R}$, to be different from zero. This is useful in Canada which has fixed its policy rate at 1 % for about 5 years in our sample.

The term structure of interest rates in the small economy is determined in a similar manner to the large economy. For any $m > 1$:

$$R_{m,t}^{\text{obs}} = R_{m,t} \exp\left(c_m \eta_t \varepsilon_{m,t}\right),$$

where $R_{m,t}$ is the interest rate on a bond that pays one unit of domestic currency in m quarters as determined by the expectations hypothesis, c_m is a constant risk premia on the m quarter interest rate, η_t is shock, common to all small economy interest rates, that follows exogenous autoregressive process and $\varepsilon_{m,t}$ is an idiosyncratic shock to the m quarter interest rate in the small economy.

3 Solution and estimation methods

3.1 Solution for a given ZLB duration

We use the solution methods proposed in Cagliarini and Kulish (2013), Kulish and Pagan (forthcoming) and Jones (2016) and the estimation method of Kulish et al. (2014). Because these methods have more general application than the context we are considering, we discuss how the methods apply to our case. The appendix provides additional details.[8]

To estimate the model we take a sample of data of size T. At a given point in the sample, the system can be in one of the following four possible regimes: i) lower bounds are non-binding, ii) only the lower bound of the large economy binds, iii) only the lower bound of the small economy binds and, iv) both lower bounds bind. Figure 1 illustrates one possibility, in which in an initial sub-sample conventional policy applies to both economies, then there is a period of time for which the ZLB binds only in the large economy. After that the ZLB binds in both economies and eventually there is a return to conventional policy which takes place out-of-sample.

[Figure 1 about here.]

We first linearize the model around the steady state for which the ZLBs do not bind and write the resulting system of equations in matrix form as:

$$\mathbf{A}x_t = \mathbf{C} + \mathbf{B}x_{t-1} + \mathbf{D}\mathbb{E}_t x_{t+1} + \mathbf{F}w_t. \tag{3}$$

where x_t is the state vector and w_t is the vector of structural shocks, which we take to be *iid* without loss of generality. If it exists and is unique, the standard rational expectations solution to (3) is $x_t = \mathbf{J} + \mathbf{Q}x_{t-1} + \mathbf{G}w_t$.

When only the *foreign* ZLB binds the structural equations are given by

[8] The approach is more general and can be applied to a range of problems in which one requires to solve a model subject to occasionally binding constraints or anticipated changes in the structural parameters efficiently.

$$\mathbf{A}^* x_t = \mathbf{C}^* + \mathbf{B}^* x_{t-1} + \mathbf{D}^* \mathbb{E}_t x_{t+1} + \mathbf{F}^* w_t. \tag{4}$$

where the only equation that has changed in the starred system relative to (3) is the equation defining the *foreign* policy interest rate rule, which is now specified such that the nominal interest rate is fixed at its bound.[9]

When only the *domestic* ZLB binds the structural equations are given by

$$\bar{\mathbf{A}} x_t = \bar{\mathbf{C}} + \bar{\mathbf{B}} x_{t-1} + \bar{\mathbf{D}} \mathbb{E}_t x_{t+1} + \bar{\mathbf{F}} w_t. \tag{5}$$

where the only equation that has changed relative to (3) is the equation defining the *domestic* policy interest rate rule, which is now specified such that the nominal interest rate is fixed at its bound.

And when both *foreign* and *domestic* lower bounds bind the structural equations are given by

$$\bar{\mathbf{A}}^* x_t = \bar{\mathbf{C}}^* + \bar{\mathbf{B}}^* x_{t-1} + \bar{\mathbf{D}}^* \mathbb{E}_t x_{t+1} + \bar{\mathbf{F}}^* w_t. \tag{6}$$

Because at least one nominal interest rate is fixed in regimes (4), (5) or (6), if any of these regimes were expected to prevail indefinitely in our model, they would be found to be inconsistent with a unique rational expectations solution.[10] However, provided both nominal interest rates are eventually expected to be governed by policy rules both consistent with a unique solution, then there is a unique path under temporarily binding constraints (Cagliarini and Kulish, 2013).

In general, the structural equations in-sample may be written

$$\mathbf{A}_t x_t = \mathbf{C}_t + \mathbf{B}_t x_{t-1} + \mathbf{D}_t \mathbb{E}_t x_{t+1} + \mathbf{F}_t w_t. \tag{7}$$

where $\{\mathbf{A}_t, \mathbf{C}_t, \mathbf{B}_t, \mathbf{D}_t, \mathbf{F}_t\}_{t=1}^{T}$ is the sequence of structural matrices over the sample. For example, if ZLBs do not bind at time t, then $\mathbf{A}_t = \mathbf{A}$, $\mathbf{C}_t = \mathbf{C}$, $\mathbf{B}_t = \mathbf{B}$, and so. If both constraints were to bind then $\mathbf{A}_t = \bar{\mathbf{A}}^*$, $\mathbf{C}_t = \bar{\mathbf{C}}^*$, $\mathbf{B}_t = \bar{\mathbf{B}}^*$, and so on.

Now suppose that at period t' ZLB constraints are not expected to bind from some future period T' onwards. We assume agents anticipate how the structural matrices will evolve until then. That is, agents anticipate that the structural equations will evolve as follows $\{\mathbf{A}_t, \mathbf{C}_t, \mathbf{B}_t, \mathbf{D}_t, \mathbf{F}_t\}_{t=t'}^{T'}$. The approach of Kulish and Pagan (forthcoming) assumes perfect foresight of the evolution of the structure and postulates a solution of the form:

$$x_t = \mathbf{J}_t + \mathbf{Q}_t x_{t-1} + \mathbf{G}_t w_t, \tag{8}$$

For this to be a rational expectations solution, expectations must satisfy (7) in all periods t. Because

[9] Notice that the notation can accomodate additional structural changes which have to be accounted for if the expansion point of the approximation changes. In our application we work around the intended steady state.

[10] This is the case in our model where monetary policy is 'active' in the terminology of Leeper (1991). As emphasised recently by Cochrane (2014), a fixed interest rate need not imply indeterminacy in models with 'active' fiscal policy.

$x_t = \mathbf{J}_t + \mathbf{Q}_t x_{t-1} + \mathbf{G}_t w_t$, expectations are $\mathbb{E}_t x_{t+1} = \mathbf{J}_{t+1} + \mathbf{Q}_{t+1} x_t$. Substituting expectations into (7) implies that at period t:

$$\mathbf{A}_t x_t = \mathbf{C}_t + \mathbf{B}_t x_{t-1} + \mathbf{D}_t \left(\mathbf{J}_{t+1} + \mathbf{Q}_{t+1} x_t \right) + \mathbf{F}_t w_t.$$

Which implies by undetermined coefficients the following equivalences:

$$\begin{aligned}
\mathbf{J}_t &= \left[\mathbf{A}_t - \mathbf{D}_t \mathbf{Q}_{t+1} \right]^{-1} \left(\mathbf{C}_t + \mathbf{D}_t \mathbf{J}_{t+1} \right) \\
\mathbf{Q}_t &= \left[\mathbf{A}_t - \mathbf{D}_t \mathbf{Q}_{t+1} \right]^{-1} \mathbf{B}_t \\
\mathbf{G}_t &= \left[\mathbf{A}_t - \mathbf{D}_t \mathbf{Q}_{t+1} \right]^{-1} \mathbf{F}_t.
\end{aligned}$$

The backward recursion for the sequence of time-varying reduced form matrices, $\{\mathbf{Q}_t\}_{t=t'}^{T'-1}$ starts from the terminal condition $\mathbf{Q}_{T'} = \mathbf{Q}$ and works its way back to period t', yielding the system of time-varying reduced form matrices corresponding to (8). With the sequence for $\mathbf{Q}_t$ in hand, the sequence for $\mathbf{G}_t$ may computed as well as the sequence for $\mathbf{J}_t$ given the terminal condition $\mathbf{J}_{T'} = \mathbf{J}$. Note that the terminal conditions, $\mathbf{J}$ and $\mathbf{Q}$, correspond to the solution matrices to the model around the steady state for which the ZLBs do not bind, consistent with the assumption that agents do not expect constraints to bind from period T' onwards.

3.2 Endogenous ZLB durations

Equation (8) is the solution for the case in which agents anticipate given durations at time period t'. These durations, however, may or may not be those implied by the constraint itself and the structural shocks that have hit the economy up to t'. Indeed, unanticipated shocks at time period $t'+1$ could imply anticipated ZLB durations at $t'+1$ that are different to those at t'. Jones (2016) proposes an efficient algorithm to find at each point in time the duration which is consistent with the constraint itself by using the solution above iteratively. We refer to this duration as the *endogenous* duration.

We compute the endogenous duration at time period t' for the large economy as follows. We know the state $x_{t'-1}$ and the shocks $w_{t'}$. We first compute a forecast under the assumption that the ZLB constraint does not bind. We then examine the forecast path of the nominal rate for violations of the constraint. If the path of the nominal interest rate contains values below the bound, we set the nominal interest rate to its bound but only for the *first* period that the bound is violated. Notice that in that first period the nominal interest rate can be thought to be governed by a different policy rule and so from the perspective of t' it corresponds to a given ZLB duration. This means the solution we discussed above can be applied. That solution implies a new forecast path for the nominal interest rate that must be examined for additional violations. Iterating in this way we arrive at a duration of the ZLB at t' which is consistent with state $x_{t'-1}$, the shocks $w_{t'}$. As shocks unfolds in $t'+1$ one recomputes the duration. The solution satisfies the constraint and captures, at each point in time, agents' forecasts of when the constraint is expected to bind. The appendix describes the algorithm in full. Jones (2016) shows that this solution constitutes a good approximation to the underlying non-linear economy.

The block-exogeneity of the two-country model allows us to find the endogenous durations of the large economy first and then take those as given when computing the durations for the small economy. This means that we must apply the algorithm that we describe below twice.

3.3 The Likelihood function

Since (8) is a linear system, the Kalman filter can be used to construct the likelihood. At the ZLB, implicit in the evolution of the structural matrices are expected sequences of ZLB durations. This means that the likelihood is a function both of the structural parameters and the sequences of expected durations. Following Kulish et al. (2014) we estimate jointly the structural parameters and anticipated durations of the fixed interest rate regimes in the large and small economies.[11] Importantly, as in Kulish et al. (2014), we do not impose in estimation the restriction that the anticipated durations must match the duration implied by shocks themselves. It is important to not impose those constraints in estimation in light of the optimal policy prescription of Eggertsson and Woodford (2003) of extending the duration of the zero interest rate policy beyond the horizon implied by the constraint itself.

3.4 Identification of forward guidance

We identify forward guidance by combining the estimation, which produces estimates of the durations and shocks, with the method in subsection 3.2 to compute endogenous durations. This decomposition characterizes monetary policy at the ZLB and is useful since the level of the nominal interest rate at the ZLB is an insufficient statistic of the stance of monetary policy.

Figure X illustrates the decomposition. Assume that at period t' the sequence of endogenous durations is $3,2,1$ and that at t' monetary policy extends the duration by 2 quarters so that the durations are given $5,4,3,2,1$. We assume no shocks from t' onwards. The estimation recovers $5,4,3,2,1$ and with the shocks the method in subsection 3.2 gives $3,2,1,0,0$.

4 The cross-country spillovers of forward guidance

Foreign forward guidance plays a key role in the response of the small open economy's policy interest rate. This section explores the response of the estimated model to an unanticipated demand shock originating in the US.

To compute impulse responses, we set the parameters of the model to the mode of the posterior distributions, as estimated in the next section. Figure 2 plots the related response of the US economy to US discount factor shocks of size -10 times the estimated standard deviation of the shock both

[11] Details of the Kalman filter and Kalman smoother are relegated to the appendix.

with and without the ZLB. The positive US demand shock raises output and inflation in both countries and appreciates the US exchange rate. The negative US demand shock is sufficiently large to cause the US nominal interest rate to be at its ZLB for six quarters, hitting the ZLB two quarters after the shock. Comparing the positive and negative impulse responses illustrates the asymmetrices induced by the lower bound constraint. When Federal Reserve policy is passively constrained with the nominal interest rate at zero, the real interest rates in the US increases, causing US inflation and output to decline.

[Figure 2 about here.]

The Federal Reserve can use forward guidance policies to alleviate the effect of the ZLB on the US. Figure 3 plots what happens in the model when, in response to the negative US demand shock, the Federal Reserves announces on impact of the shock that it will immediately lower the policy interest rate to zero and keep it there for 12 quarters. This announcement corresponds to an additional six quarters over the endogenous duration resulting from the shock, with the additional forward guidance stimulus occurring both before and after the nominal interest rate hits the ZLB in the endogenous ZLB case. The impulse response under this policy is shown as the blue dashed line. Broadly, the forward guidance policy can be used to mimic the behaviour of the economy where the US ZLB does not bind: the policy stimulates US output and raises US inflation.

[Figure 3 about here.]

Figure 4 illustrates what happens to the Canadian macroeconomic quantities under the US forward guidance policy. Figure 5 plots the response of the exchange rate. The US exchange rate depreciates by more, causing a fall in Canadian inflation. With an estimated trade elasticity around 0.7, Canadian exports do not change much from US forward guidance as the effect of stronger US demand offsets a relatively more depreciated US/CAD exchange rate. As emphasized by the theoretical literature, this result could be reversed if the elasticity between US and Canadian goods was sufficiently high, with the result that US demand for Canadian goods was more sensitive to the Canadian exchange rate appreciation (Haberis and Lipinska, 2012). Our study is the first to show that, in the context of forward guidance policy spillovers from the US to Canada, the export stimulus due to a stimulated foreign demand is not strong enough to overcome Canada-US product substitution due to the exchange rate.

[Figure 4 about here.]

The interaction between the negative US demand shock and the US ZLB has interesting cross-country effects on Canada. When the ZLB does not bind in Canada, the nominal exchange rate depreciation normally seen following a negative foreign demand shock is considerably less pronounced. [It can, for a sufficiently large negative US demand shock, be smaller than for a relatively smaller foreign negative demand shock.] This response is due to the relative stance of monetary policies across the two countries. Canada, when it is not constrained by the ZLB, can relax monetary policy by reducing nominal interest rates. Compared to the response where the US ZLB does not bind, the Canadian real interest rate is lower, so that the US/CAD real exchange rate is higher. Figure 5 plots real interest rates and the real exchange rate to make this point clear, showing that in response to the negative US demand shock, the difference between the US and Canadian real interest rates is more positive when compared to the real interest rate differences following a positive US demand shock. Because of the more muted response of the exchange rate following the negative US demand shock, inflation in imported goods for Canada is lower. This leads to a channel for the US ZLB to change the sign of the inflation response in small open economies, illustrating how the ZLB constraint can change the sign of responses to shocks (see, for other examples, Eggertsson, 2010; Wieland, 2014).

[Figure 5 about here.]

At the baseline estimation results, we see a notable effect on the endogenous level of the Canadian interest rate from US forward guidance, which in our study is a key channel. The US/CAD depreciation pushes down Canadian inflation through a decline in import price inflation. To offset an increase in the real interest rate, the Canadian central bank eases nominal interest rates by more, so that US forward guidance pushes the policy rate of Canada down relative to the endogenous, or no forward guidance, path.

In Figure 6, we show that US forward guidance policy can be large enough to drive the Canadian policy interest rate to its ZLB. In this experiment, we hit the US with a negative ten standard deviation discount factor shock, together with a negative five and a half standard deviation shock to the Canadian discount factor. These two shocks alone are not enough to cause the Canadian nominal interest rate to hit the ZLB. However, when the Federal Reserve uses its forward guidance channel in the same way as it did in the experiment of Figure 3, the Canadian nominal interest rate falls to its ZLB. When the Canadian central bank is constrained, it cannot lower the real interest rate below what it would have been absent US forward guidance, so that Canadian inflation and output fall relative to the path without US forward guidance. As a result, the CAD/USD nominal and real exchange rates appreciate substantially, reinforcing the decline in Canadian inflation, and causing Canadian exports to decline.

[Figure 6 about here.]

The extent to which this CAD/USD appreciation reinforces the negative demand shock can be measured by the gap between the endogenous (black) and forward guidance (dashed blue) responses in Figures 5 and 7. In particular, the difference is much larger in the case where Canada is at its ZLB. This illustrates the channel highlighted by Cook and Devereux (2014)–the exchange rate exacerbates country-specific shocks when nominal interest rates are at their lower bounds, potentially to the extent that it is better for countries to share a single currency and not be exposed to contractionary exchange rate movements.

