The Purchasing Power Parity Puzzle: The Dependent Economy Solution

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Abstract:

The Australian Dependent Economy model is used to explain the real exchange rate in the short, medium and long run and from it we develop a solution to the Purchasing Power Parity puzzle in which any errors in forecasting the terms of trade or export share result in transitory deviations from a relative purchasing power parity equivalent which we identify as the 'underlying real exchange rate'. These deviations decay at a rate dependent on the relative intensity of the utilisation of structures in the production of exportable and non-tradeable goods. An empirical analysis using Australian data shows strong support for the key propositions of the theory.

Keywords: real exchange rates; purchasing power parity; terms of trade; mean reversion

JEL Classification: F41, F31

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1 Introduction

The theory of purchasing power parity (PPP) has a long history. It holds that the prices of goods in different countries, when expressed in a common currency are equal. If PPP held without restriction then all real exchange rates would be equal. Therefore the study of movements in real exchange rates is also the study of deviations from purchasing power parity. Several studies, noting the poor performance of existing real exchange rate models, have concluded that additional research about the behaviour of the real exchange rate is required (Obstfeld and Rogoff 2000; Lan 2002).

Equation Chapter 1 Section 1

In this study, the Australian Neo-Classical Dependent Economy model is used to develop a revised model of movements of the real exchange rate identifying aspects of the short, medium and long-run. The solution is developed based on four propositions:

- Real exchange rate movement can be identified by movements in the relative price of non-traded goods. This is a standard result in the literature (eg Strauss (1995)).
- The long-run equilibrium relative price of non-traded goods is independent of demand, and dependent instead on technology and the cost of capital. This is a standard result in the literature (e.g. Obstfled and Rogoff (1996)). When the terms of trade are included as the relative price of exportable goods, the long-run equilibrium price of non-traded goods will also depend on the terms of trade and will move with it. We call this the 'underlying real exchange rate'.

- The short-run is determined by the balance of supply and demand, where supply is fixed by sectoral capital allocations, demand surprises will cause shocks to the relative price of non-traded goods and the real exchange rate in the short-run.
- There is medium-run transition from shocks back to the long-run equilibrium or ‘underlying real exchange rate’. This transition path is constrained by the avoidance of arbitrage opportunities between the holding of equipment, structures and foreign bonds. Stable or unstable paths will depend on structural parameters of the production function.

The neo-classical dependent economy framework was first presented by Bruno (1976) and the first order solution approach presented by Brock and Turnovsky (1994). The model in this paper expands the framework by incorporating the terms of trade based on the Salter-Swan dependent economy approach and then provides a solution to the Purchasing Power Parity Puzzle of Rogoff (1996) by identifying a source of real shocks, in both the terms of trade and export share, and a theoretical basis for an extended return to parity based on maintaining ‘no arbitrage’ conditions.

Rogoff's (1996) puzzle is identified as a combination of high-short term volatility in the real exchange rate along with a relatively slow rate of reversion back to PPP. An explanation of high short-term volatility is provided by short-run capital adjustment restrictions which cause surprises in the terms of trade and the export share driving a disproportionate adjustment in relative prices in the short-run. The export share is a measure of the relative demands for the production of exportable and non-tradeable goods in the domestic economy, determining labour allocation in the short run and driving the sectoral allocation of equipment, structures and labour in the medium and long-run. The slow rate of reversion to PPP can then be explained by the linkage between the price of non-traded goods and the utilisation of structures in the economy which results in a medium-run adjustment process back to equilibrium after the occurrence of short-term shocks. Conceptually, if adjustment processes were faster or slower than the identified rate, this would imply that an investor holding structures (capital formed from the non-tradeable good) would obtain a higher or lower rate of return than alternative investments in equipment or foreign bonds, thus breaching 'no arbitrage' conditions. The long-run solution of our model identifies that the time path of exogenous variables such as technology, cost of capital and terms of trade will determine the evolution of the underlying long-run real exchange rate rather than a singular long-run absolute parity. This supports the quasi-power purchasing parity (QPPP) approach proposed by Hegwood and Papell (2002).

Equation Section (Next)

2 History of the Purchasing Power Parity Puzzle

The Purchasing Power Parity Puzzle was coined by Rogoff:

> *"How is it possible to reconcile the extremely high short-term volatility of real exchange rates with the glacial rate (15 percent per year) at which deviations from PPP seem to die out?"* (Rogoff 1996).

This then creates a call for research "*on the economic source of deviations from PPP, which may comprise persistent and/or transitory components*" (Lan 2002). This paper is a response to the call for research, explaining both the source and persistence of deviations from PPP in the Dependent Economy Model.

Studies of real exchange variations across countries identify several important explanatory variables: Balassa (1964) and Samuelson (1964) find the distinction between non-tradeable and tradeable goods as a pathway for real exchange rate variations between countries with different levels of national wealth, while Bhagwati (1984) indicates that the difference between capital labour ratios has a powerful influence on real exchange rates measured by the relative price of services.

The Australian version of the Dependent Economy model by Salter (1959) and Swan (1960) suit Australian conditions characterised by exports of commodities with volatile prices which are reflected in exchange rate movements, imports of more advanced manufactures and recurrent current account deficits. These structural characteristics may be inappropriate for large economies or economies with a different export base. A feature of the model developed here is the behaviour of the TOT which depends on movements in the price of importable and exportable goods relative to their international and domestic counterparts. The terms of trade have a strong relationship with real exchange rates for Australia and many other commodity exporting countries (Chen and Rogoff 2003; Cashin, Cespedes et al. 2004).

While the 'Purchasing Power Parity Puzzle' is 10 years old, its origin is in Dornbusch's (1976) model of exchange rates and the failure of monetary models as effective exchange rate forecasters identified by Meese and Rogoff (1983). This failure is manifest in a unit root for the real exchange rate as a logical alternative assumption to PPP (Bleaney, 1998). The PPP puzzle represents the econometrically identified compromise between these two conflicting positions, where numerous studies have shown a mean reversion averaging 4.1 years (Lan, 2002). Research about the puzzle is focused on three areas: firstly, on the methodology for the determination and measurement of the rate of mean reversion; secondly, broadening the scope of data reviewed for evidence of PPP reversion and finally, discussion of potential solutions to the PPP Puzzle itself.

Methodological enquiries have challenged several aspects of the standard methodology for measuring the half-lives associated with the PPP Puzzle. Taylor (2001) indicates that the data frequency for half-life analyses tend to be biased in favour of finding extended half-lives, creating what is described as the temporal aggregation problem followed up in studies by Sekioua and Karansos (2006), Chambers (2005) amongst others. Imbs, Mumtaz et al (2003) go so far as to contend that by controlling for aggregation bias, the PPP puzzle may be resolved, though this is disputed by Chen and Engel (2005). Non-linearity issues are associated with speed of reversion to long-run equilibrium, with Cheung and Lai (2000) identifying that response dynamics exhibit non-monotonous behaviour, leading to substantial imprecision in half-life estimates.

An alternative specification under which aspects of the PPP puzzle may be resolved is by structural breaks. Hegwood and Papell (2002) show that when the potential for structural breaks is incorporated into exchange rate data covering the gold standard period (1810 to 1913) for industrial economies, the half-life was significantly shortened, consistent with nominal price-stickiness.

Rogoff’s (1996) original description of the puzzle was based on the broad alignment of the research on exchange rates seeking to identify the rate of convergence and whether a random-walk can be rejected. Studies by Frankel and Rose (1995), Wei and Parsley (1995), Lothian and Taylor (1996) all identify rates of convergence in the region of 15% per annum, representing a half-life of 4 to 5 years. The scope of the PPP puzzle has since been expanded by studies which examined: Big Mac currencies (Cumby, 1996); the different rates of convergence of the components of the Big Mac (Parsley and Wei, 2004); the difference between CPI and WPI based measures (Higgins and Zakrajsek, 2000); the difference between developing and industrialised countries (Cheung and Lai, 2000); whether certain countries confirm or refute the PPP Puzzle (Yazgan 2003) and examining the PPP Puzzle after controlling for indicators such as the terms of trade (Chen and Rogoff 2003; Bjornland and Hungnes 2005).

The last two studies can be seen as stepping stones in the development of the model proposed here. In Chen and Rogoff (2003), the terms of trade are utilised, as well as productivity, but no variable incorporating the real impact of capital flows. Chen and Rogoff (2003) also specifically examine the PPP Puzzle for Australia, concluding that after adjustment for the terms of trade, the puzzle remains. In the empirical section we identify no evidence for an extended rate of mean reversion, instead identifying a residual half-life of 1-2 years after adjustment for the terms of trade and export share. In Bjornland and Hungnes (2005) the interest rate differential is incorporated as capital flows resulting from differentials were identified as causing extended deviation from PPP. In the Dependent Economy formulation, the role of capital flows is subsumed into two broader frameworks, firstly the production of export goods, which decline (increase) with capital inflows (outflows) and secondly, the capital arbitrage conditions which shape the rate of mean reversion.

The potential role of the Traded-Non-Traded goods dichotomy as a solution to the PPP Puzzle has been discounted by some researchers on the basis that prices of Non-Traded goods and Traded goods appear far more strongly correlated with each other than they are with the exchange rate (Engel 1999; Chari, Kehoe et al. 2002). However, Obstfeld and Rogoff (2000) provide qualified support for the approach, noting that a substantial proportion of measured variations in traded goods prices represent non-trade components, with market control, presented as 'Pricing to Market' being a potential explanation. A 'pricing to market' approach, with its suggestion of nominal price rigidity, is not the formal basis of our model. However, we hypothesise that the concept of non-traded capital can be expanded to include branding and other methods for reducing competition, so that the net effect, an upward sloping supply curve (in the short-run) is a fundamental difference of form, rather than substance.

Equation Section (Next)

3 The Model

The real exchange rate is equated in the first instance with the weighted relative price of non-traded goods to traded goods, with weights determined by the consumer's utility function. For simplicity and following the small country assumption, the relative price of non-traded goods in the international economy is assumed to be unitary and constant. This is shown in equation [3.1] below, with lower case letters are used representing natural logarithms, q, the log of the real exchange rate and p_N the log of the relative price of non-tradeable goods. The model can therefore focus on the determinants of the domestic non-traded goods price.

$$\begin{aligned} &q = \gamma p_N \\ &where: \\ &U(C) = C_N{}^{\gamma} C_M{}^{1-\gamma} \qquad p_N^* = 1 \end{aligned} \qquad [3.1]$$

We define the neo-classical dependent economy model as a two-capital[1] and labour, two-good[2], linearly homogenous production framework. Capital is assumed to be fixed in an instantaneous short-run, but otherwise flexible. There are two real prices, both relative to the price of importable goods, which is taken as the numeraire.