[Figure 7 about here.]

The appreciation of the CAD/USD real exchange rate from the endogenous relative to the US forward guidance case also measures the incentive by the Bank of Canada to engage in forward guidance policies of its own. Canadian forward guidance would have the effect of lowering the path of the Canadian real interest rate, undoing some of the negative effect of US forward guidance. This illustrates the 'begger thy neighbor' tension inherent in competing forward guidance policies across countries in our estimated model: forward guidance policy conducted by large countries can, through the exchange rate, lead small open economies to conduct forward guidance policies which compete in their effect on exchange rates. Overall, these policies can have little net effect on the exchange rate and imply substantial periods of time at each country's ZLB.

5 Estimation

For estimation, we use US and Canadian nominal interest rates, output growth, and inflation, and changes in the US/Canadian nominal exchange rate. These seven quarterly data series are plotted in Figure 8. We also use yield curve data, using 6-month, 1-year, and 2-year nominal interest rates for both the US and Canada. The series start in 1984Q1 and end in 2014Q4. Prior to estimation, the data are demeaned by their model implied means.

[Figure 8 about here.]

5.1 Calibrated parameters

We calibrate a number of structural parameters in line with values from the literature. The quarterly discount factor β, common to both countries, is set at 0.995. The annual trend growth rate μ is calibrated to 1.4 per cent. Both central banks are assumed to target an annual inflation rate of 2.0 per cent. These three parameters imply a steady-state annual nominal interest rate in both countries of 5.5 per cent. The consumption habit formation parameter h is set at 0.71, the posterior mode of the estimated habit parameter in Smets and Wouters (2007), while the inverse of the Frisch elasticity of labor supply ϕ, is calibrated to 1.5 in both countries, close to the values estimated in Smets and Wouters (2007) and Justiniano and Preston (2010). The export share of Canadian consumption, α is calibrated to 0.29 to match the average share of exports and imports to Canadian GDP.

The calibration of the slopes of the pricing equations, κ^* for the US, and κ, κ_f and κ_x for Canada, are important for the passthrough of foreign shocks to domestic inflation and output. We calibrate κ^* and κ to 0.02, which is lower than the calibration of Ireland (2004) but is in line with estimates of the Philips curves for domestically produced goods in the US (Del Negro et al., 2014). We set κ_z to 1, which implies that roughly 60% of domestic importers can change prices optimally each quarter, in line with the estimates of Justiniano and Preston (2010). Finally, we calibrate κ_f to 0.1, which says that roughly 25% of domestic exporters can change prices optimally each quarter.

5.2 The nominal interest rate bounds

We measure anticipated interest rate durations in the US at 0.125% between 2009Q1 and the end of our sample 2014Q4. Similarly, we measure these durations in Canada at 0.5% between 2009Q2 and 2010Q2, and then at 1% between 2010Q3 and the end of the sample. This modeling choice is motivated by the clear communication of the Bank of Canada to a forward guidance fixed interest rate policy during this period.[12] Strictly speaking, the methodology we use allows the Canadian central bank to lower the nominal interest rate below 1% following the conclusion of any anticipated lower bound duration and for agents to anticipate this to happen.

5.3 Estimation results

5.3.1 Structural parameters

The prior distribution for each estimated parameter are given in Table 1. We set symmetric prior distributions for the parameters that are present in both countries. We set fairly loose priors for the standard deviations of the shock processes for each country, and choose to scale the standard deviation of demand shocks by less. The estimates of the monetary policy rule will be important for accurately estimating anticipated forward guidance durations, as it is the monetary policy regime that the economy reverts to following a period of time at the interest rate lower bound which is important for governing expectations about the path of the interest rate, and therefore in determining the stimulatory effect of forward guidance. Given the Federal Reserve's emphasis on the subdued labor market outcomes over the 2009-2015 period, we allow for a rule that responds to the growth rate of GDP and to the detrended level of output. We put loose priors over the coefficients of these variables.

The posterior estimation results are presented in Table 1. Figures 16 and 17 plot prior and posterior distributions for the structural parameters. Relative to the priors, the data finds persistent demand shocks in the US, and relatively less persistent demand and TFP shocks in Canada, consistent with the estimates on US-Australian data in Kulish and Rees (2011). US demand shocks are estimated to be roughly three times as large as permanent technology shocks, in line with the relative sizes of comparable shocks found in Ireland (2004). The posterior estimate of the trade

[12]For example, in the May 2013 press release, the Bank of Canada said that 'With continued slack in the Canadian economy, the muted outlook for inflation, and the constructive evolution of imbalances in the household sector, the considerable monetary policy stimulus currently in place will likely remain appropriate for a period of time, after which some modest withdrawal will likely be required, consistent with achieving the 2 per cent inflation target.'

elasticity τ is centered around 0.84, which is close to the estimate obtained by Justiniano and Preston (2010).

The theoretical literature emphasizes the importance of the trade elasticity τ in determining the effects of forward guidance. In Figure 9, we compare the effects of stimulatory US forward guidance on the initial response of Canadian variables under parameters randomly chosen from the prior and posterior joint distributions. In this exercise, we draw a set of parameters from the prior and posterior joint distributions and solve the model under those parameters. We then hit the US with a large negative demand shock, which for comparison we scale to be the same across the prior and posterior draws. For cases where the demand shocks causes the US to hit its ZLB, we conduct forward guidance by extending the length of the ZLB duration implied by the constraint itself by two quarters. Figure 9 plots the initial response of Canadian inflation and output across the trade elasticity. The initial responses under the prior show how the prior allows both negative and positive Canadian output and inflation responses following US forward guidance. As domestic goods become more elastic in foreign consumption baskets, the exchange rate effects caused by US forward guidance generate more negative output responses to foreign forward guidance.

[Figure 9 about here.]

It is clear from Figure 9 that the model estimates tightly prefer one interpretation of the effects of foreign forward guidance, across possible draws of the trade elasticity. In particular, for all samples of parameters from the posterior, when the US extends the ZLB duration beyond that implied by the constraint, both Canadian output and inflation decline.

5.3.2 Expected durations

The posterior distributions for the anticipated durations of the US Fed Funds rate at 0.125% between 2009Q1 and 2014Q4, and the Bank of Canada's Bank Rate at 0.25% between 2009Q2 and 2010Q2 and 1% between 2010Q3 and 2014Q4 are in Table 2.[13]

These posteriors show that the anticipated durations are well identified. Interestingly, the data puts little weight on short durations in the US throughout the sample period, with the modal value of each posterior density around 6 to 7. These values are consistent with the results of Swanson and Williams (2014), who find anticipated durations of around 7 each quarter from 2011. The mass of the posterior density shifts up in 2011Q3, with a modal anticipated duration of 9 quarters. This shift aligns with the explicit adoption of forward guidance from 2011Q3, when the Federal Reserve announced that it would maintain the interest rate at zero until mid-2013. Since then, the Federal Reserve continually pushed the explicit liftoff date back, which is consistent with our estimated posterior distributions remaining centered around 6-7 quarters. For Canada, the mass of

[13]Posterior distributions for the expected durations of the US Fed Funds rate at 0.125% are shown in Figure 18. The posteriors for Canadian durations are shown in Figure 19.

the posterior distribution of the durations shrink towards smaller durations, which is consistent with the Bank of Canada raising the interest rate to 1%.

6 ZLB duration decomposition and counterfactual paths

6.1 Decomposing the estimated lower bound durations

In this section, we decompose a lower bound duration into a component due to structural shocks and a component due to forward guidance commitment policy. The decomposition gives the component of a lower bound duration due to structural shocks, with the unexplained residual assigned to an explicit announcement by the central bank that it will maintain interest rates at the lower bound for the duration.

One might assume that the most likely sequence of durations is the median or the mode of the posterior marginal distributions. However that particular sequence might not exist in the posterior. Because of this, we define one particular draws to conduct the decomposition, the *Median-Target* draw, which is chosen as the draw which minimizes the sum of the distances between the draw's lower bound duration and the median of the posterior of lower bound durations. That is, letting $\mathbf{T}$ and $\mathbf{T}^*$ be the vector of lower bound durations for Canada and the US respectively, we choose the draw i that finds $\min\{|\mathbf{T}_i - \text{median}(\mathbf{T})| + |\mathbf{T}_i^* - \text{median}(\mathbf{T}^*)|\}$. That draw is associated with a posterior draw of the parameters–we also select that parameter draw for the decomposition.

Figure 10 illustrates the output of the decomposition algorithm for the median-target draw. The decomposition shows that the durations for the US followed roughly a humped shaped pattern increasing to 2011Q3, which is consistent with policy announcements by the Federal Reserve in that quarter, when the Fed explicitly committed to maintaining the interest rate at 0 to 0.25 basis points unitil mid-2013.[14] The decomposition, however, shows that these explicit announcements were not indicative of any additional policy easing by the Federal Reserve, and was instead primarily due to structural shocks which implied extended periods of time at the ZLB.

[Figure 10 about here.]

The second panel of Figure 10 shows the same decomposition for the Canadian policy interest rate. The draw used shows a different pattern of anticipated durations in Canada over the 2009 to 2015 period when compared to the US. Instead of a hump shape, the Canadian lower bound durations initially start quite low, at around 3 to 4 quarters, before increasing to about 6 quarters around 2013. Since then, the estimated anticipated durations have declined to about 4 quarters in early 2015. The decomposition indicates that, as for the US, a strict adherence to the estimated Canadian Taylor rule would have suggested that the Bank of Canada use positive policy interest rates from 2009Q2 to 2010Q1. This period coincides with the year that the Canadian overnight

[14] FOMC statement, August 9, 2011.

Table 1: Estimated parameters

	Prior			Posterior				Prior			Posterior		
Parameter	Dist	Mean	Std Dev	Mode	Mean	Std Dev	Parameter	Dist	Mean	Std Dev	Mode	Mean	Std Dev
ρ_ξ^*	B	0.600	0.200	0.938	0.934	0.010	$10 \times \sigma_p$	IG	1.000	2.000	0.467	0.461	0.043
ρ_r^*	B	0.600	0.200	0.863	0.860	0.019	τ	G	1.000	1.500	0.836	0.740	0.179
ϕ_π^*	N	2.000	0.300	2.251	2.111	0.243	ρ_η^*	B	0.600	0.200	0.888	0.902	0.038
ϕ_g^*	N	0.200	0.100	0.354	0.278	0.087	$100 \times \sigma_\eta^*$	IG	0.071	0.025	0.084	0.090	0.007
ϕ_y^*	G	0.250	0.130	0.091	0.129	0.051	ρ_η	B	0.600	0.200	0.690	0.661	0.084
$10 \times \sigma_\xi^*$	IG	0.429	1.414	0.277	0.301	0.033	$100 \times \sigma_\eta$	IG	0.071	0.025	0.071	0.069	0.006
$100 \times \sigma_z$	IG	0.429	1.414	1.511	1.556	0.106	$100 \times \sigma_2$	IG	0.071	0.025	0.048	0.044	0.005
$100 \times \sigma_r^*$	IG	0.429	1.414	0.157	0.163	0.015	$100 \times \sigma_4$	IG	0.071	0.025	0.029	0.033	0.005
ρ_ξ	B	0.600	0.200	0.847	0.846	0.015	$100 \times \sigma_8$	IG	0.071	0.025	0.078	0.079	0.008
ρ_a	B	0.600	0.200	0.766	0.763	0.045	$100 \times c_2$	N	0.000	2.000	0.020	0.019	0.023
ρ_p	B	0.600	0.200	0.970	0.969	0.012	$100 \times c_4$	N	0.000	2.000	0.041	0.052	0.024
ρ_r	B	0.600	0.200	0.912	0.908	0.010	$100 \times c_8$	N	0.000	2.000	0.061	0.058	0.028
ϕ_π	N	2.000	0.300	2.533	2.569	0.238	$100 \times \sigma_2^*$	IG	0.143	0.050	0.259	0.265	0.019
ϕ_g	N	0.200	0.100	0.380	0.372	0.082	$100 \times \sigma_4^*$	IG	0.071	0.025	0.035	0.037	0.005
ϕ_y	G	0.250	0.130	0.057	0.058	0.023	$100 \times \sigma_8^*$	IG	0.071	0.025	0.055	0.054	0.006
$10 \times \sigma_\xi$	IG	0.429	1.414	0.495	0.500	0.049	$100 \times c_2^*$	N	0.000	2.000	−0.114	−0.118	0.181
$100 \times \sigma_a$	IG	0.429	1.414	3.300	3.315	0.428	$100 \times c_4^*$	N	0.000	2.000	−0.047	−0.073	0.179
$100 \times \sigma_r$	IG	0.429	1.414	0.196	0.197	0.016	$100 \times c_8^*$	N	0.000	2.000	0.028	−0.001	0.179

Table 2: Forward guidance decomposition

	United States duration (quarters)						Canadian duration (quarters)					
	Posterior draw			Forward guidance			Posterior draw			Forward guidance		
Date	Median	10%	90%	Median	10%	90%	Median	10%	90%	Median	10%	90%
2009Q1	6	6	7	6	5	7						
2009Q2	7	6	9	6	4	7	4	3	5	4	3	5
2009Q3	7	6	9	4	4	6	3	2	5	3	2	5
2009Q4	7	6	9	5	4	6	4	3	5	4	3	5
2010Q1	7	6	9	4	4	4	3	2	5	3	2	4
2010Q2	7	6	9	4	4	4	2	1	3	2	1	3
2010Q3	7	6	9	4	4	4	5	3	6	4	3	5
2010Q4	7	6	9	4	3	4	5	3	7	4	3	5
2011Q1	8	6	10	4	4	4	4	3	6	4	3	4
2011Q2	8	6	10	5	4	6	5	3	7	4	3	5
2011Q3	9	7	11	4	4	5	6	4	7	5	4	5
2011Q4	8	6	9	4	4	5	6	4	8	4	4	5
2012Q1	9	7	11	4	4	5	6	4	8	4	4	5
2012Q2	9	7	11	4	4	5	6	5	8	4	4	5
2012Q3	8	7	10	4	4	4	6	4	8	4	3	4
2012Q4	9	7	11	4	4	4	6	4	8	4	3	5
2013Q1	8	7	10	4	4	4	6	5	8	4	4	5
2013Q2	8	6	10	4	3	4	6	4	8	4	3	4
2013Q3	8	6	10	4	4	4	6	4	8	4	3	4
2013Q4	8	6	10	4	4	4	6	4	7	4	3	4
2014Q1	8	6	10	4	3	4	6	5	8	4	4	5
2014Q2	8	6	10	4	4	5	7	5	8	4	4	5
2014Q3	7	6	9	4	4	4	6	4	8	4	4	5
2014Q4	6	5	8	4	3	4	6	4	8	4	4	4

Note: From 2010Q3, Canadian forward guidance is at 1%

policy rate was at 0.25 per cent. From 2010Q3 to the end of our sample 2014Q4, the policy interest rate was at one per cent, and the decomposition computed relative to that lower bound suggests that, for most periods, the majority of the estimated lower bound durations were due to explicit forward guidance policy and not to structural shocks.

To study whether the decomposition is sensitive to the particular draw chosen in computing Figure 10, we take 1,000 random draws from the posterior distributions of durations and parameters and recompute the decomposition. Figure 20 plots the joint distribution of those 1,000 draws of the duration for the US and Canada against the decomposed forward guidance duration associated with each posterior draw. A more active forward guidance commitment policy is represented by more mass of the joint density being closer to the diagonal: if, for example the posterior duration is 6 quarters, and the decomposed forward guidance duration is also 6 quarters, that point would lie on the diagonal. By contrast, durations which are driven more by structural shocks has a joint density with mass far from the diagonal, so that the decomposed forward guidance duration is closer to zero quarters.

Under this representation, Figure 20 shows that the estimated posterior durations indicate that relative to the draw of the duration, the forward guidance durations are stronger in Canada than the US. In the US, most of the mass of the joint density of the total duration and forward guidance duration lies around 8 quarters for the total duration and 4 quarters for the forward guidance duration, while for Canada, most of the mass of the joint density lies around 4–6 quarters for the total duration and 4–6 quarters for the forward guidance duration.

6.2 Counterfactual paths

In this section, we construct a number of counterfactual paths for the economy to explore alternative policy scenarios and to understand the empirical consequences of forward guidance policies. We compute these counterfactuals using the median-target posterior draw (parameters and lower bound durations) which minimizes the distance between the duration draw and the median duration draw, as discussed in the previous section.[15]

Our first two counterfactual simulations explore the role of forward guidance and the ZLB constraint. Figure 11 plots the path of observable variables under the two scenarios. With the estimated model, we extract the structural shocks and feed those shocks through the model using our ZLB algorithm, without any assumptions on the duration announcements made by either central bank. We find that without forward guidance policy in either the US and Canada, inflation and the level of output would be substantially lower, in line with the theoretical predictions of forward guidance policies. In particular, the shocks would have implied that US inflation would have been negative for all of the 2009 to 2015 period. Canadian inflation would also have fallen compared to the actual rate, but not to the same extent. Part of this inflation response is due to the relatively appreciated CAD/USD, as discussed below.

[Figure 11 about here.]

In the second experiment, we do not impose the lower bound constraints on either the US or Canadian policy interest rate, so that both central banks follow their Taylor rules without restrictions.[16] This counterfactual series highlights the deflationary forces estimated over the 2009 period, with inflation and output both flat. Forward guidance policies were stimulatory even relative to the no lower bound constraint–a result that aligns with Wu and Xia (2014), who find that monetary policy in the US since 2009 was stimulatory relative to the policy behavior of the Federal Reserve prior to 2009. We find that the lower bound of zero would not have been a binding constraint in Canada, although it is constrained relative to the one per cent level that we assume Canada conducted forward guidance at.

[Figure 12 about here.]

[15] We have experimented using other draws of the posterior–our results are not particularly sensitive to this choice.

[16] Note, this series does not reveal a shadow policy interest rate in the way Wu and Xia (2014) define it, as a measure of the stance of policy, but instead is the series that would have arisen given the structural shocks and without the lower bounds.

The counterfactual behavior of the nominal USD/CAD exchange rate indicates the relative strength of US and Canadian forward guidance policy. When forward guidance policies are not imposed in the simulation, the USD appreciates, suggesting that the combined forward guidance policies lowered the US real interest rate relative to the Canadian real interest rate. This can be partly seen from the relative movements in inflation. Under the forward guidance policies, inflation in both the US and Canada increase, but more so in the US. With the nominal interest rate in the US and Canada fixed at their lower bounds of 0.125 per cent, and 1 per cent (after 2010Q2), respectively, and a larger increase in the magnitude of US inflation relative to Canadian inflation when imposing forward guidance, risk sharing between countries implies the real exchange rate appreciates.[17]

This last counterfactual experiment empirically documents how it is important to identify forward guidance durations across countries. To illustrate the spillover effects of US forward guidance on the Canadian economy, in Figure 13 we plot the counterfactual where the US follows its estimated forward guidance policy, while Canada reacts passively in response, and does not response with its own forward guidance policies. Under this scenario, we find that Canada's one per cent lower bound would have endogenously been visited every period from late 2010 on. Both inflation and output growth under this counterfactual would have been notably lower.

[Figure 13 about here.]

This spillover operates partly through the response of the exchange rate. As the US undergoes expansionary forward guidance policy and Canada acts passively in response, the nominal and real USD/CAD exchange rates depreciate. Inflation through imported goods falls, which pushes down inflation in Canada. Because the nominal interest rate is constrained, this decline in inflation causes an increase in the real interest rate in Canada, further reducing current Canadian consumption and output. In addition, the appreciation of the CAD/USD under US forward guidance and no Canadian forward guidance causes exports to fall, with the increased US demand for exports due to higher US output not sufficiently large to offset the US substitution out of Canadian goods owing to the exchange rate appreciation. Interestingly, exports also fall when forward guidance in both countries is removed, indicating that the effect of the CAD/USD exchange rate depreciation on Canadian exports when both countries avoid forward guidance is not large enough to offset the fall in US demand for exports from a stronger US economy. These last two results highlight the complicated dynamics forward guidance policies at the ZLB.

[Figure 14 about here.]

6.3 A fixed exchange rate policy

[To be completed] The goal of this paper is to study the policy responses of small open economies. In this section we ask what happens if, instead of a nominal interest rate policy, Canada implements

[17] The log-linearized risk-sharing condition implies $q_t = \mathbb{E}_t q_{t+1} - \left[(r_t - \mathbb{E}_t \pi_{t+1}) - (r_t^* - \mathbb{E}_t \pi_{t+1}^*)\right]$.

a fixed exchange rate policy ($\Delta e = 0$), fixed at the level of the exchange rate that prevailed when the US was first constrained by the ZLB in 2009Q1. To implement this policy, we simply replace the Canadian Taylor rule with $\Delta e = 0$, remove the risk premia shock and allow the nominal interest rate to be determined endogenously. During the ZLB period, however, the nominal interest rate cannot be lowered to depreciate the currency. In the case that the ZLB binds, as was illustrated in the impulse response analysis, monetary policy can instead use calendar-based forward guidance to lower real interest rates to achieve the depreciation.

In Figure 15, we compare two counterfactual paths. In both counterfactuals, Canadian monetary policy aims to keep the nominal exchange rate constant. In the first experiment, the Canadian nominal interest rate is unconstrained by the ZLB and it implements a fixed exchange rate policy at the time period the US hits the ZLB in 2009Q1. In the second experiment, we find a sequence of ZLB durations which hold the nominal interest rate roughly constant at around 2009 levels while the US is at the ZLB. In both cases, we keep US monetary policy constant at their estimated forward guidance announcements.

[Figure 15 about here.]

To keep the exchange rate constant over the US ZLB period, Canadian monetary policy is required to be to extremely loose relative to US policy, corresponding to high counterfactual inflation and a rapid recovery in real output. In the unconstrained case, the policy interest rate becomes substantially negative, and in the ZLB constrained case, Canadian monetary policy announces long durations at the ZLB. From 2013 onwards, there are depreciating forces on the Canadian exchange rate. To keep the exchange rate constant, Canadian monetary policy tightens. In the unconstrained counterfactual, the nominal interest rate becomes less negative, while when monetary policy is conducted through calendar-based forward guidance, the length of the announcement duration slowly falls.

7 Conclusion

In this paper, we examine empirically how forward guidance of the United States Federal Funds rate at its lower bound endogenously changes the monetary policy of Canada, a small open economy. To this end, we set up and estimate a modified Galí and Monacelli (2005) model on US and Canadian data which respects the ZLB in both the US and Canada and which jointly estimates anticipated lower bound durations from 2009Q1 onwards. In estimating these durations we allow, but do not require, the two central banks to use forward guidance policy, or announcements by the central bank that it will maintain the policy interest rate at its lower bound for longer than otherwise implied by a interest rate rule.

We find anticipated lower bound durations of around 6 to 8 quarters, of which roughly half in the US and most of the Canadian durations cannot be explained by structural shocks. This suggests that both countries made use of the forward guidance channel to stimulate output and inflation. In counterfactual simulations, we find that this announcement policy was stronger in its stimulatory effect than the policy that would have arisen in the absence of the lower bound constraint. In particular, if neither country used the expectations channel of policy, output and inflation in both countries would have been substantially lower. As a result, we identify notable spillovers of US forward guidance policy on Canada. We find that forward guidance policies across countries go hand-in-hand, such that without Canadian announcement policy, US forward guidance policy operates, through the exchange rate, to decrease Canadian inflation and output.

In our estimation, the lower bound durations are unconstrained with each duration free to take any value every period the interest rate is observed to be at its lower bound. In future research, it would be useful to describe more concretely a policy rule that governs these anticipated durations, specified in a way that links the duration to the prevailing state of the economy like the rate of inflation or the exchange rate.

A The log-linear equations

This appendix describes the model's log-linear equations. For a variable X_t with steady state $\bar{X}_t$ we write $x_t = \ln X_t - \bar{X}_t$.

A.1 Large country

Log-linearized, the large country's problem can be expressed as the following four equations. From the large country's household's optimal choice of consumption:

$$(\mu - \beta h)(\mu - h)\lambda_t^* = \mu h y_{t-1}^* + \mu \beta h \mathbb{E}_t\{y_{t+1}^*\} - \left(\mu^2 + \beta h^2\right) y_t^* - \mu h z_t + (\mu - h)\left(\mu - \beta h \rho_\xi^*\right)\xi_t^*.$$

The household's intertemporal condition:

$$0 = \lambda_t^* - \mathbb{E}_t\left\{\lambda_{t+1}^*\right\} - \left(r_t^* - \mathbb{E}_t\left\{\pi_{t+1}^*\right\}\right).$$

Firms' pricing condition:

$$\pi_t^* = \beta \mathbb{E}_t \pi_{t+1}^* + \kappa^* \left[\xi_t^* + \varphi y_t^* - \lambda_t^*\right].$$

And a monetary policy rule:

$$r_t^* = \max\left\{0, \rho_R^* r_{t-1}^* + (1 - \rho_R^*)\left(\phi_\pi^* \pi_t^* + \phi_g^*(y_t^* - y_{t-1}^* + z_t)\right) + \varepsilon_{R,t}^*\right\}.$$

When habits are turned off ($h = 0$) these equations reduce to the typical three-equation New Keynesian model.

A.2 Small country

The small country's system of equations comprises of the small country's household's optimal choice of consumption:

$$(\mu - \beta h)(\mu - h)\lambda_t = \mu h c_{t-1} + \mu \beta h \mathbb{E}_t\{c_{t+1}\} - (\mu^2 + \beta h^2)c_t - \mu h z_t + (\mu - h)(\mu - \beta h \rho_\xi)\xi_t.$$

An intertemporal condition:

$$0 = \lambda_t - \mathbb{E}_t\left\{\lambda_{t+1}\right\} - (r_t - \mathbb{E}_t\{\pi_{t+1}\}),$$

a risk-sharing condition:

$$-\lambda_t = -\lambda_t^* + q_t + rp_t,$$

three pricing equations for the prices of exports, imports and domestic goods:

$$\pi_{F,t} = \kappa_f(q_t - \gamma_{F,t}) + \beta \mathbb{E}_t\{\pi_{F,t+1}\}$$
$$\pi_{X,t} = \kappa_z(\gamma_{H,t} - \gamma_{X,t}) + \beta \mathbb{E}_t\{\pi_{X,t+1}\}$$
$$\pi_{H,t} = \kappa(\xi_t - \lambda_t + \varphi y_t - (1+\varphi)a_t - \gamma_{H,t}) + \beta \mathbb{E}_t[\pi_{H,t+1}].$$

Relative prices for exports, imports and domestic goods:

$$\gamma_{F,t} = \gamma_{F,t-1} + \pi_{F,t} - \pi_t$$
$$\gamma_{X,t} = \gamma_{X,t-1} + \pi_{X,t} - \pi_t + \Delta e_t$$
$$\gamma_{H,t} = \gamma_{H,t-1} + \pi_{H,t} - \pi_t.$$

Market clearing conditions:

$$\pi_t = (1-\alpha)\pi_{H,t} + \alpha \pi_{F,t}$$
$$y_{H,t} = \frac{C_H}{Y_H}(c_t - \tau \gamma_{H,t}) + \frac{X}{Y_H}(y_t^* + \tau(q_t - \gamma_{X,t}))$$

The definition of the change in the real exchange rate:

$$q_t = q_{t-1} + \Delta e_t + \pi_t^* - \pi_t.$$

And the small country's monetary policy rule:

$$r_t = \max\left\{0, \rho_r r_{t-1} + (1-\rho_r)\left[\phi_\pi \pi_t + \phi_g(y_t - y_{t-1} + z_t)\right] + \varepsilon_{r,t}\right\}.$$

If $\alpha = 0$, then $\frac{X}{Y_H} = 0$ and $\gamma_{H,t} = 0$, so that the small country is closed and separate from the large country.

A.3 Long-Rates

To include a long-rate of maturity m, we need to include all previous $m-1$ maturity interest rates and relate them to each other by the expectations hypothesis:

$$m r_{m,t} = r_{1,t} + (m-1)\mathbb{E}_t r_{m-1,t+1}. \tag{9}$$

This is computationally more efficient than including m leads of $r_{1,t}$. We relate the observed rates $r_{j,t}^{\text{obs}}$ to the model implied rates as per Kulish et al. (2014),

$$r_{j,t}^{\text{obs}} - r = r_{j,t} + c_j + \eta_t + \varepsilon_{j,t}, \tag{10}$$

where c_j is a constant and estimated component of the term premia, $\varepsilon_{j,t}$ is a idiosyncratic and time-varying component of the term premia and η_t is an additional and persistent component of the time-varying term premia which is common to all maturities:

$$\eta_t = \rho_\eta \eta_{t-1} + \varepsilon_{\eta,t}. \tag{11}$$

There is one set of equations (9) to (11) for each of the foreign and domestic countries.

A.4 Observation Equations

US output growth:

$$\Delta y_t^{*,obs} = y_t^* - y_{t-1}^* + z_t.$$

US inflation:

$$\pi_t^{*,obs} = \pi_t^* + \pi^*.$$

US policy rate:

$$r_t^{*,obs} = r_t^* + r^*.$$

Canada output growth:

$$\Delta y_t^{obs} = y_t - y_{t-1} + z_t.$$

Canada inflation rate:

$$\pi_t^{obs} = \pi_t + \pi.$$

Canada policy rate:

$$r_t^{obs} = r_t + r.$$

Canada exchange rate:

$$\Delta e_t^{obs} = \Delta e_t.$$

B Implementation

There are two occasionally binding lower bound constraints to impose in this model, one to the large country's nominal interest rate, and one to the small country's nominal interest rate. A flexible algorithm is developed which can handle cases where shocks in the large country endogenously push the large country's interest rate to its ZLB (perhaps many time periods after a shock hits) and which subsequently causes the small country's interest rate to endogenously fall to the ZLB (perhaps many time periods periods after the large country has hit its ZLB). The algorithm relies on constructing a perfect foresight path of the nominal interest rate in both countries, and piecing together linear systems in a step-by-step way. These methods are based on the solution concepts developed in Cagliarini and Kulish (2013); Kulish and Pagan (forthcoming). As shown by Guerrieri and

Iacoviello (2014) and Jones (2016), the approximation does a good job at capturing the non-linear effects induced by the occasionally binding constraints.

B.1 Notation

Denote by x_t^* the vector of endogenous variables for the large country at time period t, one of which is the nominal interest rate in the large country, and x_t the vector of endogenous variables for the small country at time period t, one of which is the nominal interest rate R_t. The initial conditions are $[x_{t-1}^{*}{}' \; x_{t-1}{}']'$ and the initial vector of unanticipated exogenous variables, denoted by ε_t. The model is a system of n equations.

B.2 Initialization at time period t

We know, at period t:

- The shock that hits at period t: ε_t.
- The initial vector of variables x_{t-1}.

B.3 The algorithm

The steps of the algorithm are:

0. Linearize the model around the non-stochastic steady state, ignoring the ZLBs in both countries.

1. For each period t:

   For the large country:

   (a) Solve for the path $\{x_\tau\}_{\tau=t}^{T}$ with T large, using the solution of the linearized economy from step (0), given ε_t and the initial vector of variables x_{t-1}, and assuming no future uncertainty. This gives a path for the nominal interest rate, $\mathbf{i}_t^k = \{i_\tau^k\}_{\tau=t}^{T}$.

   (b) Examine the path $\mathbf{i}_t^k$. If $\mathbf{i}_t^k \geq 0$, then the ZLB does not bind, so move onto step (2). If $\mathbf{i}_t^k < 0$, then move onto step (1c).

   (c) For the *first* time period where $\mathbf{i}_t^k < 0$, set the nominal interest rate in that period to zero. This changes the anticipated structure of the economy. Under this new structure, calculate the path of all variables, including the new path for the nominal interest rate $\mathbf{i}_t^{k+1} = \{i_\tau^{k+1}\}_{\tau=t}^{T}$.

   Iterate on steps 1a and 1c until convergence of $\mathbf{i}_t^*$.

   Repeat steps 1a to 1c for the small country.

2. Increment t by one. The initial vector of variables now becomes x_t, which was solved for in step 1. Draw a new vector of unanticipated shocks ε_{t+1} and return to step 1.

To compute the path $\{x_\tau\}_{\tau=t}^{T}$ under forward guidance, compute step (1c) first, imposing the sequence of structural matrices corresponding to the ZLB and non-ZLB periods. Then examine the path $\{i_\tau\}_{\tau=t}^{T}$ for subsequent violations of the ZLB.