The first price is the relative price of exportable goods to the price of importable goods which is measurable as the commodity Terms of Trade, it is determined exogenously following the small economy assumption. The second price is the relative price of non-tradeable goods, which will be determined endogenously.

Consumption and investment of tradeable goods is in units of the internationally importable goods. Production of tradeable goods is exclusively in units of the exportable good.

Wealth can be stored as real equipment, or structures, which are created by the conversion of 1 unit of the importable and non-tradeable goods respectively, or as international bonds, which are denominated in units of importable goods and earn interest *r*, determined exogenously. There are no restrictions on international trade or capital flows, other than that the non-tradeable goods and non-tradeable capital, structures, are non-tradeable and that the Capital Account ($S - I = \dot{B}$) must balance with the Current Account ($X + rB - M = \dot{B}$).

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[1] One type of capital is derived from tradeable goods (equipment) and one type of capital is derived from non-tradeable goods (structures).
[2] A combination of labour, structures and equipment can be used to produce tradeable goods, and a combination of labour, structure and equipment can be used to produce non-tradeable goods.

Production is assumed to be undertaken by competitive firms with perfectly mobile labour, such that the utilisation of labour is determined by the instantaneously equating the marginal productivity of labour (times price) with the real wage. Wages will be equal in both sectors. Investment in capital will be based on competitive firms establishing the marginal return on capital equal to the cost of capital.

The cost of capital for equipment (tradeable capital) will be set on the world market through arbitrage with the same rate of return on international bonds adjusted for depreciation, in other words, real interest parity holds. The cost of capital for structures will include the expected rate of change in the price of non-tradeable goods and the international price of bonds[3].

The conditions of the supply problem are as follows:

$$\dot{E} = I_s - \delta_E E \quad [3.2]$$

$$\dot{S} = I_s - \delta_S S \quad [3.3]$$

$$L = L_N + L_X = 1 \quad [3.4]$$

$$S = S_N + S_X \quad [3.5]$$

$$E = E_N + E_X \quad [3.6]$$

$$\begin{aligned} R_S &= P_N(r + \delta_S) - \dot{P}_N = P_N r_S \\ r_S &= r + \delta_S + \frac{\dot{P}_N}{P_N} \end{aligned} \quad [3.7]$$

$$R_E = r_E = r + \delta_E \quad [3.8]$$

$$F(E_X, S_X, L_X) = A_X E_X{}^{\alpha_X(1-\kappa_X)} S_X{}^{\alpha_X \kappa_X} L_X{}^{1-\alpha_X} \quad [3.9]$$

[3] Through arbitrage.

$$G(E_N, S_N, L_N) = A_N E_N^{\alpha_N(1-\kappa_N)} S_N^{\alpha_N \kappa_N} L_N^{1-\alpha_N} \qquad [3.10]$$

Equations [3.2] and [3.3] show the stock of structures (*S*) and equipment (*E*) evolving based on investment (*I*) and depreciation rates (δ_S, δ_E). Equations [3.4], [3.5]and [3.6] show that the supply of Labour (which is normalised to 1), stock of Equipment and stock of Structures must be allocated to either the non-tradeable goods (subscript *N*) or Exportable goods (subscript *X*) production sectors.

Equations [3.7] and [3.8] are the components of the rental cost of capital, R_S and R_E[4], which are determined by arbitrage conditions. Equations [3.9] and [3.10] are Cobb-Douglas representations of the supply function.

The solution assumes utility maximising behaviour but does not require a full solution to the inter-temporal utility maximisation problem. The demand side is not required for the long-run solution, while shocks in the short-run may be treated as exogenous. Endogeneity of the export share on the non-traded goods price should be considered as a component of empirical testing. The Neo-Classical demand system requires a utility function $U(C_N, C_M)$ and a budget constraint (see Appendix 1).

As a standard cost minimisation problem, where the market price is the shadow-price of the volume constraint, the solution is developed as follows:

$$Z_X = wL_X + R_E E_X + R_S S_X - P_X \left(Y_X - A_X E_X^{\alpha_X(1-\kappa_X)} S_X^{\alpha_X \kappa_X} L_X^{1-\alpha_X} \right) \qquad [3.11]$$
$$Z_N = wL_N + R_E E_N + R_S S_N - P_N \left(Y_N - A_N E_N^{\alpha_N(1-\kappa_N)} S_N^{\alpha_N \kappa_N} L_N^{1-\alpha_N} \right) \qquad [3.12]$$

Applying standard inter-temporal optimisation conditions:

[4] The capital cost of equipment is measured in units of the imported tradeable goods which is taken as the numeraire good ($P_M = 1$)

$$P_X Y_X = \frac{wL_X}{1-\alpha_X} = \frac{R_E E_X}{\alpha_X (1-\kappa_X)} = \frac{R_S E_S}{\alpha_X \kappa_X} \quad [3.13]$$

$$P_N Y_N = \frac{wL_N}{1-\alpha_N} = \frac{R_E E_N}{\alpha_N (1-\kappa_N)} = \frac{R_S E_S}{\alpha_N \kappa_N} \quad [3.14]$$

Using [3.4], [3.13], [3.14] and defining the total economy $Y_t = P_X Y_X + P_N Y_N$ and the ratio shares by T whereby $P_X Y_X = TY_t$ and $P_N Y_N = (1-T) Y_t$ we can then describe ex poste equilibrium real wages as a function of *T* and create an expression for the share of labour that is purely relative to *T* and the productivity variables.

$$L_X = \frac{(1-\alpha_X) TY_t}{w} = \frac{(1-\alpha_X) T}{\left((1-\alpha_N)(1-T) + (1-\alpha_X) T\right)} \quad [3.15]$$

$$L_N = \frac{(1-\alpha_N)(1-T) Y_t}{w} = \frac{(1-\alpha_N)(1-T)}{\left((1-\alpha_N)(1-T) + (1-\alpha_X) T\right)} \quad [3.16]$$

$$w = \left((1-\alpha_N)(1-T) + (1-\alpha_X) T\right) Y_t \quad [3.17]$$

Capital stocks (which will be based on the prior expectations of *T* and Y_t, denoted ET_0 and EY_t respectively) from [3.13] and [3.14] are as follows:

$$E_X = \frac{\alpha_X (1-\kappa_X) ET_0 EY_t}{r_E} \quad E_N = \frac{\alpha_N (1-\kappa_N)(1-ET_0) EY_t}{r_E}$$
$$S_X = \frac{\alpha_X \kappa_X ET_0 EY_t}{P_{N,t-1} r_{S,\mathrm{E}t}} \quad S_N = \frac{\alpha_X \kappa_X (1-ET_0) EY_t}{P_{N,t-1} r_{S,\mathrm{E}t}} \quad [3.18]$$

We can also describe real wages w with respect to the marginal productivity of the production function:

$$w = (1-\alpha_X) P_X A_X E_X^{\alpha_X (1-\kappa_X)} S_X^{\alpha_X \kappa_X} L_X^{-\alpha_X} = (1-\alpha_N) P_N A_N E_N^{\alpha_N (1-\kappa_N)} S_N^{\alpha_N \kappa_N} L_N^{-\alpha_N}$$
[3.19]

Expression [3.19] can be rearranged to describe the relative price of non-traded goods P_N as a function of P_X the capital stocks and T. This can then be transformed by replacing the capital stock variables (E_X, E_N, S_X, S_N) with their expected values from [3.18] Using [3.17], and [3.19], output related terms ($\mathrm{E}Y_t$) can be cancelled to give an expression purely in terms of P_N, P_X, T_t and $\mathrm{E}T_t$ as well as the cost of capital r_E and r_S. (see Appendix 2) This is expressed in log-terms below:

$$p_{N,t} = A(r, E\dot{p}_{N,t}, p_{N,t-1}, E\dot{p}_X) + B(T, ET_0) + C\left(p_X, Ep_X\right) \quad [3.20]$$

$$A(r, E\dot{p}_{N,t}, p_{N,t-1}, E\dot{p}_X) = \frac{1}{1-\alpha_X}\left(B_X - B_N + B_E \ln r_E - B_S \ln r_{S,\mathrm{E}t} - B_S p_{N,t-1} + \left(1-\alpha_N\right) Ep_X\right)$$

[3.21]

$$B(T, ET_0) = \alpha_X \ln\left(\frac{ET_0}{T}\right) + \alpha_N \ln\left(\frac{1-T}{1-ET_0}\right) + \left(\alpha_X - \alpha_N\right)\ln\left(\frac{\left(1-\alpha_N\right)\left(1-T\right)+\left(1-\alpha_X\right)T}{\left(1-\alpha_N\right)\left(1-\mathrm{E}T\right)+\left(1-\alpha_X\right)\mathrm{E}T}\right)$$

[3.22]

$$C\left(p_X, Ep_X\right) = p_X - Ep_X \quad [3.23]$$

Where:

$$B_E = \alpha_N(1-\kappa_N)\left(1-\alpha_X\right) - \alpha_X(1-\kappa_X)\left(1-\alpha_N\right)$$

$$B_S = \alpha_X\kappa_X\left(1-\alpha_N\right) - \alpha_N\kappa_N\left(1-\alpha_X\right)$$

$$B_X = \left(1-\alpha_N\right)\left(a_X + \alpha_X \ln\alpha_X + \left(1-\alpha_X\right)\ln(1-\alpha_X) + \alpha_X\left(\kappa_X \ln\kappa_X + (1-\kappa_X)\ln(1-\kappa_X)\right)\right)$$

$$B_N = \left(1-\alpha_X\right)\left(a_N + \alpha_N \ln\alpha_N\left(1-\alpha_N\right)\ln(1-\alpha_N) + \alpha_N\left(\kappa_N \ln\kappa_N + (1-\kappa_N)\ln(1-\kappa_N)\right)\right)$$

$$\ln r_E = \ln(r + \delta_E)$$

$$\ln r_{S,\mathrm{E}t} = \ln(r + \delta_S + E\dot{p}_{N,t})$$

[3.24]

The expression for the log of the relative price for non-traded goods in [3.20] brings together the key elements that will allow us to describe the movement of the real exchange rate.