B.4 Details of each step

At the following steps:

0. Write the n equations of the linearized model at t as:

$$\mathbf{A}x_t = \mathbf{C} + \mathbf{B}x_{t-1} + \mathbf{D}\mathbb{E}_t x_{t+1} + \mathbf{E}\varepsilon_t, \tag{SM}$$

where x_t is a $n \times 1$ vector of state and jump variables and w_t is a $l \times 1$ vector of exogenous variables. Use standard methods to obtain the reduced form:

$$x_t = \mathbf{J} + \mathbf{Q}x_{t-1} + \mathbf{G}\varepsilon_t \tag{RF}$$

1. For each period t:

   (a) Using (RF), obtain the path $\{x_\tau\}_{\tau=t}^{T}$ given ε_t. Set T to be large. Assume $\{\varepsilon_\tau\}_{\tau=t+1}^{T} = 0$ (no future uncertainty). So:

$$\begin{aligned} x_t &= \mathbf{J} + \mathbf{Q}x_{t-1} + \mathbf{G}\varepsilon_t \\ x_{t+1} &= \mathbf{J} + \mathbf{Q}x_t \\ &\vdots \\ x_T &= \mathbf{J} + \mathbf{Q}x_{T-1} \end{aligned}$$

   This step gives us a path $\mathbf{i}_t = \{i_\tau\}_{\tau=t}^{T}$

   (b) Examine the path $\{i_\tau\}_{\tau=t}^{T}$

   - if $i_\tau \geq 0$ for all $t \leq \tau < T$, accept $\{x_\tau\}_{\tau=t}^{T}$
     - i_t path does not violate ZLB today or in future
   - if $i_\tau < 0$ for any $t \leq \tau < T$, move to step (1c)

   (c) Update the path of $\{i_\tau\}_{\tau=t}^{T}$ for the ZLB. For the *first* time period t^* where $i_{t^*} < 0$, set $i_{t^*} = 0$. The model system at t^* therefore becomes:

$$\mathbf{A}^* x_{t^*} = \mathbf{C}^* + \mathbf{B}^* x_{t^*-1} + \mathbf{D}^* \mathbb{E}_{t^*} x_{t^*+1} + \mathbf{E}^* \varepsilon_{t^*},$$

Compute the new path $\{i_\tau\}_{\tau=t}^{T}$. This involves computing $\{x_\tau\}_{\tau=t}^{t^*}$ and $\{x_\tau\}_{\tau=t^*+1}^{T}$. At t^*, $\mathbb{E}_{t^*}x_{t^*+1}$ is computed using the the reduced form solution (RF) and $\varepsilon_{t^*+1}=0$. This expresses x_{t^*} as a function of x_{t^*-1}. Proceeding in this way with the correct structural matrices (either ZLB $(*)$ or no ZLB at each time period), we can compute the path $\{i_\tau\}_{\tau=t}^{T}$. See Jones (2016) for more details.

B.5 Output of the algorithm

The algorithm yields a set of time-varying structural matrices:

$$x_t = \mathbf{J}_t + \mathbf{Q}_t x_{t-1} + \mathbf{G}_t w_t,$$

from which we get the path of $\{x_\tau\}_{\tau=t}^{\infty}$ where the nominal interest rate is subject to the ZLB. Both the current value of the nominal interest rate, and expectations of the ZLB binding, affect current values of state variables.

B.6 Identifying forward guidance

Here, we explain how to use the algorithm in Appendix B.3 to decompose an anticipated duration of the ZLB into a component due to structural shocks, and a component due to Odyssean or commitment forward guidance. Assume that at period t, the ZLB binds and we have used procedures to estimate the model parameters and the anticipated length of the ZLB at period t. We have in hand at period t:

1. An estimated duration $\widetilde{T}$ of the ZLB at t, so that the interest rate is expected to stay at zero until time period $t+\widetilde{T}$.

2. An estimate of the history of the states $\{x_\tau\}_{\tau=0}^{t-1}$ and an estimate of the structural shocks $\{w_\tau\}_{\tau=1}^{t}$, computed using the Kalman smoother.

The estimated parameters, durations and shocks recover the observed series and give an estimate of the model's state variables x_t. To decompose the proportion of the estimated duration due to structural shocks, so that the remainder is due to forward guidance policies, at each point of time:

1. Use the state x_{t-1} and the structural shock w_t to compute, using the ZLB algorithm of Appendix B.3, the endogenous duration of the ZLB.

2. If the computed endogenous duration is less than the estimated duration, then the additional time is assigned to commitment forward guidance policy.

The endogenous duration is the duration that would have occured had the central bank simply set the nominal interest rate to zero in periods where the policy rule would have specified that it be negative, and set the interest rate to its positive value when the policy rule specifies that it be positive.

B.7 Kalman filter

The model in state space representation is:

$$x_t = \mathbf{J}_t + \mathbf{Q}_t x_{t-1} + \mathbf{G}_t w_t \qquad \text{(State Eqn)}$$

$$z_t = \mathbf{H}_t x_t. \qquad \text{(Obs Eqn)}$$

The structural shocks are Gaussian, so that $w_t \sim N(0, \mathbf{Q})$, where $\mathbf{Q}$ is the covariance matrix of w_t. There is no observation error by assumption. The Kalman filter recursion is given by the following equations. The state of the system is the state vector and its covariance matrix $(\hat{x}_t, P_{t-1})$. The predict step involves using the structural matrices $\mathbf{J}_t$, $\mathbf{Q}_t$ and $\mathbf{G}_t$:

$$\hat{x}_{t|t-1} = \mathbf{J}_t + \mathbf{Q}_t \hat{x}_t$$

$$\mathbf{P}_{t|t-1} = \mathbf{Q}_t \mathbf{P}_{t-1} \mathbf{Q}_{t|t-1}^\top + \mathbf{G}_t \mathbf{Q} \mathbf{G}_t^\top.$$

This formulation differs from the time-invariant Kalman filter step because in the forecast stage the structural matrices $\mathbf{J}_t$, $\mathbf{Q}_t$ and $\mathbf{G}_t$ can vary over time. We update these forecasts with imperfect observations of the state vector. Also note that $\mathbf{H}_t$ is time-varying, reflecting that when the nominal interest rate is at its lower bound, we lose it as an observable variable. The update step involves computing forecast errors $\widetilde{y}_t$ and its associated covariance matrix $\mathbf{S}_t$:

$$\widetilde{y}_t = z_t - \mathbf{H}_t \hat{x}_{t|t-1}$$

$$\mathbf{S}_t = \mathbf{H}_t \mathbf{P}_{t|t-1} \mathbf{H}_t^\top.$$

The Kalman gain matrix is given by:

$$\mathbf{K}_t = \mathbf{P}_{t|t-1} \mathbf{H}_t^\top \mathbf{S}_t^{-1}.$$

With $\widetilde{y}_t$, $\mathbf{S}_t$ and $\mathbf{K}_t$ in hand, the optimal update of the state x_t and its associated covariance matrix is:

$$\hat{x}_t = \hat{x}_{t|t-1} + \mathbf{K}_t \widetilde{y}_t$$

$$\mathbf{P}_t = (I - \mathbf{K}_t \mathbf{H}_t) \mathbf{P}_{t|t-1}.$$

The Kalman filter is initialized with x_0 and $\mathbf{P}_0$ computed from their unconditional moments. The recursion is computed until the final time period T of data.

B.8 Kalman smoother

With the estimates of the parameters and durations in hand at time period T, the Kalman smoother gives an estimate of $x_{t|T}$, or an estimate of the state vector at each point in time given all available information. With $\hat{x}_{t|t-1}$, $\mathbf{P}_{t|t-1}$, $\mathbf{K}_t$ and $\mathbf{S}_t$ in hand from the filter, the vector $x_{t|T}$ is computed by:

$$x_{t|T} = \hat{x}_{t|t-1} + \mathbf{P}_{t|t-1} r_{t|T},$$

where the vector $r_{T+1|T} = 0$ and is updated with the recursion:

$$r_{t|T} = \mathbf{H}_t^\top \mathbf{S}_t^{-1} \left(z_t - \mathbf{H}_t \hat{x}_{t|t-1} \right) + \left(I - \mathbf{K}_t \mathbf{H}_t \right)^\top \mathbf{P}_{t|t-1}^\top r_{t+1|T}.$$

Finally, to get an estimate of the shocks to each state variable, denoted by e_t, we compute:

$$e_t = \mathbf{G}_t w_t = \mathbf{G}_t r_{t|T}.$$

B.9 Sampler

This section describes the sampler used to obtain the posterior distribution of interest. Denote by ϑ the vector of parameters to be estimated and $\mathbf{T}$ the vector of durations to be estimated. Contained in $\mathbf{T}$ are a set of durations for both the foreign and domestic countries. Denote by $\mathscr{Z} = \{z_\tau\}_{\tau=1}^T$ the sequence of observable vectors. The posterior $\mathscr{P}(\vartheta, \mathbf{T} \mid \mathscr{Z})$ satisfies:

$$\mathscr{P}(\vartheta, \mathbf{T} \mid \mathscr{Z}) \propto \mathscr{L}(\mathscr{Z} \mid \vartheta, \mathbf{T}) \times \mathscr{P}(\vartheta, \mathbf{T}).$$

With Gaussian errors, the likelihood function $\mathscr{L}(\mathscr{Z} \mid \vartheta, \mathbf{T})$ is computed using the appropriate sequence of structural matrices and the Kalman filter:

$$\log \mathscr{L}(\mathscr{Z} \mid \vartheta, \mathbf{T}) = -\left(\frac{N_z T}{2} \right) \log 2\pi - \frac{1}{2} \sum_{t=1}^{T} \log \det \mathbf{H}_t \mathbf{S}_t \mathbf{H}_t^\top - \frac{1}{2} \sum_{t=1}^{T} \widetilde{y}_t^\top \left(\mathbf{H}_t \mathbf{S}_t \mathbf{H}_t^\top \right)^{-1} \widetilde{y}_t.$$

The prior is simply computed using priors over ϑ which are consistent with the literature, and with flat priors over $\mathbf{T}$.[18]

The Markov Chain Monte Carlo posterior sampler has two blocks, corresponding to ϑ and $\mathbf{T}$. Initialize the sampler at step j with the last accepted draw of the structural parameters, the period of the breaking parameters and durations, denoted by ϑ_{j-1} and $\mathbf{T}_{j-1}$ respectively. The blocks are, in order of computation:

1. In the first block, randomly choose up to $\bar{T}$ durations to test in each country, corresponding to up to $\bar{T}$ time periods that each economy is at the ZLB. For each of those time periods, randomly choose a duration in the interval $[1, T^*]$ for each country to generate a new $\mathbf{T}_j$ proposal. Recompute the sequence of structural matrices associated with $(\vartheta_{j-1}, \mathbf{T}_j)$, compute the posterior $\mathscr{P}(\vartheta_{j-1}, \mathbf{T}_{j-1} \mid \mathscr{Z})$, and accept the proposal $(\vartheta_{j-1}, \mathbf{T}_j)$ with probability

[18] For practical convenience, we require that each estimated duration lies below some maximum value T^* which, in practice, is rarely visited by the sampler.

$\frac{\mathscr{P}(\vartheta_{j-1},\mathbf{T}_j|\mathscr{Z})}{\mathscr{P}(\vartheta_{j-1},\mathbf{T}_{j-1}|\mathscr{Z})}$. If $(\vartheta_{j-1},\mathbf{T}_j)$ is accepted, then set $\mathbf{T}_{j-1}=\mathbf{T}_j$.

2. The second block is a more standard Metropolis-Hastings random walk step. Start by selecting which structural parameters to propose a new value for. For those parameters, draw a new proposal ϑ_j from a proposal density centered at ϑ_{j-1} and with thick tails to ensure sufficient coverage of the parameter space and an acceptance rate of roughly 20%. The proposal ϑ_j is accepted with probability $\frac{\mathscr{P}(\vartheta_j,\mathbf{T}_{j-1}|\mathscr{Z})}{\mathscr{P}(\vartheta_{j-1},\mathbf{T}_{j-1}|\mathscr{Z})}$. If $(\vartheta_j,\mathbf{T}_{j-1})$ is accepted, then set $\vartheta_{j-1}=\vartheta_j$.

C Additional figures

[Figure 16 about here.]

[Figure 17 about here.]

[Figure 18 about here.]

[Figure 19 about here.]

[Figure 20 about here.]

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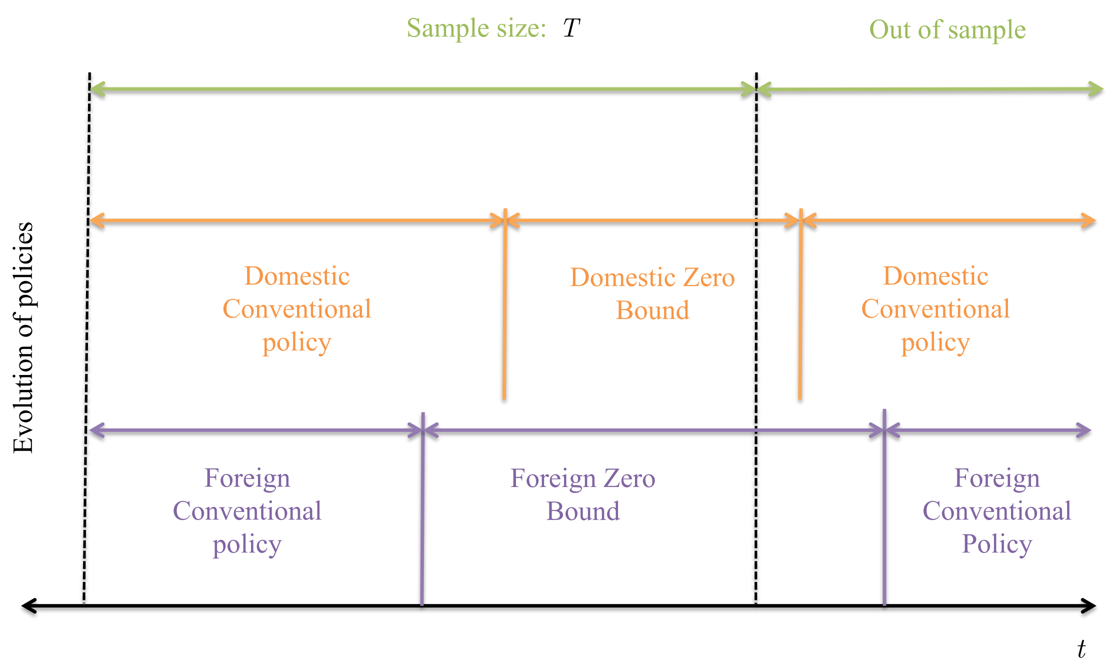

Figure 1: Timing

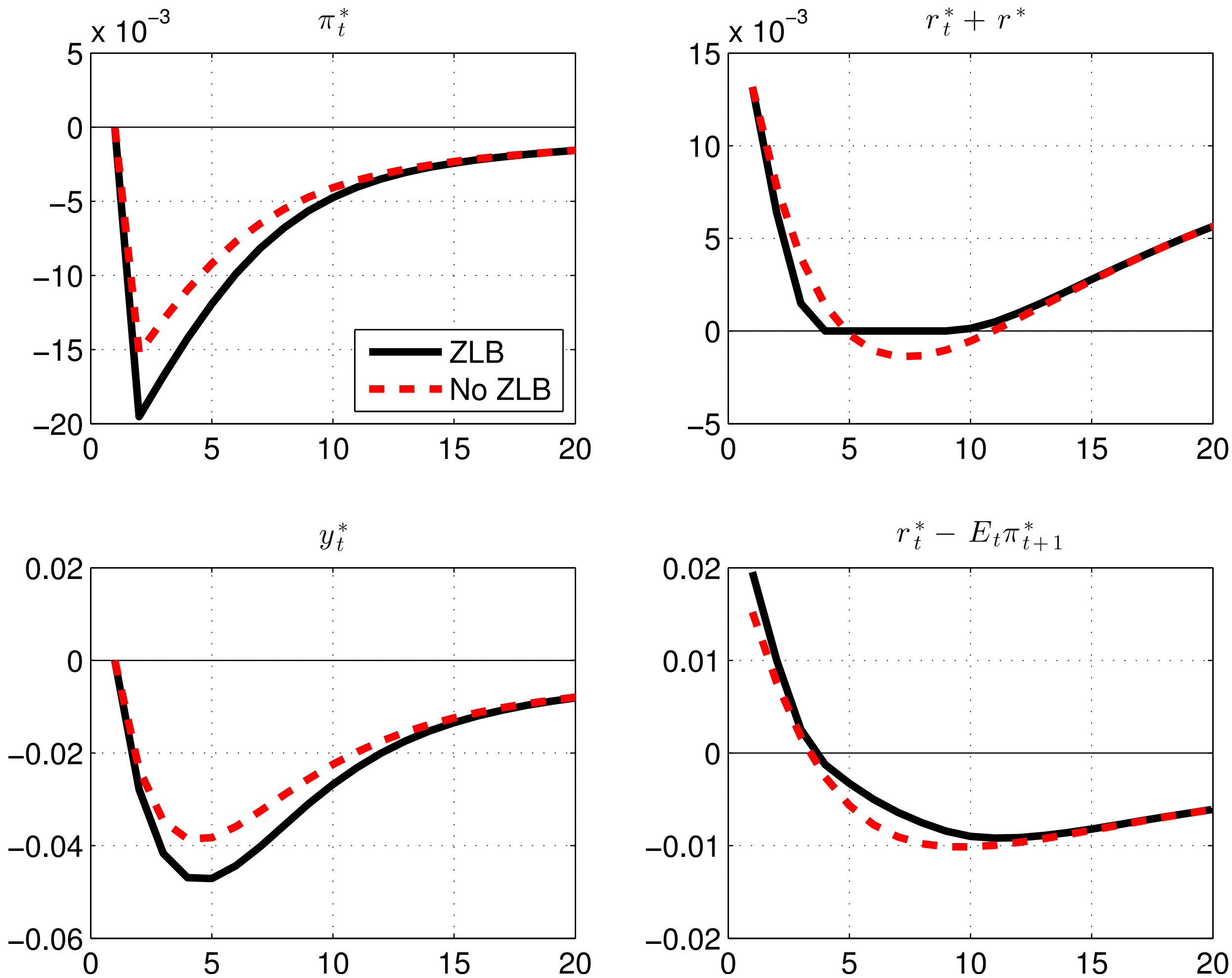

Figure 2: Asymmetry from ZLB. Negative US demand shock.