The first terms $A(r, E\dot{p}_{N,t}, p_{N,t-1}, E\dot{p}_X)$ [3.21], describe the underlying technology and cost of capital parameters that in turn define the underlying real exchange rate. They are akin to the capital intensity and productivity issues identified by Balassa (1964), Samuelson (1964) and Bhagwati (1984).

It also includes the role of the previous relative price of non-traded goods ($B_S p_{N,t-1}$) which is incorporated through the rental capital cost of structures in the production optimisation problem. This term (along with $B_S r_{S,\mathrm{E}t}$) ensures that the relative price of non-traded goods (and therefore the real exchange rate) will have a 'memory' in the medium term and does not immediately revert to parity after shocks. This is a key component of our solution to the Purchasing Power Parity Puzzle. There is also a role for expected changes in the terms of trade.

The second term $B(T, ET_0)$ [3.22], is a function of the actual T and expected ET and identifies the role of surprises to the export share in shifting the relative price of non-traded goods. When T = ET the function is zero, so with perfect expectations, there is no role for the demand side of the economy. This is another component of our solution to the Purchasing Power Parity Puzzle as it helps explain volatility in the real exchange rate.

Lastly the third term $C\left(p_X, Ep_X\right)$ [3.23] relates to the relative price of the exportable good, the commodity terms of trade. There is a slightly different role for the expected terms of trade to surprises changes in the terms of trade, depending on whether the exportable or non-tradeable goods sector are more labour intensive ($1-\alpha_X > 1-\alpha_N$). This means that there is potentially a transitory role for surprises or shocks in the terms of trade, as well as a permanent role for the terms of trade in the underlying real exchange. The terms of trade have previously been identified empirically as having a role in determining real exchange rates by for example, Chen and Rogoff (2003).

Starting from [3.21] and setting expectations equal to actuals so that the transitory terms [3.22] and [3.23] are zero; we can describe the 'underlying real exchange rate' in the following manner:

$$\overline{q} = \frac{\beta}{B_P}\left(\left(1-\alpha_N\right) p_X + B_S \ln r_S - B_E \ln r_E + B_X - B_N\right)$$
$$where:$$
$$B_P = \left(1-\alpha_N\right)\left(\alpha_X \kappa_X\right) + \left(1-\alpha_X\right)\left(1-\alpha_N \kappa_N\right) = \left(1-\alpha_X\right) + B_S \quad [3.25]$$

The coefficient β indicates the weight of non-traded goods in the consumers utility function and consumer price index (Obstfeld and Rogoff 1996).

We can then describe the expected change in the real exchange rate based on the 'underlying real exchange rate' $\overline{q}$ and a rate of return coefficient B_3, along with surprises in the terms of trade and export share:

$$\Delta q \approx B_1 \tilde{p}_X - B_2 \ln \tilde{T} - B_3\left(q_{t-1} - \overline{q}\right)$$
$$where:$$
$$B_1 \approx \beta$$
$$B_2 \approx \frac{\left(1-\alpha_N\right)\alpha_X + \mathrm{E}T_0\left(\alpha_N - \alpha_X\right)}{\left(1-\mathrm{E}T_0\right)\left(\left(1-\alpha_N\right) + \mathrm{E}T_0\left(\alpha_N - \alpha_X\right)\right)}$$
$$B_3 \approx \beta \frac{B_P}{B_S}\left(r + \delta_S\right) \quad [3.26]$$
$$\Delta q = q_t - q_{t-1}$$
$$\ln \tilde{T} = \ln \frac{\hat{T}}{\mathrm{E}T_0}$$
$$\tilde{p}_X = \ln \frac{\hat{P}_X}{\mathrm{E}P_X}$$

The coefficient on the terms of trade B_1 assumes that all terms of trade innovations ($\tilde{p}_X$) are surprises. Where this is not the case, the coefficient with respect to changes in the terms of trade will be that of the underlying real exchange rate $\beta \frac{1-\alpha_N}{B_P}$ in equation [3.25]. The coefficient on the export share surprises B_2 utilises a 1st order Taylor linearization of the relationship in [3.22]. The coefficient on the adjustment process B_3 represents the process of adjustment back to long-run equilibrium after shocks, it is determined based on a transformation of the relationship between the rate of change of the price of non-traded goods ($\dot{p}_{N,t}$) from [3.21] and [3.24] as a Bernoulli equation and solved for the dynamic form (see Appendix 3).

Equation [3.26] is clearly amenable to empirical analysis, based on a two step process involving identification of the underlying real exchange rate via equation [3.25] and estimation of the surprise components relating to the export share and/or the terms of trade.

Equation Section (Next)

4 Empirical Analysis

Two empirical approaches were used to test the validity of the neo-classical dependent economy model for the Australian real exchange rate. The first involves application of a two-step ordinary least squares approach to the model in [3.26] testing whether the data fits the model in its purest theoretical form. The second tests the data for cointegration, in which two cointegrating vectors are discovered. From this we create a vector error correction model. The cointegration and vector error correction approach avoids making assumptions with respect to endogenous variables and serial correlation which are required for the direct two step OLS approach and is therefore more robust. Both approaches provide similar parametric results and support the validity of the theory.

Properties of the data set

Quarterly data is drawn from the IMF International Financial Statistics (IFS) database for Australia over the quarters Q1 1980 to Q2 2005. This is the period during which a Real Effective Exchange Rate (REEF) index has been published in IFS for Australia

(*IFS Series Code 193..RECZF...*). As a result of using the REEF for *q*, the real exchange rate is a multi-lateral trade weighted index. This is in contrast to many studies that undertake bilateral exchange rate investigations, utilising the US dollar or German Deutschemark as a reference currency.

For other variables:

- The terms of trade (P_X) are determined as the ratio of the Export Price Index (*IFS Series Code 19376...ZF...*) to the Import Price Index (*IFS Series Code 19376.X.ZF...*) published in IFS.
- The export share, (*T*) is the export of goods and services in domestic currency (*IFS Series Code 19390C.CZF...*) divided by the gross domestic product in domestic currency (*IFS Series Code 19399B.CZF...*).
- The Real Interest Rate (*r*) is calculated utilising the Medium-Term Treasury Bond rate (*IFS Series Code 19361A..ZF...*) less the actual change in Consumer Price Index (*IFS Series Code* 19364...ZF...) for the following 4 quarters, assuming 12 months perfect inflation foresight.

For econometric calculations these time-series are converted into natural logarithms.

Eviews 5.01 is the econometric package used to analyse the data. In Eviews the following codes are used to represent the 4 time-series:

AUSLNRER – The log of the real exchange rate for Australia, corresponding to *q*.

AUSLNXY – The log of the export share (X/Y) for Australia, corresponding to *T*

AUSLNTOT – The log of the terms of trade for Australian, corresponding to P_X

AUSLN_MR – The log of the real medium-run interest rate for Australia, corresponding to *r*.

The time-series for the real exchange rate and real interest rate are shown graphically in Chart 5.1 and for the terms of trade and export share in Chart 5.2.

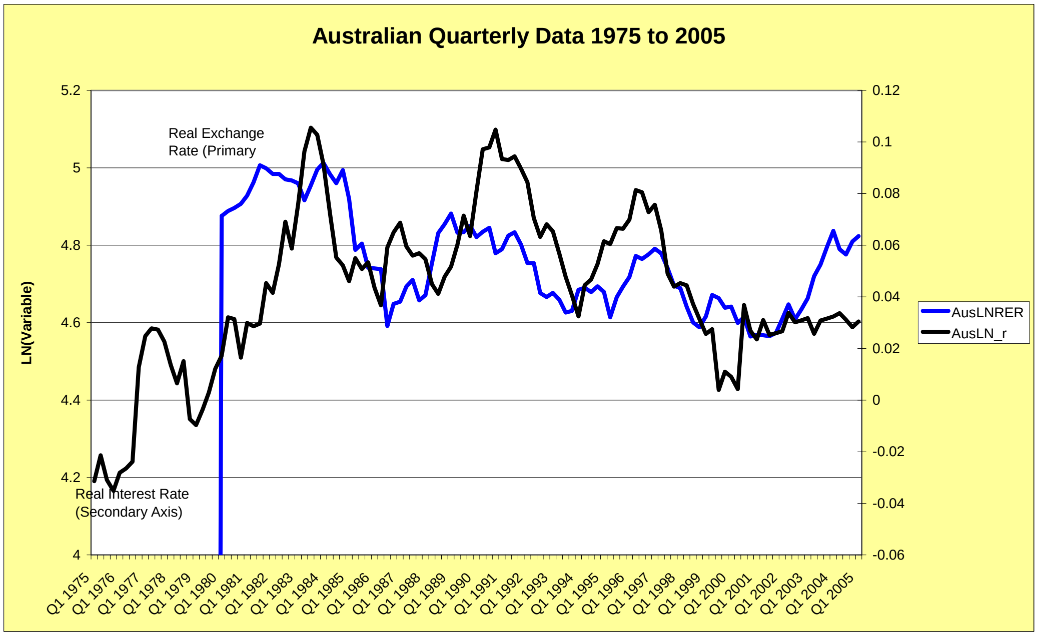

Chart 5.1 Australian Real Exchange Rate and Log Real Interest Rate Q1 1980 to Q2 2005

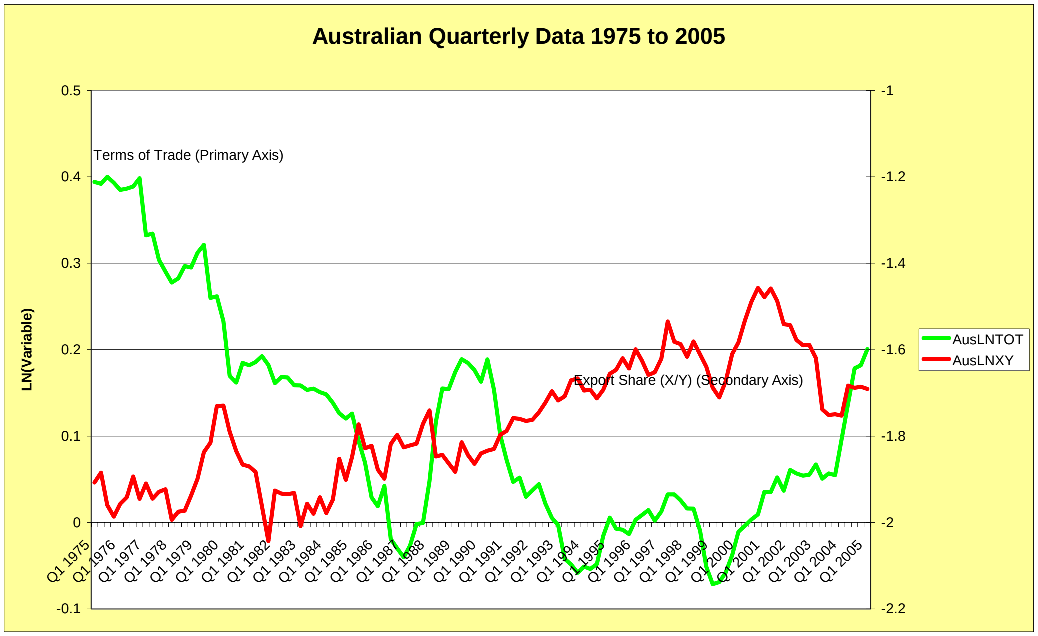

Chart 5.2 Australian Log Export Share (LNXY) and Log Terms of Trade (LNTOT) Q1 1980 to Q2 2005

When an individual time-series is non-stationary, OLS regression may result in spurious results. Non-stationary series which are cointegrated enables super-consistent estimation of the cointegrating vector using OLS regression.

Analysis of the 4 individual time-series indicates a strong possibility that the time-series are non-stationary (utilising KPSS and Augmented Dickey Fuller tests in Table 4.1). The null hypothesis of the Augmented Dickey Fuller test is a unit root (non-stationarity), while the null hypothesis for the Kwiatkowski-Phillips-Schmidt-Shin (1992) (KPSS) is that the series is stationary. Cointegration testing indicates that there are two cointegrating vectors (see Table 4.2) using 8 lags in estimation.

Table 4.1 Unit Root Tests on Australian Time-series

Test Statistic	Sample	AuslnRER	AuslnTOT	Auslnxy	AUSLN_MR
Augmented Dickey Fuller	1984:1 to 2005:2	-2.584	-1.112	-2.052	-2.351
KPSS		0.453 *	0.187	1.014 ***	1.010 ***

*** - significant at the 1% confidence level
* - significant at the 10% confidence level

Table 4.2 Trace Test of 4 Time-series 1985-2005 with 8 lags
Unrestricted Cointegration Rank Test (Trace)

Hypothesized No. of CE(s)	None *	At most 1 *	At most 2	At most 3
Eigenvalue	0.32*	0.246*	0.095	0.078
Trace Statistic	69.556*	37.968*	14.836	6.66

* denotes rejection of the hypothesis at the 0.05 level

A common response to non-stationarity is to estimate the equation in first difference form (as the series are stationary in first differences), which is consistent with the theoretical model in [3.26].

An alternative question relates to the consistency of the time-series date generation process over time and whether it is subject to structural breaks. For Australia this may be caused by de-regulation of the nominal exchange rate and nominal interest rates in the early 1980s.

Zivot and Andrews (1992) test builds on the Perron (1990) test by utilising iterative methods, thereby avoiding the necessity of pre-determining the break-point. The null hypothesis of the Zivot and Andrews test is a unit root without any exogenous change in the process, with the alternative hypothesis being a single unknown break around a trend stationary process.

Table 4.3 shows the results of sequential testing on the 4 time series. Critical values are taken from Zivot and Andrews (1992), who derived the values based on Monte Carlo simulation. The test statistic is a t-stat, with critical values determined by reference to the timing of the break relative to the total sample size (Tb/T). The results indicate that the null hypothesis of no structural break can be rejected for both the real exchange rate and the real interest rate. Both structural breaks appear to be associated with financial reform and exchange rate de-regulation, and the latter is used to define an unbroken series for analysis starting from the first quarter of 1995.

Table 4.3 Zivot and Andrew Test Statistics

Series	Code:	t-stat	Tb/T	Lags	TbTstat
Log Real Exchange Rate	auslnrer	-4.519**	0.331	11	1984Q4
Log Export Share	auslnxy	-3.125	0.570	9	1992Q1
Log Real Interest Rate	ausln_mr	-4.266**	0.174	3	1980Q1
Log Terms of Trade	auslntot	-2.285	0.223	3	1981Q3

** Significant at the 5% confidence level

Now we turn to estimating the model from Section 3 utilising a two step OLS process that tests the model in its purest theoretical form.

OLS Regression Results

The OLS regression results are reported in Table 4.4 for the underlying real exchange rate (corresponding to equation [3.25] and estimated as equation [4.1]) and Table 4.5 for the change in the real exchange rate (corresponding to equation [3.26] and estimated as equation [4.2]) which uses the parameter estimates from Table 4.4 as an input. In Table 4.5 and equation [4.2], the surprise or shock components for the export share and terms of trade assume static expectations are based on the first order change in these variables. We start by estimating the underlying real exchange rate:

$$\begin{array}{ll} \textit{Estimated Equation} & q_t = A_0 + A_1 p_{X,t} + A_2 r_t + \varepsilon_{Q,t} \\ \textit{where}: & \\ \varepsilon_{Q,t} = \rho \cdot \varepsilon_{Q,t-1} + u_t & \end{array} \qquad [4.1]$$

Table 4.4: Regressing the Underlying Real Exchange Rate $\overline{q}$ Equation [4.1]

Variable	Sample	C	AUSLNTOT	AUSLN_MR	AR(1)	Adjusted R-squared
Ordinary Least Squares		A_0	A_1	A_2	ρ	
Structural # Break	1985:1 to 2005:2	4.638 *** (0.022)	0.666 *** (0.142)	0.906 * (0.490)	0.722 *** (0.077)	0.887
Diagnostic Tests	White's Heteroscedasticity		Breusch-Godfrey Serial LM		Durbin-Watson	Normality
	F-stat	Prob	F-stat	Prob	Statistic	Jacques-Bera
Statistics	0.434	0.824	1.200	0.307	1.755	3.639

*** - significant at the 1% confidence level
** - significant at the 5% confidence level
* - significant at the 10% confidence level
# Standard Errors (in parentheses) are White Heteroscedasticity-Consistent Standard Errors

The results in Table 4.4 indicate that all the variables are significant at the 10% level of confidence and all except the real interest rate parameter are significant at the 1% level. The lower significance level (10%) with respect to parameter of the real interest rate is consistent with this parameter being derived from the difference in capital intensity measure between the exportable and non-traded sectors ($\alpha_X - \alpha_N$).[5] Diagnostic tests do not indicate any issues of significant concern.

Due to the non-stationarity identified in Table 4.1, the results in Table 4.4 may be spurious. However this is not the case if equation [4.1] corresponds to a cointegrating vector of the system, in which case OLS estimation is super-consistent (Davidson and Mackinnon 1993). The first cointegrating vector is estimated in Table 4.6, and is very similiar to the estimation in Table 4.4, indicating that this result is not spurious.

[5] Due to the Cobb-Douglas functional form, the production coefficients/exponents determine both the elasticity of output with respect to changes in inputs and the intensity of capital utilisation in an economic optimum.

Using the estimation of the from Table 4.4 we then estimate the change in the real exchange rate corresponding to equation [3.26] in the previous section. By estimating in first difference form we address the issue of non-stationarity identified in Table 4.1.

$$\begin{aligned} &\Delta q_t = C_1 \hat{\tilde{P}}_{X,t} + C_2 \hat{\tilde{T}} - C_3 q_{t-1} + C_4 \hat{\tilde{q}}_{t-1} + \varepsilon_t \\ &\textit{where } H_0 : \\ &C_3 = C_4 \end{aligned} \qquad [4.2]$$

Table 4.5 Regressing Changes in the Real Exchange Rate $\Delta\hat{q}$ Equation [4.2]

Variable		D(AUSLNTOT)	D(AUSLNXY)	AUSLNRER (-1)	Underlying RER (-1)	Adjusted R-squared
Ordinary Least Squares		C_1	C_2	$-C_3$	C_4	
Structural Break	1985:1 to 2005:2	0.738 *** 0.142	-0.503 *** 0.096	-0.149 ** 0.064	0.149 ** 0.064	0.456
		White's Hetero-skedasticity Test:	Breusch-Godfrey LM Test	Durbin's Serial CorrelationTests		Normality Test:
Diagnostic Tests		F-statistic	F-statistic	Durbin-Watson Stat	Durbin's h Statistic	Jacques-Bera Statistic
Statistic		1.242	1.593	1.700#	1.734*	1.164
Probability		[0.267]	[0.210]	(1.54/1.75)	[0.082]	[0.559]

*** - significant at the 1% confidence level
** - significant at the 5% confidence level
* Null hypothesis ($\rho = 0$) rejected at 10% confidence interval
# No conclusion able to be drawn

Table 4.5 shows the estimated coefficients using Ordinary Least Squares for the change in the real exchange rate. Estimated coefficients correspond to equation [4.2]. Coefficients for the terms of trade and the change in the export share were significant in both cases. The estimates of the parameter for the change in the terms of trade was close to that in Table 4.4 for the terms of trade in estimating the underlying real exchange rate (which excluded export shares but include real interest rates). A difference in these parameters requires that both a significant proportion of terms of variation was a surprise and that there was a significant difference in the capital intensity between the two sectors (as measured by ($\alpha_X - \alpha_N$)).

The coefficients C_3, C_4 in Table 4.5, relating to the impact of the underlying the real exchange rate component and the previous period real exchange rate component, sum to 1. This was not pre-specified and indicates that the system effectively describes an error-correction framework that reverts to the underlying real exchange rate over time. The rate of reversion indicated was statistically significant when estimated over the period 1985 to 2005. The estimated coefficient for the sample was 0.149, implying that approximately 15% of the difference between the transitory real exchange rate and the underlying real exchange rate should disappear each quarter, subject to no further surprises in the export share.