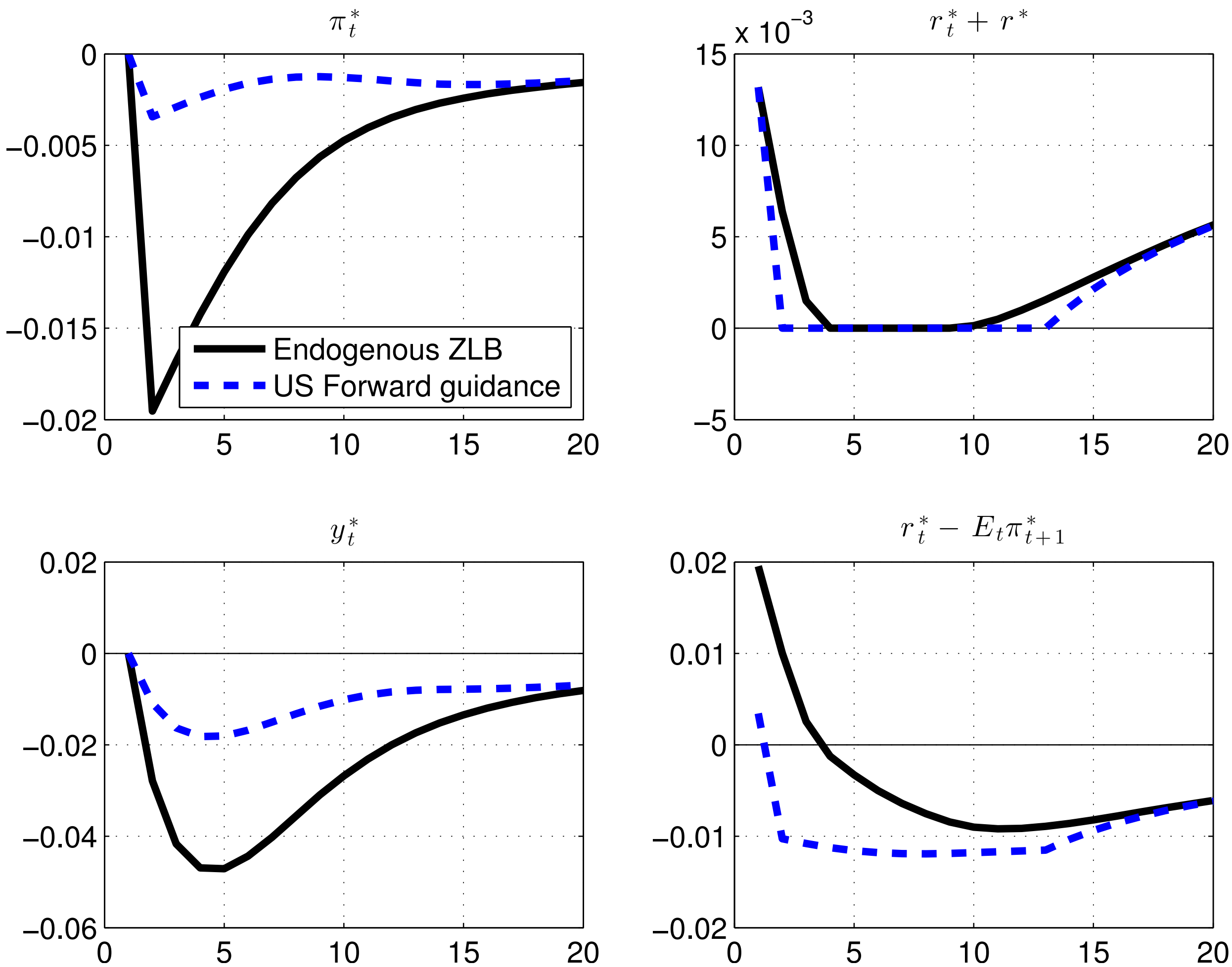

Figure 3: Impulse response, US demand shock. US variables.

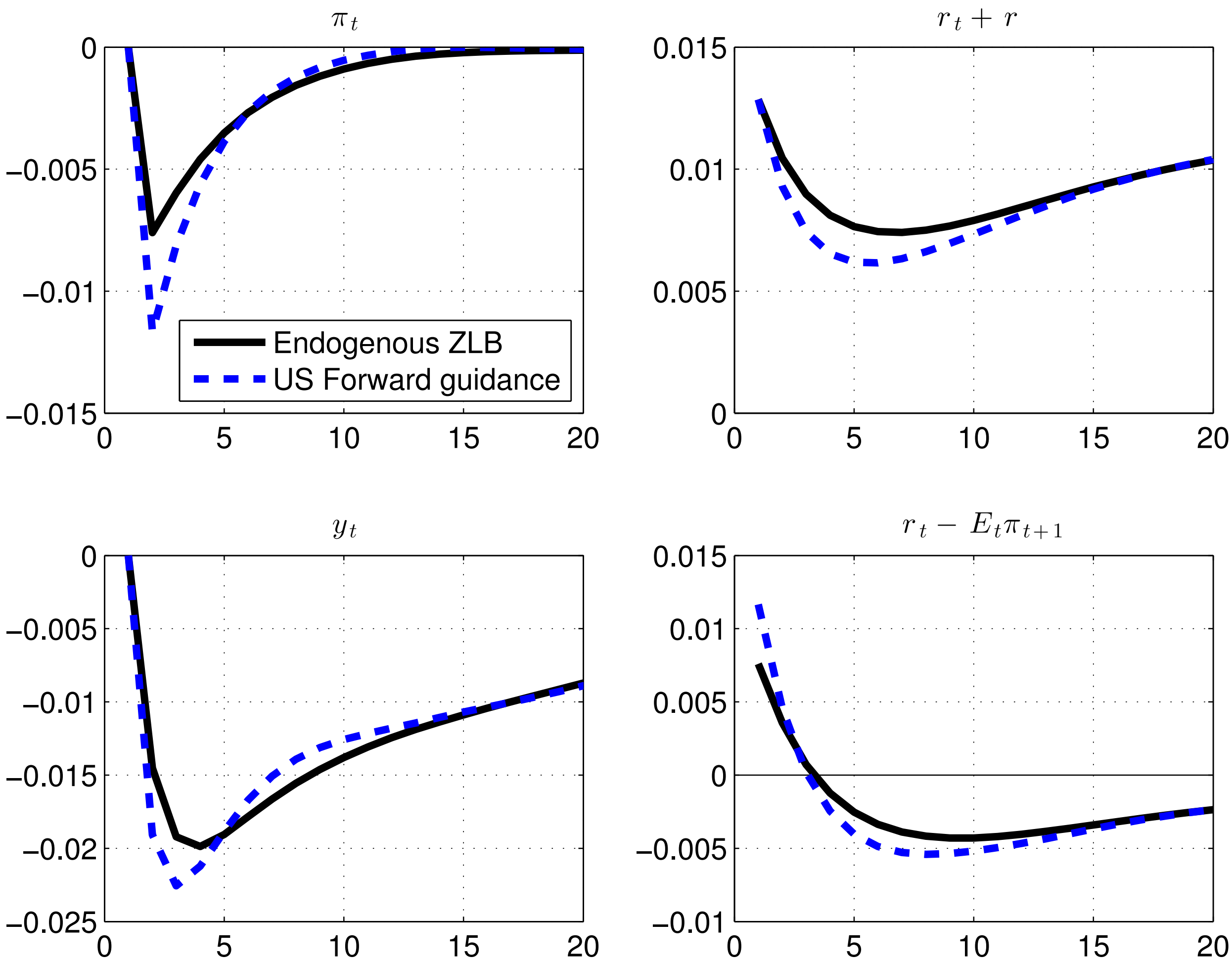

Figure 4: Impulse response, US demand shock. Canadian variables.

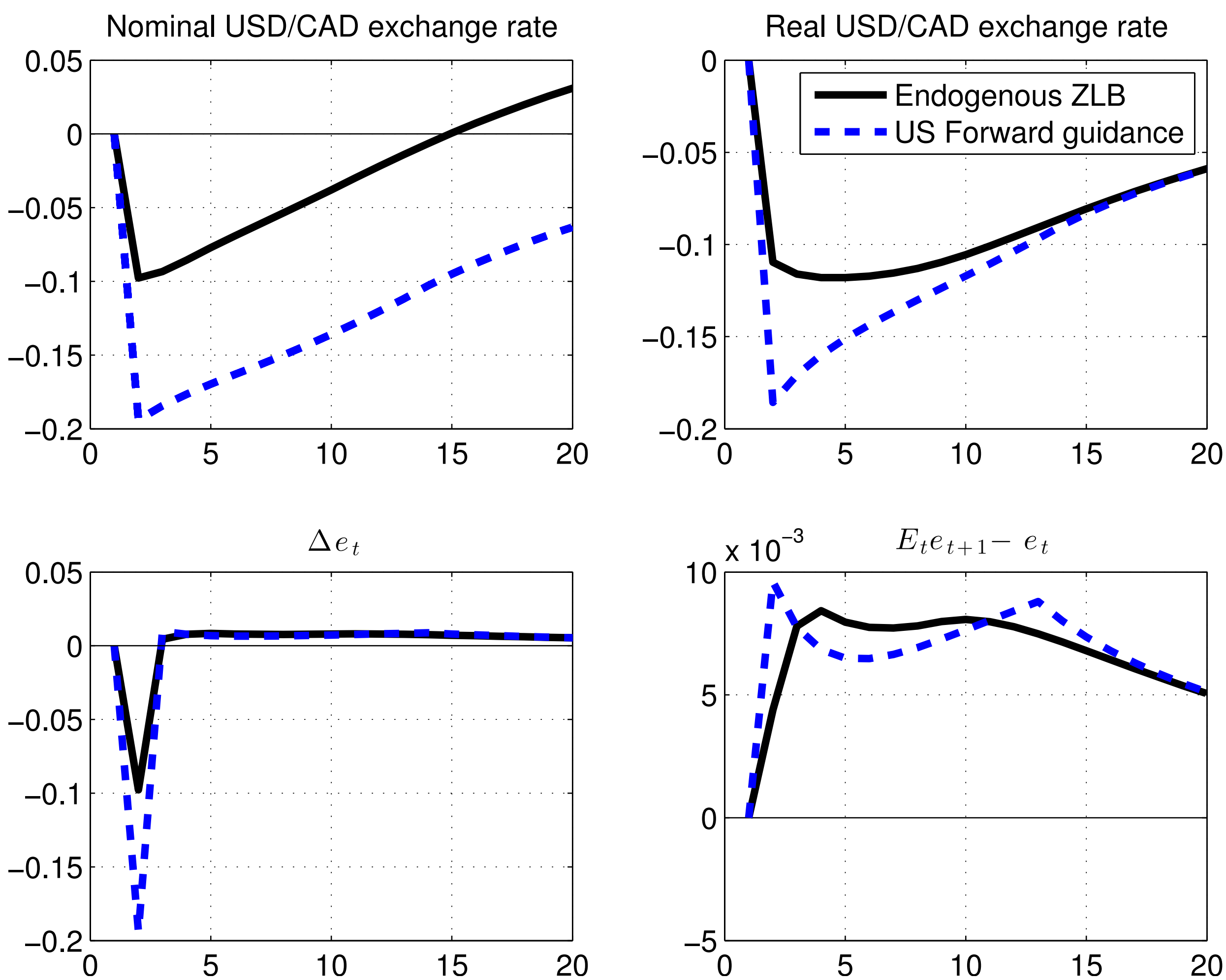

Figure 5: Impulse response, US demand shock. Exchange rate.

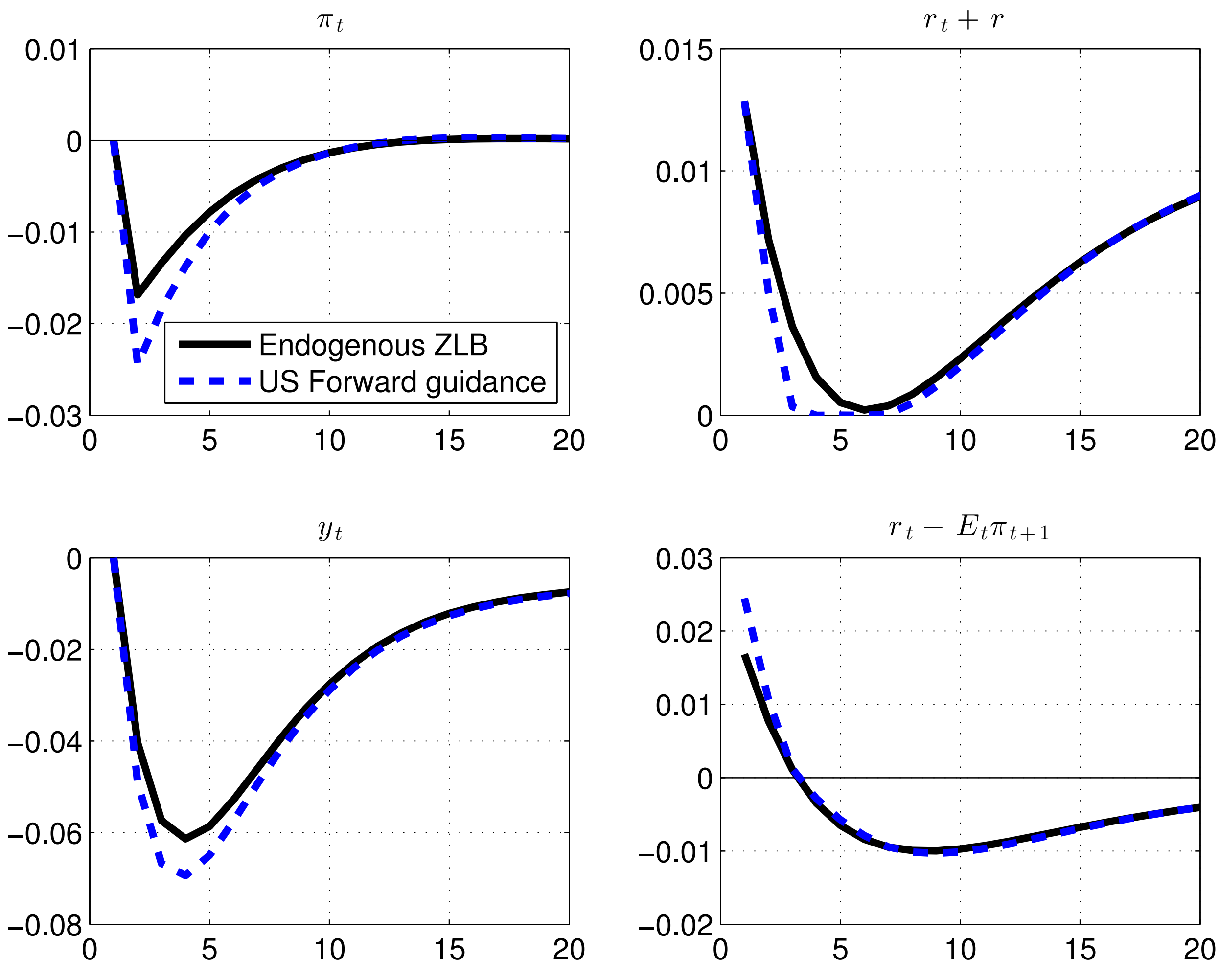

Figure 6: Impulse response, US and Canadian demand shock. Canadian variables.