A rate of mean-reversion of 15% *per quarter* compares to the mean reversion rates of 15% *per year* noted by Rogoff (1996) in his Purchasing Power Puzzle, albeit with a moving target in the underlying real exchange rate. This is substantially faster, to be potentially consistent with the nominal rigidities that were thought to cause purchasing power parity deviation in Rogoff (1996). However it should be noted that the neo-classical dependent economy model provided in Section 3 does not require nominal rigidities for there to be a delayed reversion to (quasi-)purchasing power parity, instead requiring consistency between capital returns adjusted for price changes and international interest rates. The result of a faster rate of mean reversion when the expected exchange rate incorporates the terms of trade is consistent with the results of Chen and Rogoff (2003) for Australian data.

Diagnostic tests provide limited evidence of serial correlation in the residuals. No evidence was identified by the Breusch-Godfrey serial LM test or the Durbin-Watson statistic. A null hypothesis of no serial correlation was rejected by the Durbin h

statistic at a 10% level of confidence, but could not be rejected at the 5% significance level.

Cointegration and Vector Error Correction Approach

A vector error correction approach allows for endogeneity between the export share, the terms of trade, the real interest rate and the real exchange rate, as well as for serial correlation through the use of lagged variables. Identification of the cointegrating vector with the underlying exchange rate supports the consistency of the estimate in Table 4.4, overcoming the potential for non-stationarity leading to a spurious regression.

From the cointegration test in Table 4.2, we estimate the parameters of the cointegrating vectors as a vector error correction model of the form in equation [4.3]

$$\Delta X_t = \alpha\beta' + \sum_{i=1}^{k-1} \Gamma_i \Delta X_{t-i} + \varepsilon_t \qquad [4.3]$$

The α represents the speed of adjustment back to the underlying cointegration vectors represented by β. Γ_i indicates the error correction components associated with previous changes in the X vector. Table 4.6 shows the estimate of the cointegrating vector (β).

Table 4.6 Cointegrating Vectors (β) 1985-2005

	Coint 1	**Coint 2**
Real Exchange Rate	1	0
Export Share	0	1
Real interest Rate	**-1.775 *****	**5.044 *****
Terms of Trade	**-0.552 ****	**1.179 *****
C	**-4.611 *****	**1.404 *****

*** - significant at the 1% confidence level
** - significant at the 5% confidence level

The two cointegrating vectors are normalised with respect to two of the time-series to create the expressions in Table 4.6. Of the six possible combinations for normalisation, the approach which normalises with respect to the real exchange rate and export share is most useful as, unlike other combinations, each variable is strongly significant. This provides empirical support for the concept of the 'underlying real exchange rate' as one of the cointegrating vectors of the relationship between the real exchange rate, export share, real interest rate and terms of trade representing long-run supply conditions. Parameters (with the exception of the real interest rate) are similar to those identified in Table 4.4. The second cointegrating vector combines the export share, real interest rate and terms of trade, and could represent a long-run demand relationship.

Table 4.7 Speed of Adjustment (α) of time-series 1985-2005

	CointEq1	CointEq2	Rsquared	Adj. Rsquared
D(RER)	**-0.423 *****	**-0.278 *****	68.3%	46.5%
D(XY)	**0.474 *****	0.134	50.9%	17.1%
D(MR)	**0.069 ****	-0.014	62.6%	36.8%
D(TOT)	-0.027	**-0.215 *****	71.8%	52.3%

*** Significant at the 1% threshold of significance
** Significant at the 5% threshold of significant

Table 4.7 reports the estimates of the adjustment coefficients *α*. The first cointegrating equation is significant for the real exchange rate, export share and real interest rate, but not the terms of trade. When the cointegrating equation is normalised by the real exchange rate and the export share, then the value of the first vector represents the deviation from the underlying real exchange rate, therefore the negative sign of D(RER) with respect to 'CointEq1' represents the speed of adjustment back to equilibrium. The positive linkage with the real interest rate is consistent with expectations of a future depreciation being associated with both real exchange rates and real interest rates above underlying levels.

The second cointegrating equation is significant for predicting changes in the real exchange rate and terms of trade, but not the real interest rate or export share. This is interesting as the second vector integrated the export share, terms of trade and real interest rate, in a possible demand relationship so that the effect on the real exchange rate could represent changes in the export share, or predictable components associated with expectations of future terms of trade movement. Both of these hypotheses are consistent with the model described in Section 3 in equations [3.26] and [3.25].

The responses of the real exchange rate and other cointegrating variables to previous changes (representing the $\sum_{i=1}^{k-1} \Gamma_i \Delta X_{t-i}$ component from equation [4.3]) are reported in Table 4.8A to 4.8D in Appendix 4. Results were generally mixed, with only 25 out of 128 parameters statistically significant at the 10% level of confidence - only twice what would be expected based on chance alone. Highly significant responses (at the 1% level of confidence) occurred on 5 occasions. Four related to the response of the change in series to the real interest rate, where the real exchange rate and terms of trade react significantly to the 1 period delayed real interest rate change and the export share and real interest rate responded significantly to the 7 quarter and 4 quarter delayed responses respectively (see Table 4.8C). The other strongly significant response (1% threshold of significance) occurred between the eight quarter delayed export share and the terms of trade.

The overall effectiveness of the regressions is indicated by the adjusted r-squared measure in Table 4.7. The performance of the real exchange rate model was similar to the OLS result in Table 4.5 with an adjusted r^2 of 46.5%. The export share performed poorly with only 17% of the change correlated with the model. The real interest rate and terms of trade both performed relatively well at 36.8% and 52.3% respectively on an adjusted r^2 basis.

5 Conclusion

The Purchasing Power Parity Puzzle of Rogoff (1996) has two aspects: a high level of short-term real exchange rate volatility combined with a relatively slow rate of mean reversion. To this is added a further question as to whether the long-run real exchange rate should be static or evolve over time (Hegwood & Papell, 2002). Obstfeld and Rogoff (2000) suggest that a solution to the puzzle lies in the distinction between traded and non-traded goods. Starting from the two capital goods dependent economy structure proposed by Bruno (1976) and Brock and Turnovsky (1994) we include a short-run restriction on capital adjustment and the terms of trade following Salter (1959) and Swan (1960) to extract a solution to the Purchasing Power Parity Puzzle in which surprises in the terms of trade or export share result in transitory deviations from purchasing power parity which decay at a rate dependent on the relative intensity of the utilisation of structures (non-tradeable capital) in the production of Exportable and Non-tradeable goods. This rate of decay is determined by the necessity of avoiding arbitrage opportunities between different forms of capital. We identify an 'underlying real exchange rate' which incorporates productivity parameters consistent with Balassa (1964) and Samuelson (1964) as well as the terms of trade.

Applying this theoretical structure to Australian data for the period 1985 to 2005, the model was tested based on a two step ordinary least squares model and as a cointegrated system. The former supported the role of the terms of trade and export share as a source of deviation from the underlying real exchange rate, with the underlying real exchange rate identified as a cointegrating vector in the cointegrated system. The error correction component of the model indicates a rate of decay that was faster than the slow rate which inspired Rogoff's puzzle. Instead it supports Hegwood & Papell (2002) and Chen and Rogoff (2003) where movements in the underlying real exchange rate are accompanied by faster rates of decay.

The following fundamental conclusion applies: the dependent-economy model with traded and non-traded capital identifies sources of deviations from purchasing price parity and the rate of decay of real exchange rate deviations, thus solving the Purchasing Power Parity Puzzle.

A forthcoming paper will expand the set of countries to be empirically analysed to other potential dependent economies.

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Appendix 1: Demand Solution Equations

The following equations define the demand or utility maximisation side of solving the problem. Except for the avoidance of arbitrage on the holding of capital stocks, they are not required for the supply-side solution which can be derived based on cost-minimisation.

$$\int_0^\infty U\left(C_M, C_N\right) \cdot e^{-\beta t} dt \qquad \text{[A1.4]}$$

$$H = U\left(C_M, C_N\right) \cdot e^{-\beta t} + \lambda_B \cdot \dot{B} + \lambda_S \cdot \dot{S} + \lambda_E \cdot \dot{E} \qquad \text{[A1.5]}$$

$$\dot{B} = \tau + P_X \cdot F(E_X, S_X, L_X) + rB - C_M - I_E + P_N \cdot \left(G(E_N, S_N, L_N) - C_N - I_S\right) \qquad \text{[A1.6]}$$

$$G(E_N, S_N, L_N) - C_N - I_S = 0 \qquad \text{[A1.7]}$$

Equation [A1.4] represents the utility function, based on consumption of imported goods (CM) and non-traded goods with a rate of time preference β.

Equation [A1.5] is a Hamiltonian presentation of the inter-temporal optimisation problem which can be solved applying Pontryagin's Maximum Principle. The presentation identifies 3 state variables: B for foreign bonds, S for the stock of structures and E for the stock of Equipment. Explicit identification enables the equality of the return on capital to be demonstrated.

Equation [A1.6] is the instantaneous budget constraint where income, comprised of international transfers (τ), income from production of exportable goods ($P_X \cdot F(E_X, S_X, L_X)$), income from foreign bonds (rB) and income from production of non-tradeable goods ($P_N \cdot G(E_N, S_N, L_N)$), can be expended on consumption of importable and non-tradeable goods (C_M, C_N), investment in equipment (I_E) or structures (I_S), with any surplus or deficit representing a change in foreign bond holdings ($\dot{B}$). Consumption of tradeable goods are denominated in units of importable goods, with a unitary price, while non-tradeable goods in production, consumption or investment have price (P_N). Production of tradeable goods is denominated in exportable goods which reflect the economies comparative advantages with price (P_X) equal to the terms of trade.

The final equation [A1.7] ensures that all non-traded goods produced are either consumed or invested, with net exports or net imports ($\dot{B}$) comprised entirely by tradeable goods.