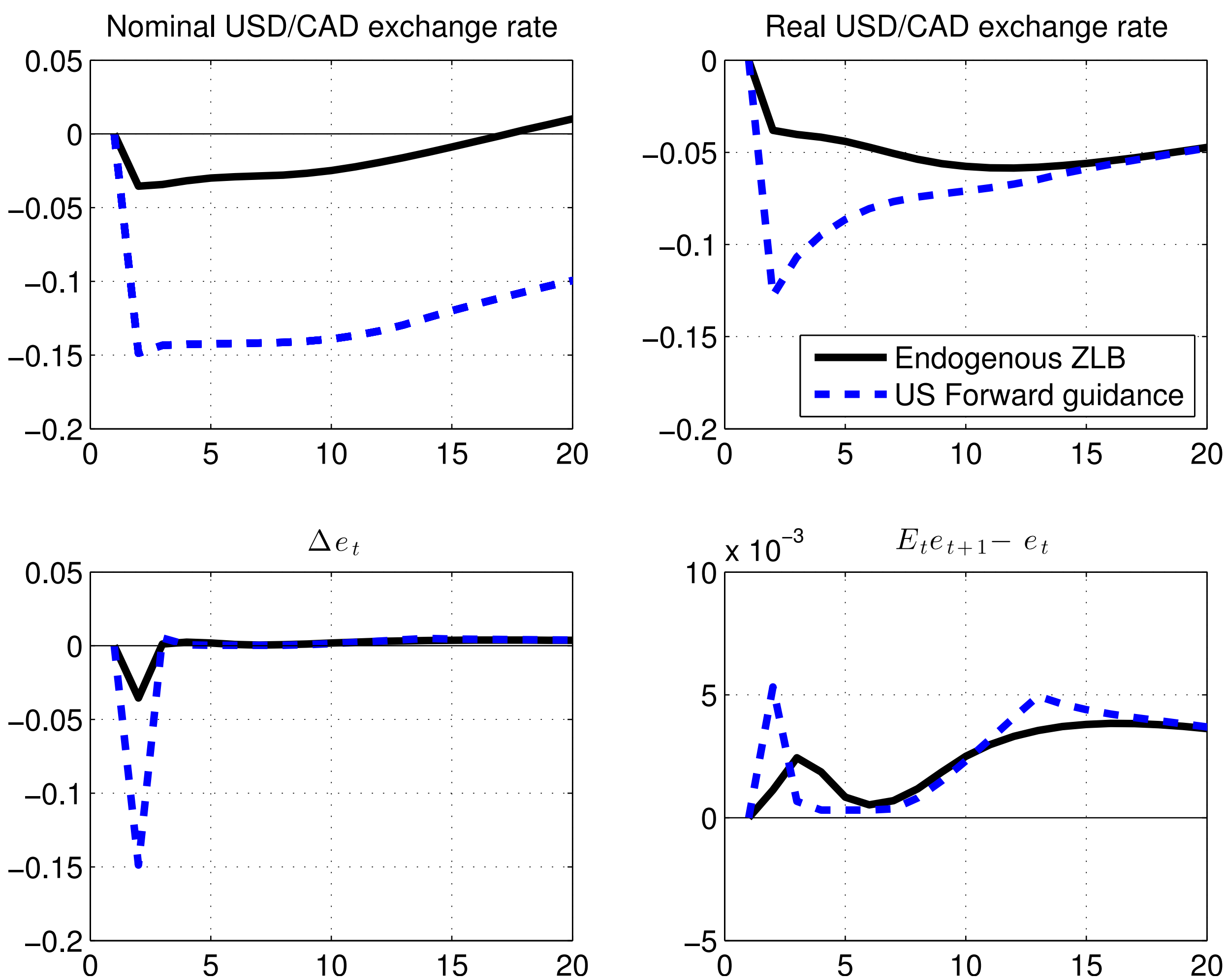

Figure 7: Impulse response, US and Canadian demand shock. Exchange rate.

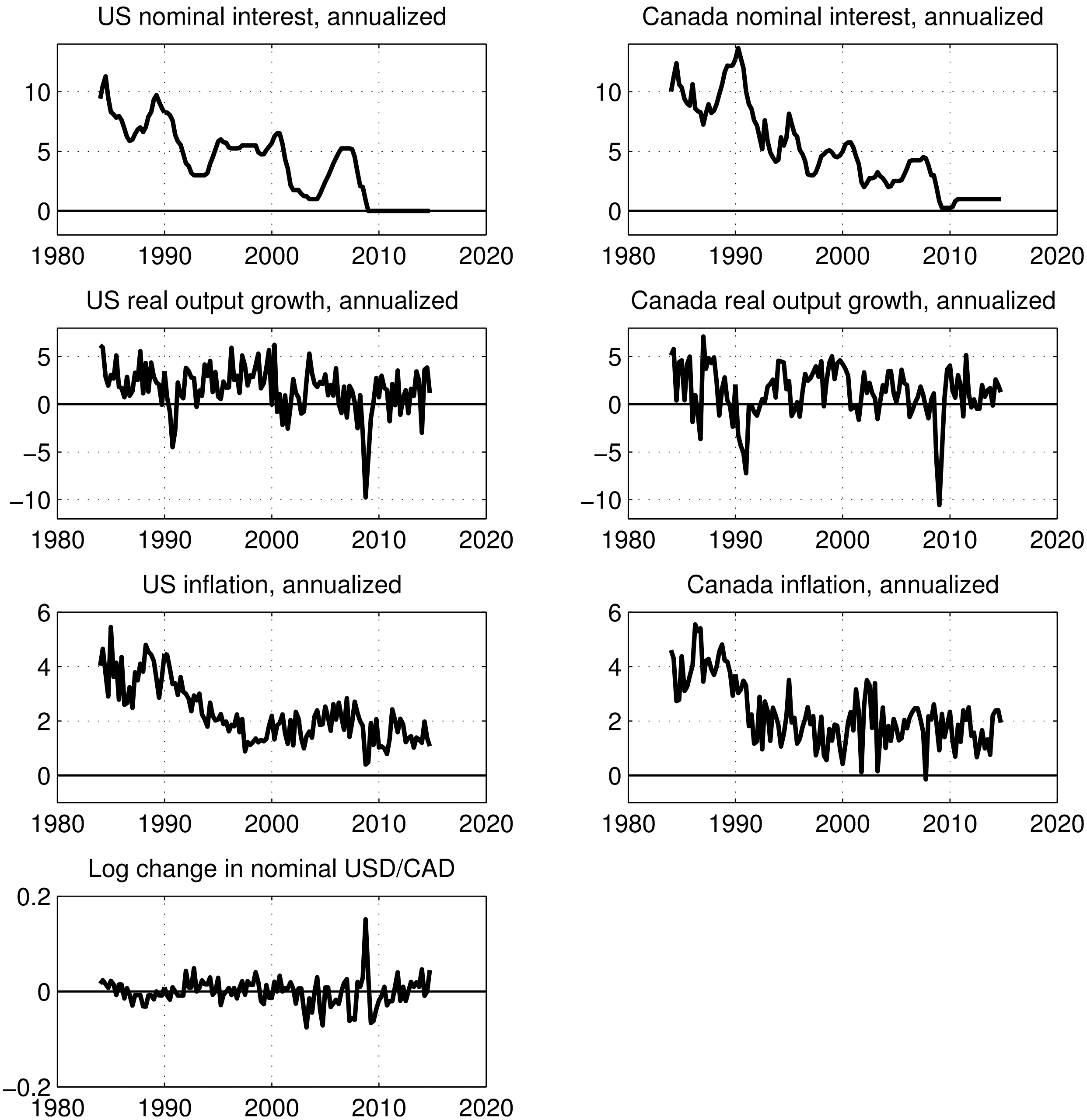

Figure 8: Quarterly data used in estimation. We also use 2Q, 1Y and 2Y Treasury yields in estimation.

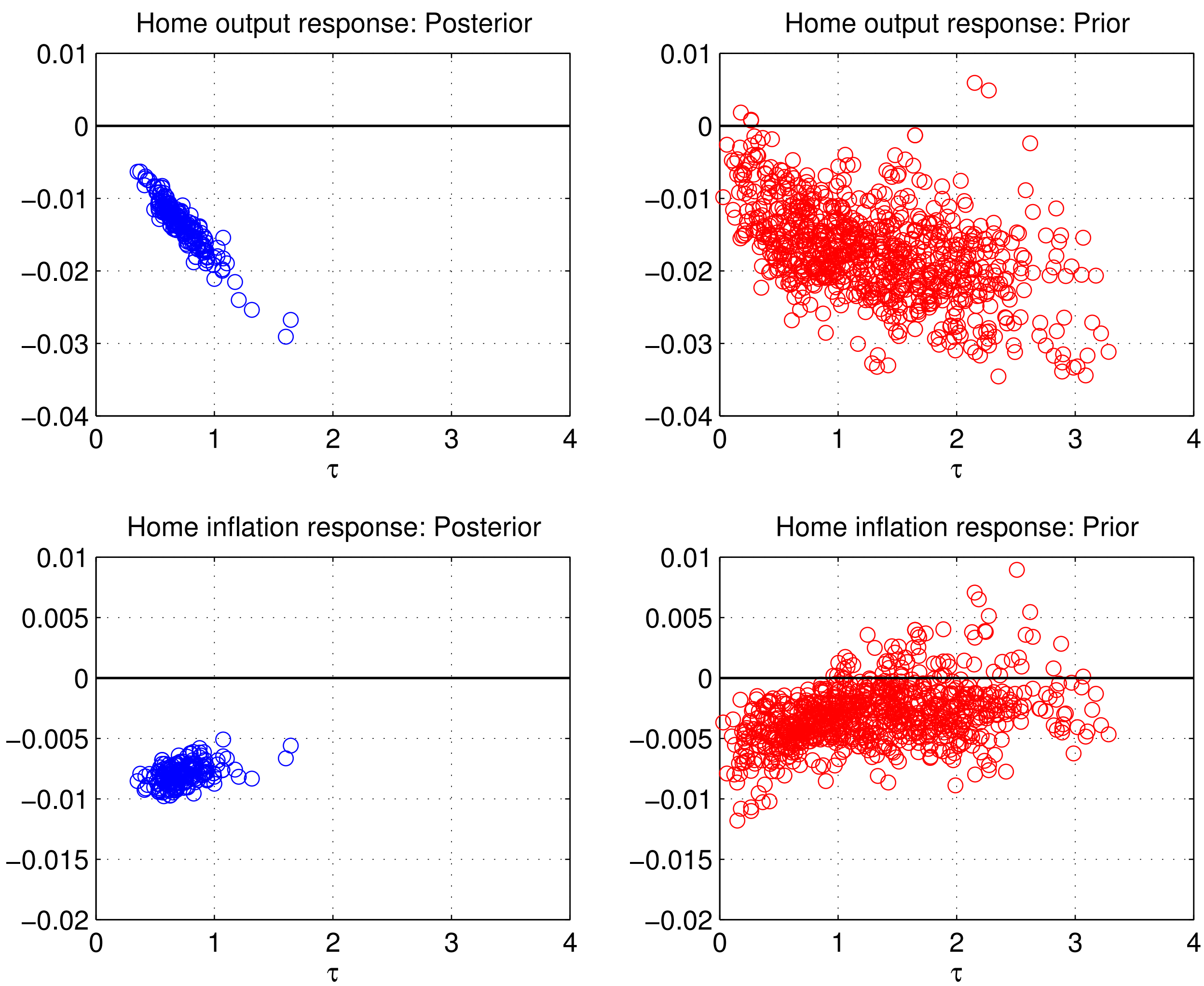

Figure 9: Effect of forward guidance. Prior and posterior.

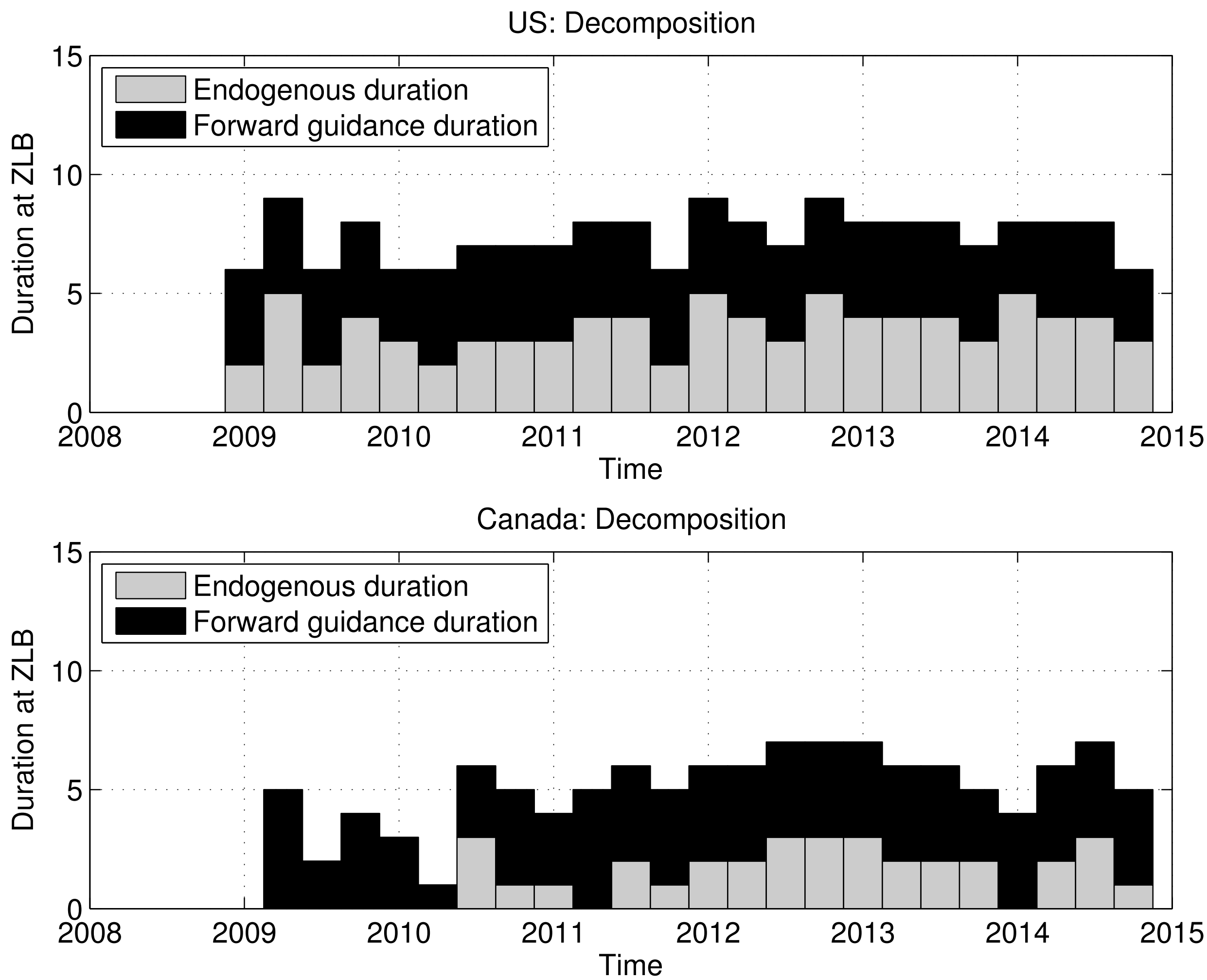

Figure 10: ZLB duration decomposition at the target median draw. This figure shows the decomposition of lower bound durations for the target median draw of the posterior distribution. That draw is chosen as the one which minimizes the sum of distances from the median draw of the posterior of durations for the US and Canada.

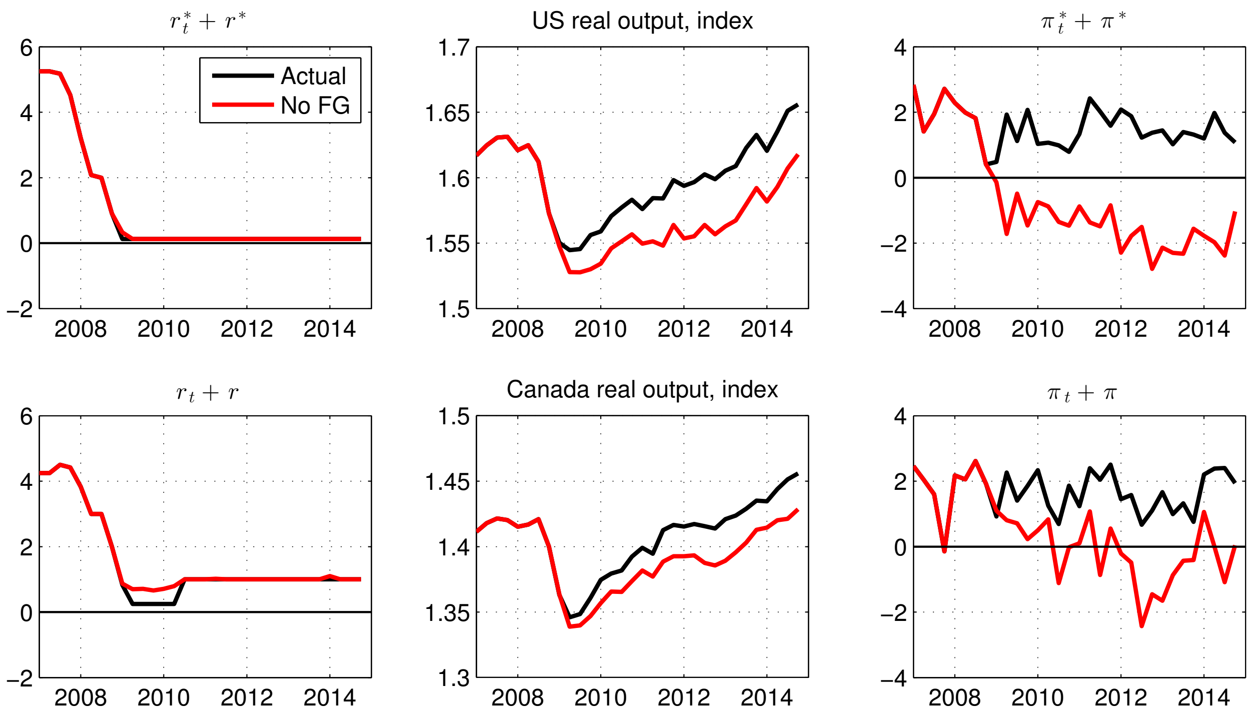

Figure 11: Removing forward guidance. This figure shows counterfactual series when both the US and Canada do not use forward guidance policies and instead act passively in response to the estimated structural shocks. Inflation and the nominal interest rate are annualized.

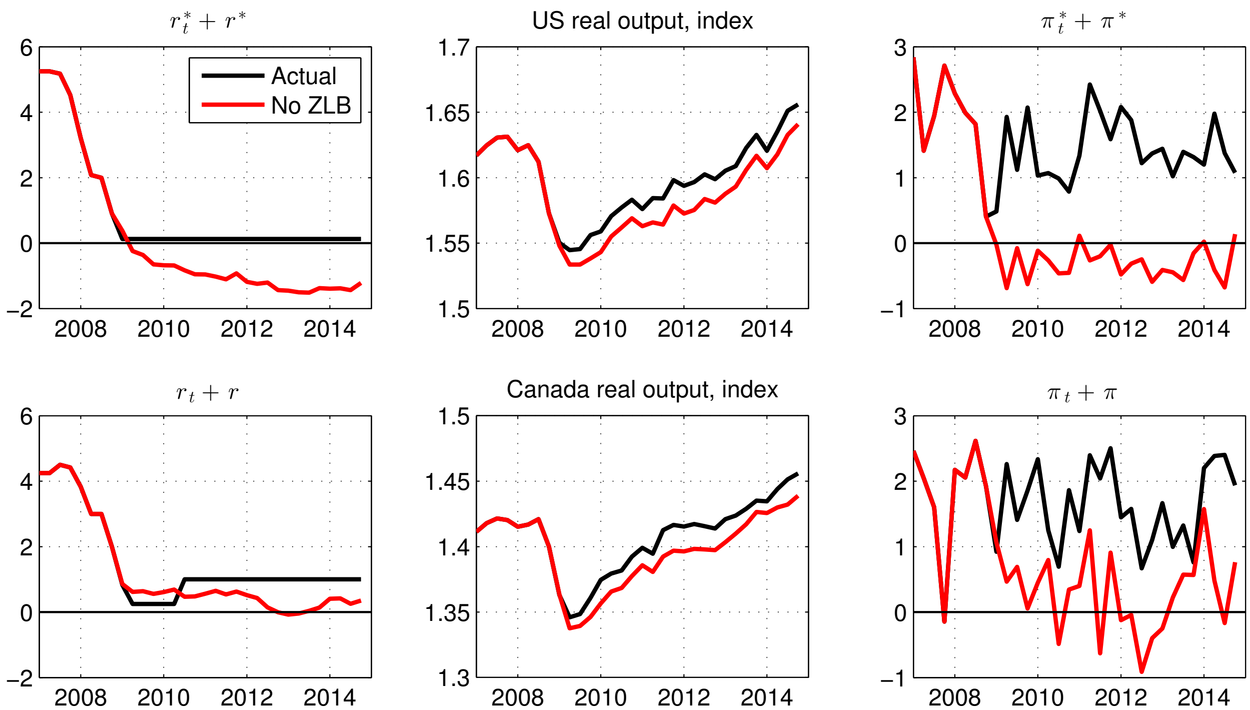

Figure 12: No ZLB constraint. This figure shows counterfactual series when the ZLB is removed as a constraint on both US and Canadian policy interest rates. Inflation and the nominal interest rate are annualized.

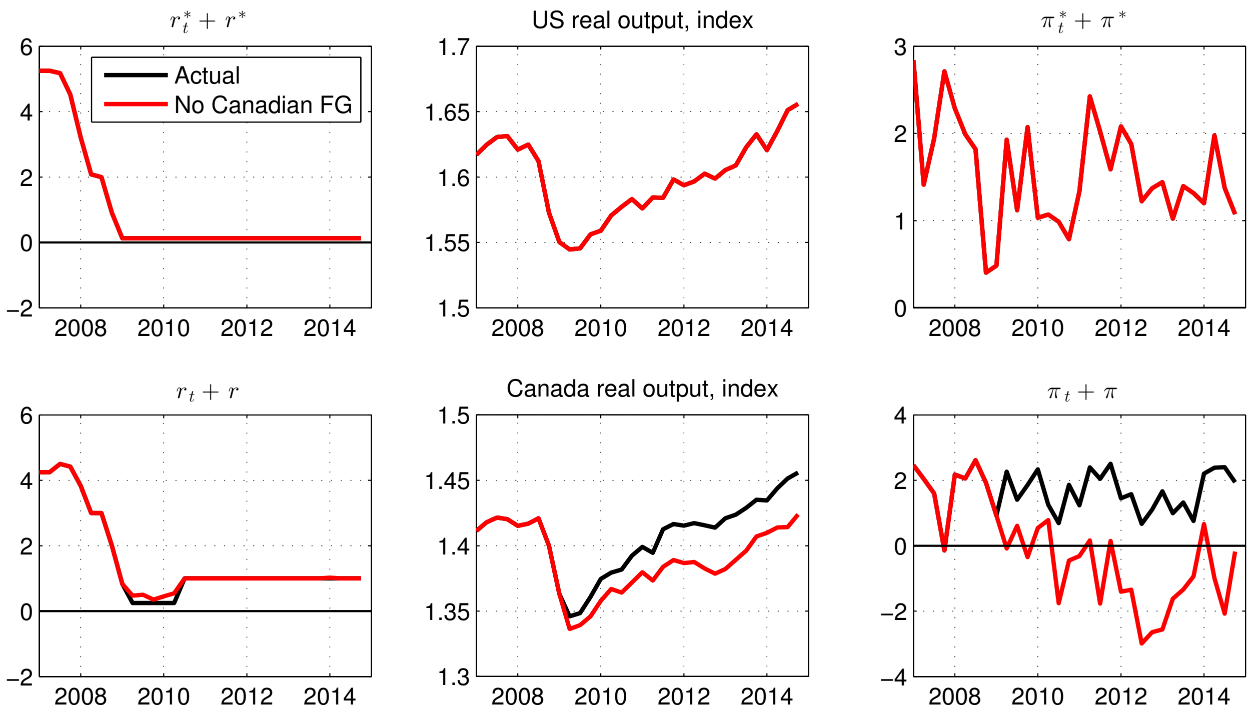

Figure 13: Removing Canadian forward guidance. This figure shows an additional counterfactual experiment, imposing estimated forward guidance in the US but not in Canada, so that the ZLB durations arising in Canada are those which are implied by the structural shocks with the Bank of Canada acting passively in response to those shocks. Inflation and the nominal interest rate are annualized.

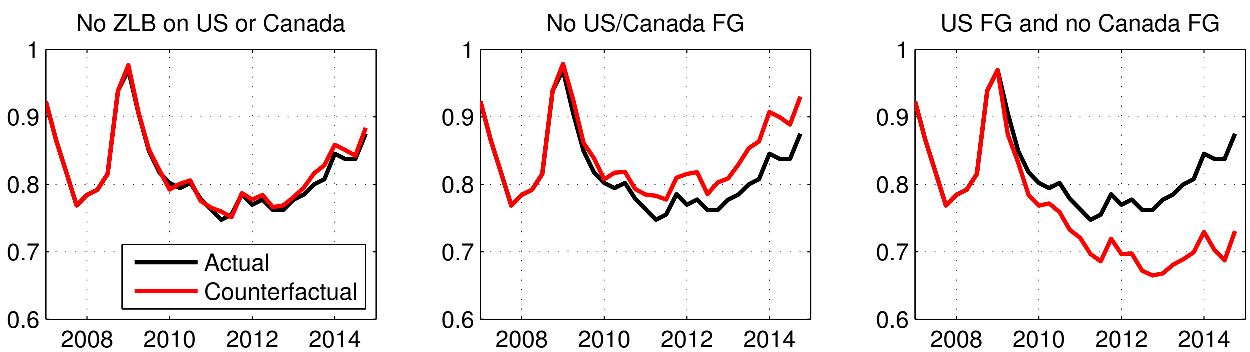

Figure 14: Index of nominal exchange rate under counterfactuals. The black line is the observed nominal interest rate.

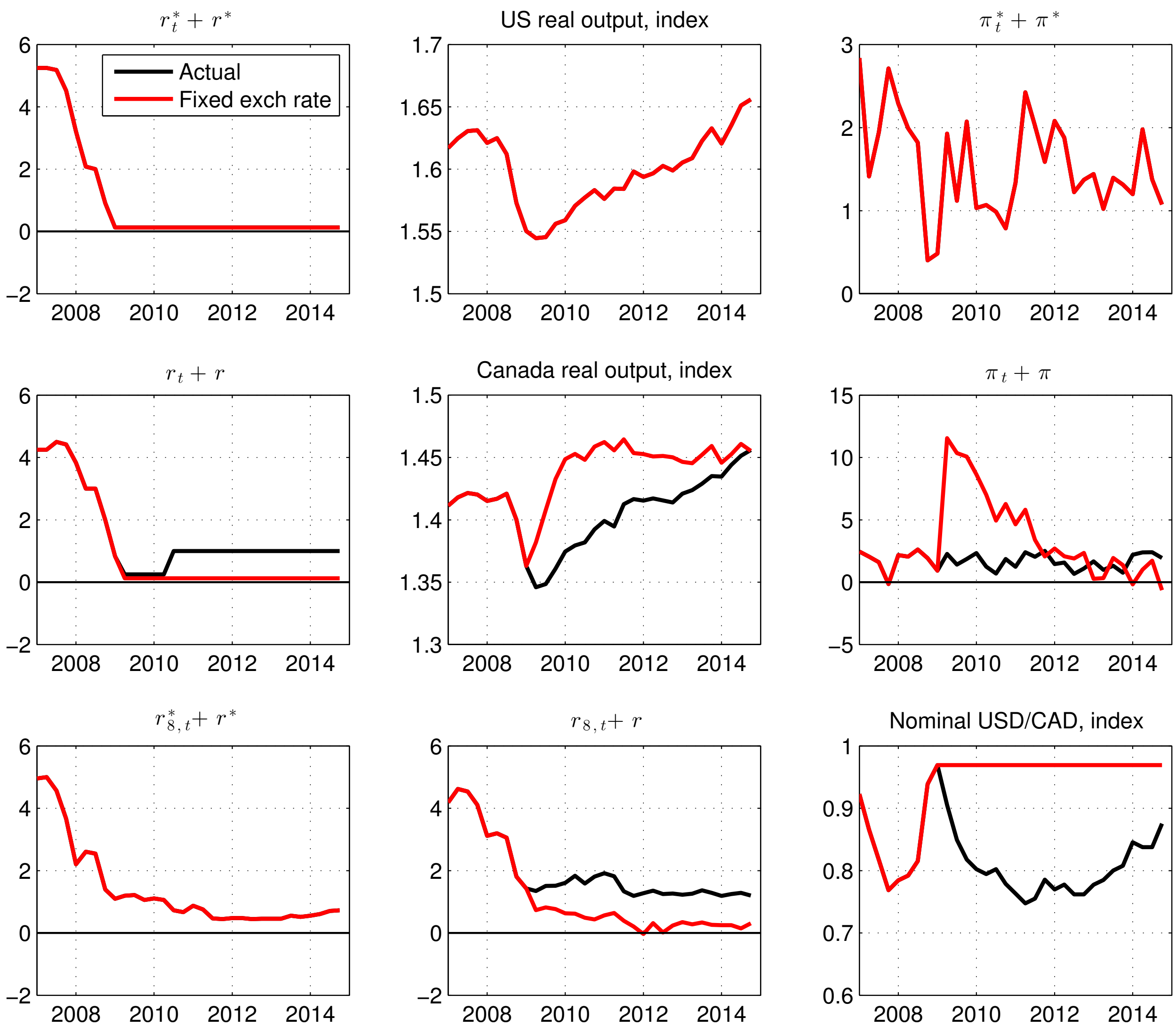

Figure 15: Fixed nominal exchange rate counterfactual. In this figure we compute a counterfactual where Canadian monetary policy is aimed at keeping the nominal exchange rate constant. We shut off the risk premium shock..