Equation Chapter 1 Section 1

Appendix 2: Mathematical solution

Starting from a cost-minimising constrained optimisation we can define the input usage relationships as follows:

$$Z = wL_X + R_E E_X + R_S S_X - P_X \left(Y_X - A_X E_X{}^{\alpha_X(1-\kappa_X)} S_X{}^{\alpha_X \kappa_X} L_X{}^{1-\alpha_X} \right) \qquad [A2.1]$$

$$P_X Y_X = \frac{wL_X}{1-\alpha_X} = \frac{R_E E_X}{\alpha_X (1-\kappa_X)} = \frac{R_S E_S}{\alpha_X \kappa_X} \qquad [A2.2]$$

And similarly

$$P_N Y_N = \frac{wL_N}{1-\alpha_N} = \frac{R_E E_N}{\alpha_N (1-\kappa_N)} = \frac{R_S E_S}{\alpha_N \kappa_N} \qquad [A2.3]$$

Using $L_N + L_X = 1$ and defining the total economy $Y_t = P_X Y_X + P_N Y_N$ and the ratio shares by $T: P_X Y_X = TY_t,\ P_N Y_N = (1-T) Y_t$ we can then describe real wages as a function of *T*.

$$\begin{aligned} L_N &= \frac{(1-\alpha_N)(1-T)Y_t}{w}, L_X = \frac{(1-\alpha_X)TY_t}{w} \\ &\left((1-\alpha_N)(1-T) + (1-\alpha_X)T\right)Y_t = w \end{aligned} \qquad [A2.4]$$

From [3.16] we can create an expression for the share of labour that is purely relative to *T* and the productivity variables, with the Y_t cancelling.

$$L_X = \frac{(1-\alpha_X)T}{\left((1-\alpha_N)(1-T) + (1-\alpha_X)T\right)} \qquad [A2.5]$$

$$L_N = \frac{(1-\alpha_N)(1-T)}{\left((1-\alpha_N)(1-T) + (1-\alpha_X)T\right)} \qquad [A2.6]$$

We can also describe real wages w with respect to the marginal productivity of the production function:

$$w = (1-\alpha_X) P_X A_X E_X{}^{\alpha_X(1-\kappa_X)} S_X{}^{\alpha_X \kappa_X} L_X{}^{-\alpha_X} = (1-\alpha_N) P_N A_N E_N{}^{\alpha_N(1-\kappa_N)} S_N{}^{\alpha_N \kappa_N} L_N{}^{-\alpha_N}$$

[A2.7]

This can then be rearranged to create an expression for the relative price of non-traded goods P_N as a function of P_X the capital stocks and *T*.

$$P_N = P_X \frac{(1-\alpha_X)^{1-\alpha_X}}{(1-\alpha_N)^{1-\alpha_N}} \frac{A_X E_X{}^{\alpha_X(1-\kappa_X)} S_X{}^{\alpha_X \kappa_X}}{A_N E_N{}^{\alpha_N(1-\kappa_N)} S_N{}^{\alpha_N \kappa_N}} \frac{T^{-\alpha_X}}{(1-T)^{-\alpha_N}} \left((1-\alpha_N)(1-T) + (1-\alpha_X)T\right)^{\alpha_X - \alpha_N}$$

[A2.8]

This expression defines the relative price of non-traded goods in the short-term.

For the long-run we can go back to the expressions [3.13] and [3.14] using the following definitions for R_S and R_E[6].

$$R_S = P_N(r+\delta_S) - \dot{P}_N = P_N r_S$$
$$r_S = r + \delta_S + \frac{\dot{P}_N}{P_N} \qquad \text{[A2.9]}$$

$$R_E = r_E = r + \delta_E \qquad \text{[A2.10]}$$

Thus capital stocks (which will be based on the prior expectations of T and Y_t, denoted ET_0 and EY_t respectively) are as follows:

$$E_X = \frac{\alpha_X(1-\kappa_X)ET_0EY_t}{r+\delta_E} \qquad E_N = \frac{\alpha_N(1-\kappa_N)\left(1-ET_0\right)EY_t}{r+\delta_E}$$
$$S_X = \frac{\alpha_X\kappa_X ET_0EY_t}{P_{N,t-1}\left(r+\delta_S\right)+E\dot{P}_{N,t}} \qquad S_N = \frac{\alpha_X\kappa_X\left(1-ET_0\right)EY_t}{P_{N,t-1}\left(r+\delta_S\right)+E\dot{P}_{N,t}} \qquad \text{[A2.11]}$$

We then substitute the relationships from [A2.11] into [A2.8]

$$P_N = \frac{P_X \dfrac{\left(1-\alpha_X\right)^{1-\alpha_X}}{\left(1-\alpha_N\right)^{1-\alpha_N}} \dfrac{A_X\left(\dfrac{\alpha_X(1-\kappa_X)ET_0EY_t}{r+\delta_E}\right)^{\alpha_X(1-\kappa_X)}\left(\dfrac{\alpha_X\kappa_X ET_0EY_t}{P_{N,t-1}\left(r+\delta_S\right)+E\dot{P}_{N,t}}\right)^{\alpha_X\kappa_X}}{A_N\left(\dfrac{\alpha_N(1-\kappa_N)\left(1-ET_0\right)EY_t}{r+\delta_E}\right)^{\alpha_N(1-\kappa_N)}\left(\dfrac{\alpha_X\kappa_X\left(1-ET_0\right)EY_t}{P_{N,t-1}\left(r+\delta_S\right)+E\dot{P}_{N,t}}\right)^{\alpha_N\kappa_N}}}{} \times \frac{T^{-\alpha_X}}{\left(1-T\right)^{-\alpha_N}}\left(\left(1-\alpha_N\right)\left(1-T\right)+\left(1-\alpha_X\right)T\right)^{\alpha_X-\alpha_N}$$

[A2.12]

And then consolidate terms

[6] The capital cost of equipment is measured in units of the imported tradeable goods which is taken as the numeraire good ($P_M = 1$)

$$
P_N = \frac{P_X \frac{(1-\alpha_X)^{1-\alpha_X}}{(1-\alpha_N)^{1-\alpha_N}} \frac{A_X {\alpha_X}^{\alpha_X} (1-\kappa_X)^{\alpha_X(1-\kappa_X)} {\kappa_X}^{\alpha_X\kappa_X} (r+\delta_E)^{\alpha_N(1-\kappa_N) - \alpha_X(1-\kappa_X)} EY_t^{\alpha_X-\alpha_N}}{A_N {\alpha_N}^{\alpha_N} (1-\kappa_N)^{\alpha_N(1-\kappa_N)} {\kappa_X}^{\alpha_N\kappa_N} \left(P_{N,t-1}(r+\delta_S) + E\dot{P}_{N,t}\right)^{\alpha_X\kappa_X - \alpha_N\kappa_N}}}{} \\
\times \frac{ET_0^{\alpha_X}}{(1-ET_0)^{\alpha_N}} \frac{T^{-\alpha_X}}{(1-T)^{-\alpha_N}} \left((1-\alpha_N)(1-T) + (1-\alpha_X)T\right)^{\alpha_X-\alpha_N}
$$

[A2.13]

To solve the expected output EY we use the [3.16] expression linking output (Y) to the real wage (w) and then replace the real wage with the marginal productivity of labour in the export sector. The labour allocation to the export sector can be equated via the export share variable T.

$$
EY_t^{\alpha_X-\alpha_N} = \frac{\left((1-\alpha_X)\mathrm{E}P_X A_X {E_X}^{\alpha_X(1-\kappa_X)} {S_X}^{\alpha_X\kappa_X} \mathrm{E}{L_X}^{-\alpha_X}\right)^{\alpha_X-\alpha_N}}{\left((1-\alpha_N)(1-\mathrm{E}T) + (1-\alpha_X)\mathrm{E}T\right)^{\alpha_X-\alpha_N}}
$$

$$
\mathrm{E}L_X = \frac{(1-\alpha_X)\mathrm{E}T}{\left((1-\alpha_N)(1-\mathrm{E}T) + (1-\alpha_X)\mathrm{E}T\right)}
$$

[A2.14]

This expression is then simplified

$$
EY_t^{\alpha_X-\alpha_N} = \frac{(1-\alpha_X)^{(1-\alpha_X)(\alpha_X-\alpha_N)} \mathrm{E}T^{-\alpha_X(\alpha_X-\alpha_N)}}{\left((1-\alpha_N)(1-\mathrm{E}T) + (1-\alpha_X)\mathrm{E}T\right)^{(1-\alpha_X)(\alpha_X-\alpha_N)}} (\mathrm{E}P_X A_X)^{\alpha_X-\alpha_N} \left({E_X}^{\alpha_X(1-\kappa_X)} {S_X}^{\alpha_X\kappa_X}\right)^{\alpha_X-\alpha_N}
$$

The capital terms again replaced using [A2.11]

$$
EY_t^{\alpha_X-\alpha_N} = \frac{(1-\alpha_X)^{(1-\alpha_X)(\alpha_X-\alpha_N)} \mathrm{E}T^{-\alpha_X(\alpha_X-\alpha_N)}}{\left((1-\alpha_N)(1-\mathrm{E}T) + (1-\alpha_X)\mathrm{E}T\right)^{(1-\alpha_X)(\alpha_X-\alpha_N)}} (\mathrm{E}P_X A_X) \\
\times \left(\left(\frac{\alpha_X(1-\kappa_X)ET_0EY_t}{r+\delta_E} \right)^{\alpha_X(1-\kappa_X)} \left(\frac{\alpha_X\kappa_X ET_0EY_t}{P_{N,t-1}(r+\delta_S) + E\dot{P}_{N,t}} \right)^{\alpha_X\kappa_X} \right)^{\alpha_X-\alpha_N}
$$

$$
EY_t^{(\alpha_X-\alpha_N)} = \frac{(1-\alpha_X)^{(1-\alpha_X)(\alpha_X-\alpha_N)} \mathrm{E}T^{-\alpha_X(\alpha_X-\alpha_N)}}{\left((1-\alpha_N)(1-\mathrm{E}T) + (1-\alpha_X)\mathrm{E}T\right)^{(1-\alpha_X)(\alpha_X-\alpha_N)}} (\mathrm{E}P_X A_X)^{\alpha_X-\alpha_N} \\
\times \left(\left(\frac{(1-\kappa_X)}{r+\delta_E} \right)^{\alpha_X(1-\kappa_X)} \left(\frac{\kappa_{Xt}}{P_{N,t-1}(r+\delta_S) + E\dot{P}_{N,t}} \right)^{\alpha_X\kappa_X} \right)^{\alpha_X-\alpha_N} (\alpha_X ET_0 EY_t)^{\alpha_X(\alpha_X-\alpha_N)}
$$

$$
EY_t^{\alpha_X-\alpha_N} = \left(\begin{array}{l} \dfrac{(1-\alpha_X)\mathrm{E}{P_X}^{\frac{1}{1-\alpha_X}} {A_X}^{\frac{1}{1-\alpha_X}}}{\left((1-\alpha_N)(1-\mathrm{E}T) + (1-\alpha_X)\mathrm{E}T\right)} \\ \times \left(\alpha_X \left(\dfrac{(1-\kappa_X)}{r+\delta_E} \right)^{(1-\kappa_X)} \left(\dfrac{\kappa_{Xt}}{P_{N,t-1}(r+\delta_S) + E\dot{P}_{N,t}} \right)^{\kappa_X} \right)^{\frac{\alpha_X}{1-\alpha_X}} \end{array} \right)^{\alpha_X-\alpha_N}
$$

Leading to a single expression for the output component from [A2.13] that does not contain endogenous variables.

$$EY_t^{\alpha_X-\alpha_N}=\frac{\left(\begin{array}{l}\left(1-\alpha_X\right)^{1-\alpha_X}A_X\left(\alpha_X(1-\kappa_X)^{(1-\kappa_X)}\kappa_{Xt}{}^{\kappa_X}\right)^{\alpha_X}\\ \times\mathrm{E}P_X\left(\left(r+\delta_E\right)^{-(1-\kappa_X)}P_{N,t-1}{}^{-\kappa_X}\left(r+\delta_S+E\dot{P}_{N,t}\right)^{-\kappa_X}\right)^{\alpha_X}\end{array}\right)^{\frac{\alpha_X-\alpha_N}{1-\alpha_X}}}{\left(\left(1-\alpha_N\right)\left(1-\mathrm{E}T\right)+\left(1-\alpha_X\right)\mathrm{E}T\right)^{\alpha_X-\alpha_N}} \quad \text{[A2.15]}$$

Equation [A2.15] is then substituted back into [A2.13] and the expression further simplified.

$$P_N=\begin{array}{l}P_X\dfrac{\left(1-\alpha_X\right)^{1-\alpha_X}}{\left(1-\alpha_N\right)^{1-\alpha_N}}\dfrac{A_X\alpha_X{}^{\alpha_X}(1-\kappa_X)^{\alpha_X(1-\kappa_X)}\kappa_X{}^{\alpha_X\kappa_X}ET_0{}^{\alpha_X}\left(r+\delta_E\right)^{\alpha_N(1-\kappa_N)-\alpha_X(1-\kappa_X)}}{A_N\alpha_N{}^{\alpha_N}\left(1-\kappa_N\right)^{\alpha_N(1-\kappa_N)}\kappa_X{}^{\alpha_N\kappa_N}\left(1-ET_0\right)^{\alpha_N}\left(P_{N,t-1}\left(r+\delta_S\right)+E\dot{P}_{N,t}\right)^{\alpha_X\kappa_X-\alpha_N\kappa_N}}\\ \times\dfrac{T^{-\alpha_X}}{\left(1-T\right)^{-\alpha_N}}\left(\left(1-\alpha_N\right)\left(1-T\right)+\left(1-\alpha_X\right)T\right)^{\alpha_X-\alpha_N}\\ \times\dfrac{\left(\begin{array}{l}\left(1-\alpha_X\right)^{1-\alpha_X}A_X\left(\alpha_X(1-\kappa_X)^{(1-\kappa_X)}\kappa_{Xt}{}^{\kappa_X}\right)^{\alpha_X}\\ \times\mathrm{E}P_X\left(\left(r+\delta_E\right)^{-(1-\kappa_X)}P_{N,t-1}{}^{-\kappa_X}\left(r+\delta_S+E\dot{P}_{N,t}\right)^{-\kappa_X}\right)^{\alpha_X}\end{array}\right)^{\frac{\alpha_X-\alpha_N}{1-\alpha_X}}}{\left(\left(1-\alpha_N\right)\left(1-\mathrm{E}T\right)+\left(1-\alpha_X\right)\mathrm{E}T\right)^{\alpha_X-\alpha_N}}\end{array}$$

The expression can then be reformed into components comprising parameters, the export share (T), the relative price of exportable goods or Terms of Trade and the costs of capital for structures and equipment.

$$P_N=\begin{array}{l}\dfrac{\left(\left(1-\alpha_X\right)^{1-\alpha_X}A_X\left(\alpha_X(1-\kappa_X)^{(1-\kappa_X)}\kappa_{Xt}{}^{\kappa_X}\right)^{\alpha_X}\right)^{1+\frac{\alpha_X-\alpha_N}{1-\alpha_X}}}{A_N\left(1-\alpha_N\right)^{1-\alpha_N}\alpha_N{}^{\alpha_N}\left(1-\kappa_N\right)^{\alpha_N(1-\kappa_N)}\kappa_X{}^{\alpha_N\kappa_N\alpha_N}}\\ \times\left(\dfrac{ET_0}{T}\right)^{\alpha_X}\left(\dfrac{1-T}{1-ET_0}\right)^{\alpha_N}\left(\dfrac{\left(1-\alpha_N\right)\left(1-T\right)+\left(1-\alpha_X\right)T}{\left(1-\alpha_N\right)\left(1-\mathrm{E}T\right)+\left(1-\alpha_X\right)\mathrm{E}T}\right)^{\alpha_X-\alpha_N}\\ \times P_X\mathrm{E}P_X{}^{\frac{\alpha_X-\alpha_N}{1-\alpha_X}}\\ \times\dfrac{\left(r+\delta_E\right)^{\alpha_N(1-\kappa_N)-\alpha_X(1-\kappa_X)\left(1+\frac{\alpha_X-\alpha_N}{1-\alpha_X}\right)}}{\left(P_{N,t-1}\left(r+\delta_S+E\dot{P}_{N,t}\right)\right)^{\alpha_X\kappa_X\left(1+\frac{\alpha_X-\alpha_N}{1-\alpha_X}\right)-\alpha_N\kappa_N}}\end{array} \quad \text{[A2.16]}$$

This can then be reformed into expected or permanent components and surprise components.

$$
\begin{aligned}
P_N = \; & \left(\begin{array}{l} \dfrac{\left(\left(1-\alpha_X\right)^{1-\alpha_X} A_X \left(\alpha_X (1-\kappa_X)^{(1-\kappa_X)} {\kappa_{Xt}}^{\kappa_X}\right)^{\alpha_X}\right)^{1-\alpha_N}}{\left(A_N \left(1-\alpha_N\right)^{1-\alpha_N} {\alpha_N}^{\alpha_N} \left(1-\kappa_N\right)^{\alpha_N (1-\kappa_N)} {\kappa_X}^{\alpha_N \kappa_N \alpha_N}\right)^{1-\alpha_X}} \\
\times \dfrac{\left(r+\delta_E\right)^{\alpha_N (1-\kappa_N)\left(1-\alpha_X\right)-\alpha_X (1-\kappa_X)\left(1-\alpha_N\right)}}{\left(r+\delta_S+E\dot{P}_{N,t}\right)^{\alpha_X \kappa_X (1-\alpha_N)-\alpha_N \kappa_N (1-\alpha_X)}} \\
\times {P_{N,t-1}}^{-\left(\alpha_X \kappa_X (1-\alpha_N)-\alpha_N \kappa_N (1-\alpha_X)\right)} \\
\times \mathrm{E}{P_X}^{\frac{1-\alpha_N}{1-\alpha_X}}
\end{array} \right)^{\frac{1}{1-\alpha_X}} \\
& \times {P_X}\big/{\mathrm{E}P_X} \\
& \times \left(\frac{ET_0}{T}\right)^{\alpha_X} \left(\frac{1-T}{1-ET_0}\right)^{\alpha_N} \left(\frac{\left(1-\alpha_N\right)\left(1-T\right)+\left(1-\alpha_X\right)T}{\left(1-\alpha_N\right)\left(1-\mathrm{E}T\right)+\left(1-\alpha_X\right)\mathrm{E}T}\right)^{\alpha_X-\alpha_N}
\end{aligned}
$$

This can then be expressed in log-form:

$$
\begin{aligned}
p_{N,t} = \; & \frac{1}{1-\alpha_X}\left(B_X - B_N + B_E \ln r_E - B_S \ln r_S - B_S p_{N,t-1} + \left(1-\alpha_N\right) Ep_X\right) \\
& + \left(\alpha_X \ln\left(\frac{ET_0}{T}\right) + \alpha_N \ln\left(\frac{1-T}{1-ET_0}\right) + \left(\alpha_X - \alpha_N\right) \ln\left(\frac{\left(1-\alpha_N\right)\left(1-T\right)+\left(1-\alpha_X\right)T}{\left(1-\alpha_N\right)\left(1-\mathrm{E}T\right)+\left(1-\alpha_X\right)\mathrm{E}T}\right)\right) \\
& + \left(p_X - Ep_X\right) \\
B_E & = \alpha_N (1-\kappa_N)\left(1-\alpha_X\right) - \alpha_X (1-\kappa_X)\left(1-\alpha_N\right) \\
B_S & = \alpha_X \kappa_X \left(1-\alpha_N\right) - \alpha_N \kappa_N \left(1-\alpha_X\right) \\
B_X & = \left(1-\alpha_N\right)\left(a_X + \alpha_X \ln \alpha_X + \left(1-\alpha_X\right) \ln(1-\alpha_X) + \alpha_X \left(\kappa_X \ln \kappa_X + (1-\kappa_X) \ln(1-\kappa_X)\right)\right) \\
B_N & = \left(1-\alpha_X\right)\left(a_N + \alpha_N \ln \alpha_N \left(1-\alpha_N\right) \ln(1-\alpha_N) + \alpha_N \left(\kappa_N \ln \kappa_N + (1-\kappa_N) \ln(1-\kappa_N)\right)\right) \\
\ln r_E & = \ln(r + \delta_E) \\
\ln r_S & = \ln(r + \delta_S + E\dot{P}_{N,t}) \\
F(T, ET_0) & =
\end{aligned}
$$

[A2.17]

The long-run can be established by setting the surprise components equal to the actuals and setting the current value equal to previous values:

$$
\begin{aligned}
& \overline{p}_N = \frac{1}{B_P}\left(\left(1-\alpha_N\right) p_X + B_S \ln r_S - B_E \ln r_E + B_X - B_N\right) \\
& where: \\
& B_P = \left(1-\alpha_N\right)\left(\alpha_X \kappa_X\right) + \left(1-\alpha_X\right)\left(1-\alpha_N \kappa_N\right) = \left(1-\alpha_X\right) + B_S
\end{aligned}
$$