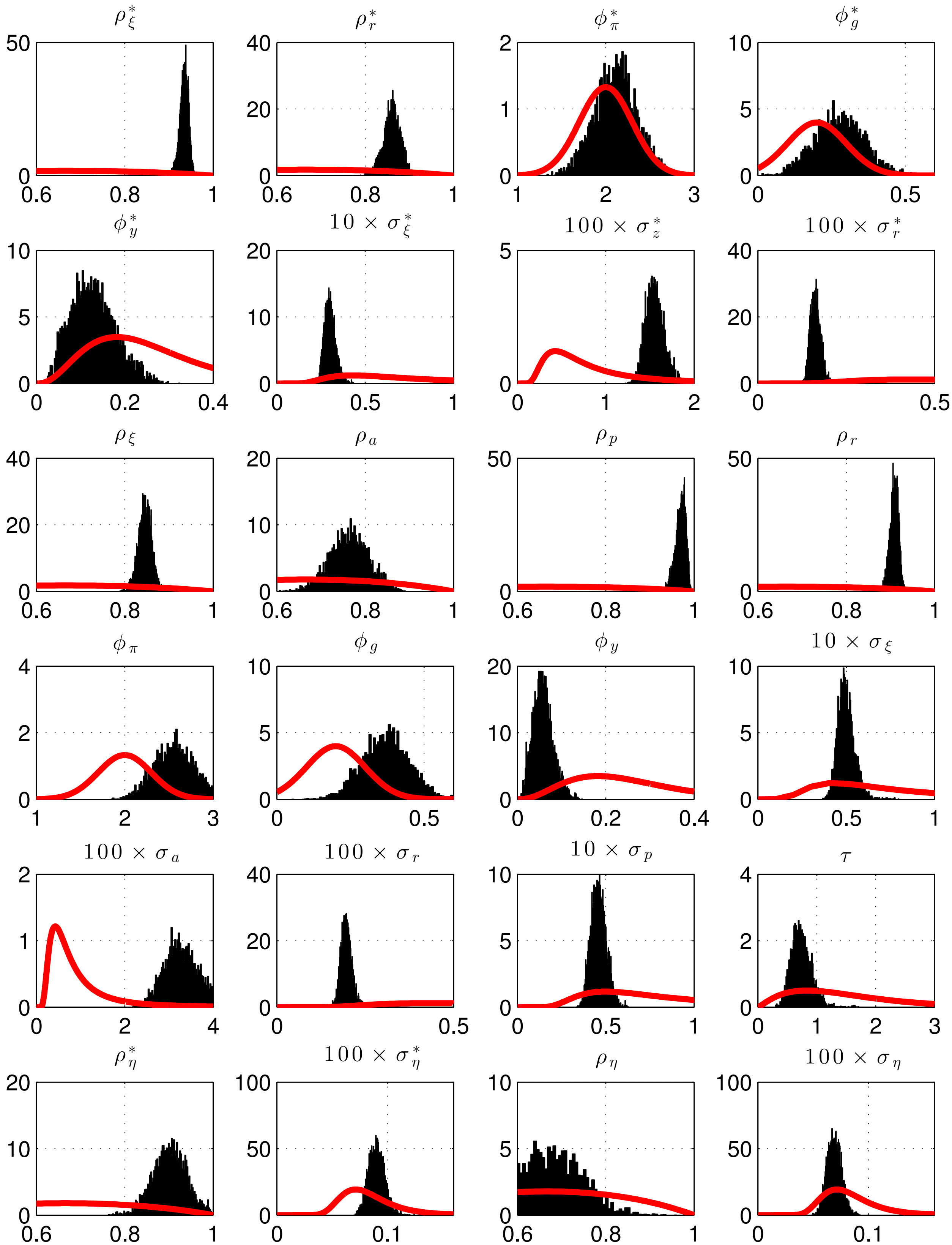

Figure 16: Prior and posterior distributions. Structural parameters.

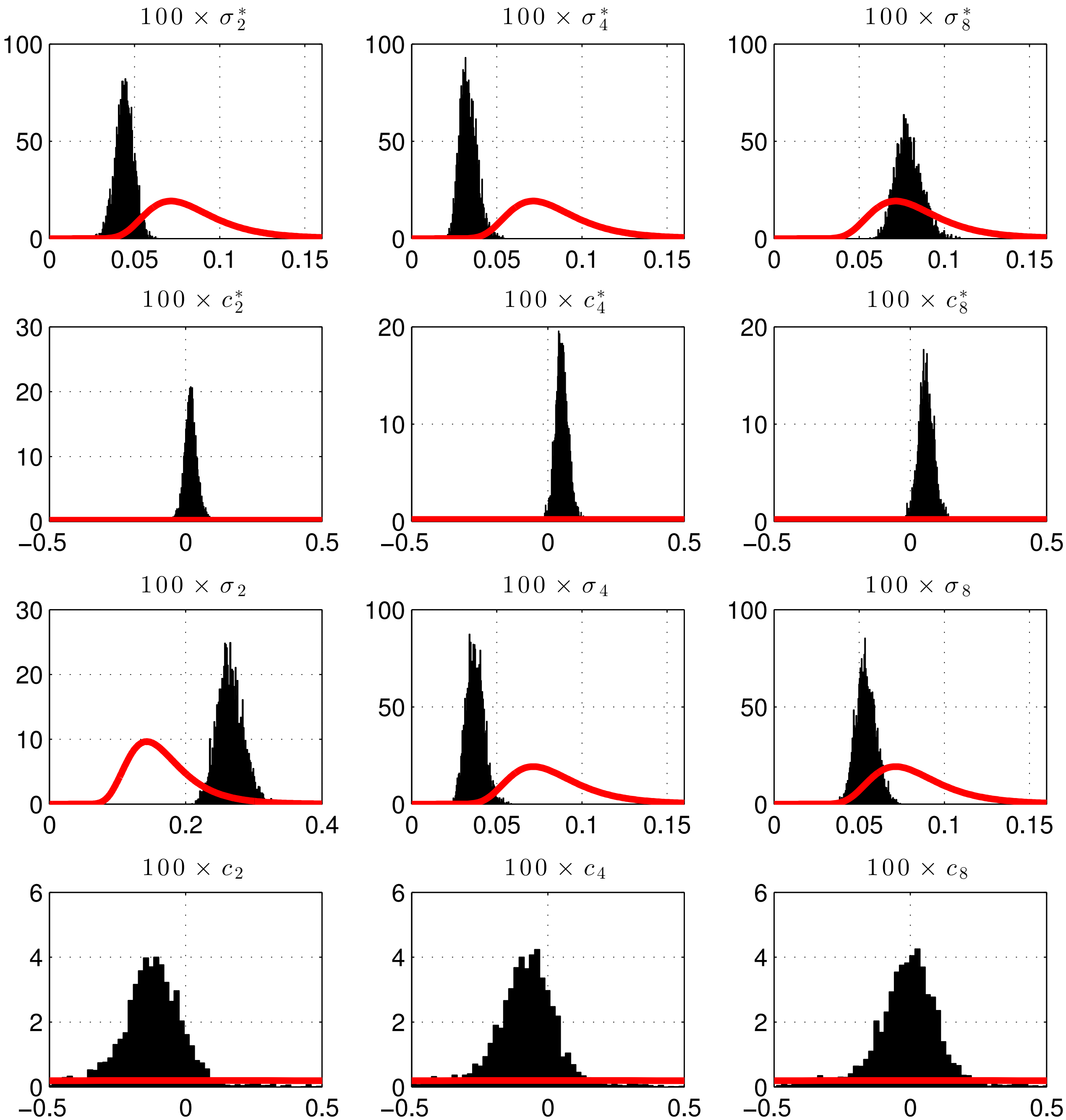

Figure 17: Prior and posterior distributions. Structural parameters for long-rates.

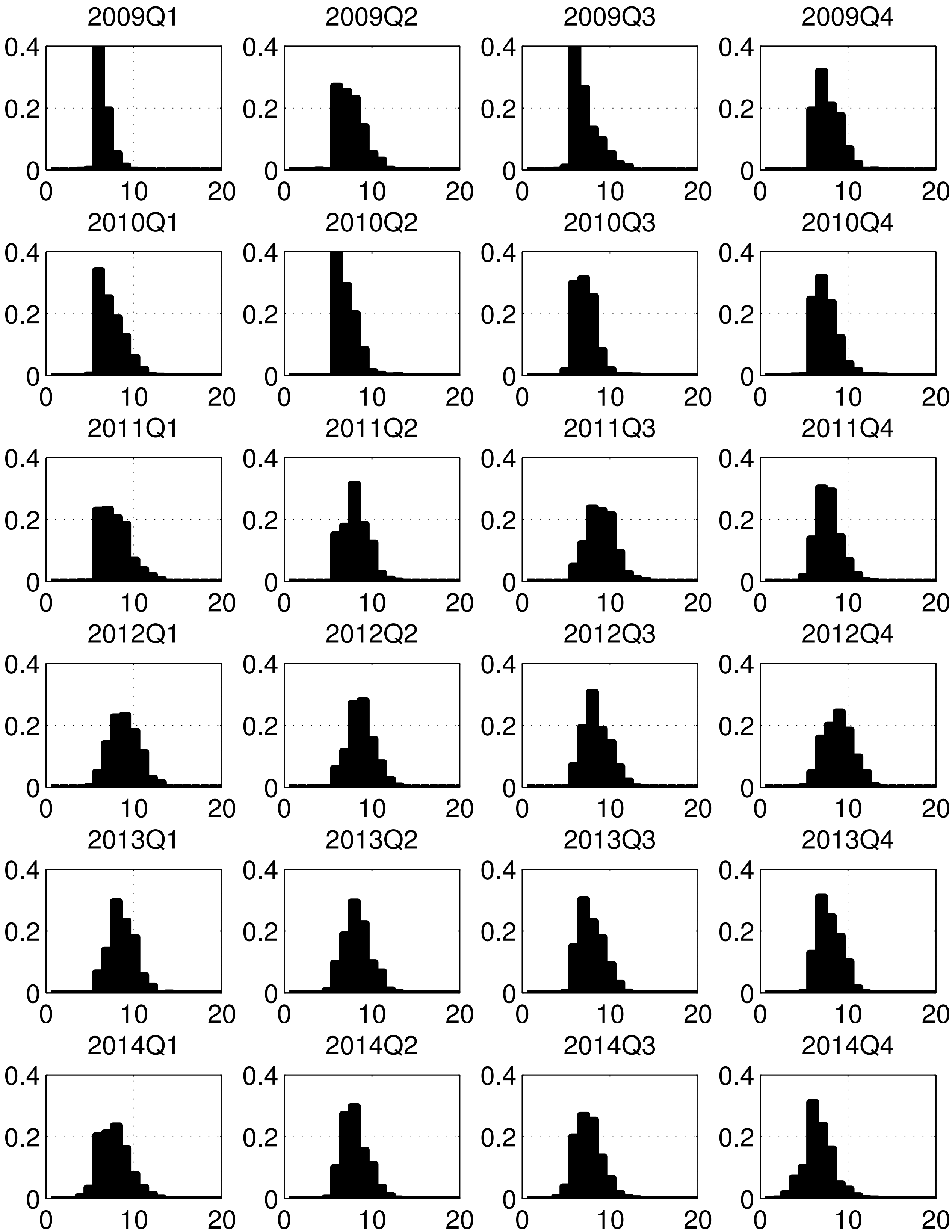

Figure 18: Posterior of expected durations in quarters, US.

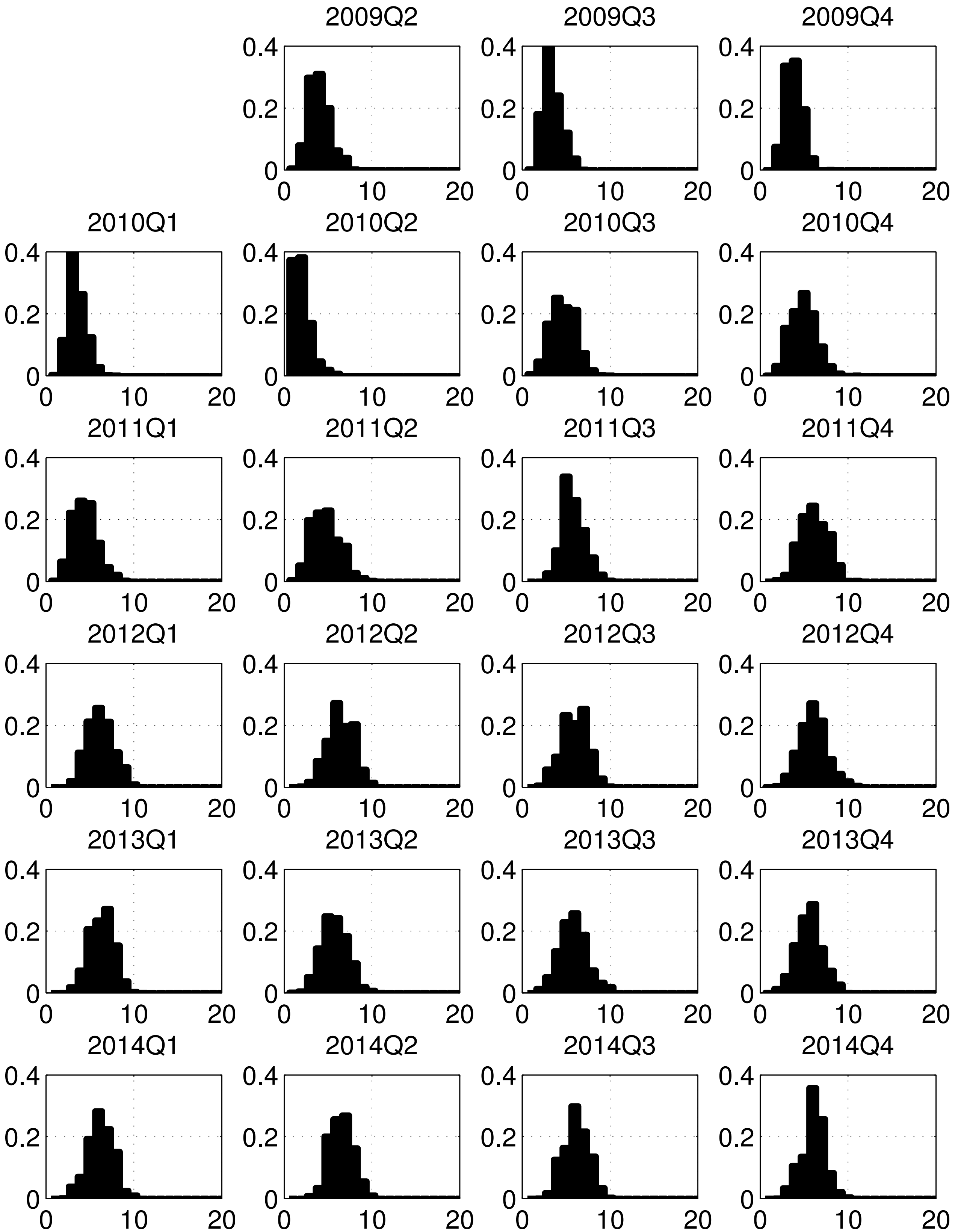

Figure 19: Posterior of expected durations in quarters, Canada.

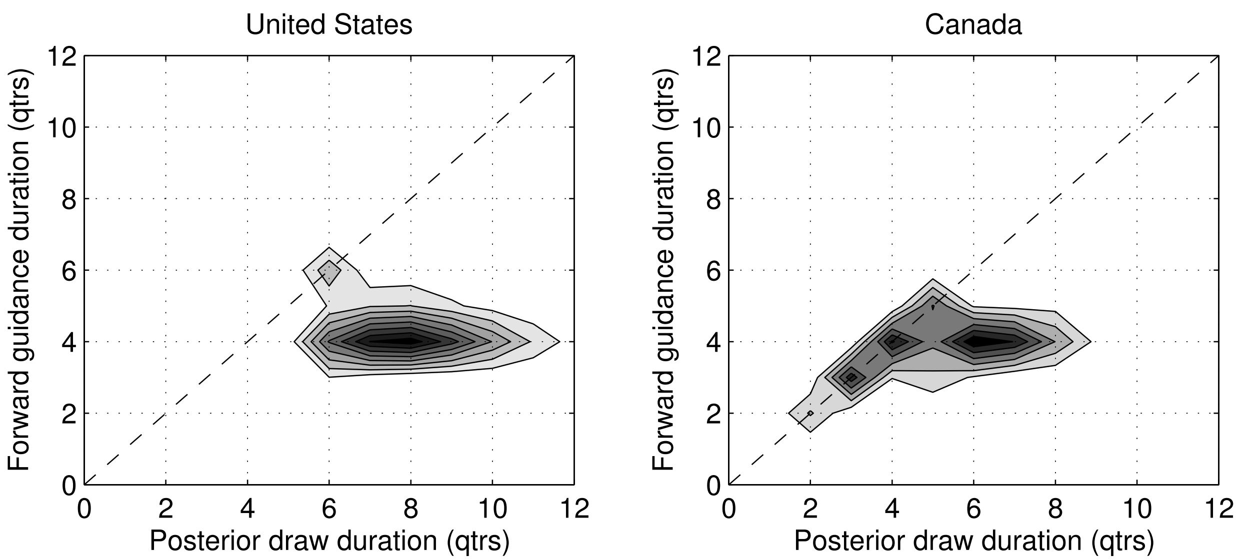

Figure 20: ZLB duration decomposition. This figure shows the joint distribution of draws from the estimated posterior distribution of ZLB durations against the decomposed forward guidance duration associated with that draw. The draws and decomposed durations are aggregated over the 2009-2015 time period. A flatter density suggests ZLB durations are associated less with forward guidance policy and more with structural shocks. A density that lies closer to the diagonal suggests ZLB durations are associated more with forward guidance commitments.