[A2.18]

Equation Section (Next)

Appendix 3: Solving the Difference Equation

Part A: Differencing the medium and long-run to generate a difference equation:

Step 1 [from [3.21] and [3.25]]

$$\ln\left(r+\delta_S+E\dot{p}_t\right)=\left(\begin{array}{l}\frac{B_X-B_N}{B_S}+\\ \frac{1-\alpha_N}{B_\vartheta}Ep_X\\ +\frac{B_E}{B_S}\ln\left(r+\delta_E\right)\end{array}\right)-\frac{B_P}{B_S}\left(\hat{p}_N-\bar{p}_N+\left(\begin{array}{l}\frac{B_X-B_N}{B_P}\\ +\frac{1-\alpha_N}{B_P}Ep_X\\ +\frac{B_E}{B_P}\ln\left(r+\delta_E\right)\end{array}\right)-\frac{B_S}{B_P}\ln\left(r+\delta_S\right)\right)$$

$$\bar{p}_N=\ln\bar{P}_N=\frac{B_X-B_N}{B_P}+\frac{1-\alpha_N}{B_P}Ep_X+\frac{B_2}{B_P}\ln\left(r+\delta_E\right)-\frac{B_\vartheta}{B_P}\ln\left(r+\delta_S\right)$$

[A3.1]

Step 2

$$\ln\left(r+\delta_S+E\dot{p}_t\right)=-\frac{B_P}{B_S}\left(\hat{p}_N-\bar{p}_N\right)+\ln r_S \qquad \text{[A3.2]}$$

Step 3

$$\ln\left(\frac{r+\delta_S+\dot{\hat{P}}_N/\hat{P}_N}{r+\delta_S}\right)=-\frac{B_P}{B_S}\ln\left(\frac{\hat{P}_N}{\bar{P}_N}\right) \qquad \text{[A3.3]}$$

Step 4.

$$\frac{r+\delta_S+\dot{\hat{P}}_N/\hat{P}_N}{r+\delta_S}=\left(\frac{\hat{P}_N}{\bar{P}_N}\right)^{-\frac{B_P}{B_\vartheta}} \qquad \text{[A3.4]}$$

Step 5 (Final)

$$\dot{\hat{P}}_N/\hat{P}_N=\left(\left(\frac{\hat{P}_N}{\bar{P}_N}\right)^{-\frac{B_P}{B_S}}-1\right)\left(r+\delta_S\right)$$

[A3.5]

where:

$$B_P=\left(\left(1-\alpha_X\right)\left(1-\alpha_N\kappa_N\right)+\left(1-\alpha_N\right)\alpha_X\kappa_X\right)$$

$$B_S=\alpha_X\kappa_X\left(1-\alpha_N\right)-\alpha_N\kappa_N\left(1-\alpha_X\right)$$

Part B: Using the difference equation to create a dynamic equation

$$\dot{P} + \left(r+\delta_S\right)\hat{P}_N = \hat{P}_N^{\;1-\frac{B_P}{B_S}}\,\overline{P}_N^{\;\frac{B_P}{B_S}}\left(r+\delta_S\right) \qquad \text{[A3.6]}$$

This can then be solved into the dynamic form as a function of time of the following form.

$$P_N(t) = \left(\left(\hat{P}_N \cdot e^{-\left(r+\delta_S\right)t}\right)^{\frac{B_P}{B_S}} + \overline{P}^{\frac{B_P}{B_S}}\left(1 - e^{-\frac{B_P}{B_S}\left(r+\delta_S\right)t}\right)\right)^{\frac{B_S}{B_P}} \qquad \text{[A3.7]}$$

As follows: the general solution to a *Bernoulli* of the form[7]:

$$\begin{aligned}
&\frac{dP_N}{dt} + RP_N = TP_N^{\;m} \\
&where: \\
&R = r + \delta_S \\
&T = \overline{P}^{\frac{B_P}{B_S}} \cdot \left(r+\delta_S\right) \\
&m = 1 - \frac{B_P}{B_S}
\end{aligned} \qquad \text{[A3.8]}$$

This can then be transformed into the form[8]:

$$\begin{aligned}
&dz + \left[\left(1-m\right)Rz - \left(1-m\right)T\right]dt = 0 \\
&where: \\
&z = P_N^{\;1-m} = P_N^{\;\frac{B_P}{B_S}}
\end{aligned} \qquad \text{[A3.9]}$$

This can then be expressed in the format[9]:

$$\begin{aligned}
&dz + \left[uz - w\right]dt = 0 \\
&where: \\
&u = \left(1-m\right)R \\
&w = \left(1-m\right)T
\end{aligned} \qquad \text{[A3.10]}$$

Which then gives the dynamic form[10]:

---

[7] Chiang & Wainwright 2005, Eq (15.24) p 493
[8] Chiang & Wainwright 2005, Eq (15.24') p 494
[9] Chiang & Wainwright 2005, Eq (15.20) p 490
[10] Chiang & Wainwright 2005, Eq (15.21) p 491

$$z(t) = e^{-\int u dt}\left(A + \int w e^{\int u dt} dt \right) \quad [A3.11]$$

Replacing the variables u, w, R and T with their components gives:

$$z(t) = e^{-\int \frac{B_P}{B_S}(r+\delta_S)dt}\left(A + \int \frac{B_P}{B_S} \overline{P}^{\frac{B_P}{B_S}} \cdot \left(r + \delta_S \right) e^{\int \frac{B_P}{B_S}(r+\delta_S)dt} dt \right)$$

$$1 - m = \frac{B_P}{B_S} \quad [A3.12]$$

$$P_N(t)^{\frac{B_P}{B_S}} = A \cdot e^{-\frac{B_P}{B_S}(r+\delta_S)t} + \overline{P}^{\frac{B_P}{B_S}} \quad [A3.13]$$

$$A = \hat{P}_N^{\frac{B_P}{B_S}} - \overline{P}_N^{\frac{B_P}{B_S}} \quad [A3.14]$$

$$P_N(t) = \left(\left(\hat{P}_N \cdot e^{-(r+\delta_S)t} \right)^{\frac{B_P}{B_S}} + \overline{P}^{\frac{B_P}{B_S}} \left(1 - e^{-\frac{B_P}{B_S}(r+\delta_S)t} \right) \right)^{\frac{B_S}{B_P}} \quad [A3.15]$$

Dynamic adjustment based on starting value PN-hat and long-run value PN-bar

Appendix 4: Cointegration Lagged Variable Response Results

Table 4.8A Real Exchange Rate Responses 1985-2005

Real Exchange Rate Responses

	D(RER(1))	D(RER(2))	D(RER(3))	D(RER(4))	D(RER(5))	D(RER(6))	D(RER(7))	D(RER(8))
D(RER)	**0.390 ****	0.0938	**0.335 ****	0.2330	0.2350	**0.367 ****	**0.307 ***	0.0130
D(XY)	**-0.52 ****	-0.008	-0.135	-0.086	-0.305	-0.257	-0.121	-0.177
D(MR)	0.0129	-0.021	0.0274	0.0565	0.0024	0.0359	0.0034	0.0076
D(TOT)	0.0152	-0.026	0.1297	-0.038	0.1538	**0.182 ***	0.1285	-0.034

For the real exchange rate response there are positive correlations between previous and current changes that are significant at the 10% level for periods 5,6 and 7. The sum of these adjustments are greater than one, implying that the system may be unstable. The real interest rate appears to be unaffected by previous changes in the real exchange rate.

Table 4.8B Real Export Share Responses 1985-2005

Export Share Responses

	D(XY(1))	D(XY(2))	D(XY(3))	D(XY(4))	D(XY(5))	D(XY(6))	D(XY(7))	D(XY(8))
D(RER)	0.2420	0.1358	0.1271	**0.221 ***	-0.095	-0.093	**0.250 ***	0.0521
D(XY)	-0.298	-0.088	-0.005	-0.192	-0.079	-0.059	**-0.32 ****	-0.228
D(MR)	-0.020	-0.043	-0.036	0.0166	0.0492	-0.014	0.0466	0.0537
D(TOT)	0.1142	**0.156 ****	0.0794	0.1064	0.0682	0.0805	0.0916	**-0.21 *****

There are several econometrically significant responses to changes in the export share that occur in the final considered lags (see Table 4.7B). The sum of own-lagged adjustment (XY on XY) is less than negative one which may result in an unstable process.

Table 4.8C Real Interest Rate Responses 1985-2005

Real Interest Rate Responses

	D(_MR(1))	D(_MR(2))	D(_MR(3))	D(_MR(4))	D(_MR(5))	D(_MR(6))	D(_MR(7))	D(_MR(8))
D(RER)	**1.790 *****	-0.346	0.3066	-0.493	0.3907	-0.001	-0.011	0.5429
D(XY)	-0.875	-0.439	-0.933	0.3051	-0.812	-0.458	**-1.53 *****	-0.790
D(MR)	0.0393	**-0.30 ***	-0.089	**-0.52 *****	-0.019	-0.146	0.0329	**-0.22 ***
D(TOT)	**1.026 *****	0.5461	**0.749 ****	0.0595	0.2601	-0.246	0.0847	-0.191

There is a significant response of the real exchange rate to previous changes in the real interest rate, and a significant positive impact on the terms of trade. There is a statistically significant relationship between changes in the real interest rate on itself with a 4 quarter lag, and another, less significantly, 4 quarters later.

Table 4.8D Terms of Trade Responses 1985-2005

Terms of Trade Responses

	D(TOT(1))	D(TOT(2))	D(TOT(3))	D(TOT(4))	D(TOT(5))	D(TOT(6))	D(TOT(7))	D(TOT(8))
D(RER)	**-0.53 ****	0.0566	-0.036	-0.072	**0.499 ***	**-0.52 ***	-0.106	**0.526 ****
D(XY)	0.1833	-0.129	0.2597	0.1610	0.3301	-0.004	-0.230	-0.188
D(MR)	-0.055	0.0842	0.0299	-0.049	0.0597	-0.008	0.0019	0.0435
D(TOT)	**0.285 ****	0.0960	-0.116	**0.344 ****	0.0904	**-0.26 ***	0.0627	0.2234