Optimal Resource Allocation on Physical and Human Capital: Theoretical Modelling and Empirical Case Study of the United States

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Abstract

This paper utilizes the latest growth-accounting dataset constructed by Feenstra *et al.* (2015) to measure the joint contribution of physical capital and human capital in the United States from 1950 onwards. To be consistent with growth theories, this paper extends the deterministic growth model of Mankiw *et al.* (1992) to a stochastic version and further relaxes their assumption of constant and exogenous fractions of investment in physical and human capital. Both endogenous and exogenous growth factors are nested in the model, depending on parametric combinations. These implied parameters are consistently estimated for the United States. Furthermore, these long-run elasticity estimates inform the short-run transitional dynamics of the model, and structurally rationalize other empirical findings of bi-directional short-run effects between the three variables: physical capital, human capital, and aggregate output.

Keywords: Human Capital; Endogenous Growth; Optimal Growth Path

JEL code: O41; O47

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1. Introduction

In a majority of endogenous growth theories, non-diminishing returns to reproducible factor inputs in the production process have always been regarded as the fundamental mechanism of long-run growth of per-capita output. This mechanism includes the concept of broadly-defined aggregate capital stock, the spillover effects of public capital investment, the resource allocation on research and development, and most importantly, the introduction of human capital in the production process.

The role of human capital, as distinct from physical capital, in boosting growth has received attention from endogenous-growth theorists in as early as Frankel (1962) and Uzawa (1965). There are numerous studies on how human capital creates spillover effects in endogenous growth models. For instance, Griliches (1979) and Romer (1986) suggested that individual firms have specific knowledge, but such aggregate level of knowledge in an industry is beneficial yet exogenous in an individual firm's profit maximizing problem. The study made by Lucas (1988) documented that human capital creates and transmits knowledge across firms. The knowledge stock possessed by skilled workers can be accumulated in the education sector using as an input of a fraction of the current level of knowledge that is not used to produce final goods and services in the output sector. The optimal transition and the steady-state value of this fractional variable were derived in Lucas' (1988) two-sector model, but it was determined by multiple exogenous parameters in the system. There is neither a guarantee that its steady-state value is between 0 and 1, nor an empirical capability of cross-country estimation and comparison in a two-sector model.

The counterpart of Lucas' (1988) two-sector model is the one-sector system studied by Mankiw, *et al.* (1992). In the latter framework, both human capital and physical capital accumulate by forgoing consumption. In other words, with two exogenous fractional parameters, s_K and s_H, final output is distributed to physical capital investment, human capital investment, and consumption, by s_K, s_H, and $(1 - s_K - s_H)$, respectively. The subsequent empirical results show that about 80% of cross-country differences in per-capita income can be explained by their theory-consistent regressions. In the subsequent two decades, the empirical findings of Mankiw, *et al.* (1992) have become a milestone reference for both theoretical and empirical growth studies on the role of human capital.

This paper aims to relax the assumption of constant and exogenous s_K and s_H made by Mankiw, *et al.* (1992) and provide a solid micro-foundation in this series of literature. A representative household-worker optimally allocates resources on consumption, physical capital accumulation and human capital development, in order to maximize his or her lifetime utility. In addition, we extend the deterministic growth model of Mankiw, *et al.* (1992) to a stochastic discrete-time version that nests both exogenous and endogenous growth factors. This approach yields theory-consistent regressions for us to estimate the share of factor inputs as well as the optimal fraction of resource allocation between physical capital and human capital. We present policy implications in the empirical part of this paper by a case study of the United States.

This paper is arranged as follows. Section 2 proposes the theory-consistent empirical model derived from the stochastic growth model of Mankiw, *et al.* (1992), which motivates the time-series basis of the cointegration growth-accounting framework. Section 3 describes the aggregate data of our system variables from the United States. Empirical analyses, including cointegration tests, Granger causality tests, and the study of the short-run and impulse response analyses are conducted in Section 4 using the structural model that is informed by the long-run parameter estimates. This paper concludes with Section 5.

2. One-sector stochastic growth model with physical and human capital

2-1. Theory

At any time t, let Y_t be the aggregate output, K_t be the aggregate physical capital stock, H_t be the aggregate human capital stock, L_t be the total hours worked by employed workers, and A be the constant level of total factor productivity (TFP). The conventional labor-augmented production technology takes the Cobb–Douglas form

In equation (1), we extend the production function constructed by Mankiw, *et al.* (1992); in addition to the constant returns to scale, we include an exogenous Harrod-neutral rate of technological progress, x, as well as multiplicative stochastic shocks, ϵ_t^P, to model possible short-run business cycles. There is no restriction on the exogenous parameters, α, β, and x, because their empirical estimates inform the nature of endogenous and/or exogenous growth in an economy. These parametric combinations are discussed later. Equation (1) can be represented in per-working-hour terms as

$$y_t = A(1+x)^{[1-(\alpha+\beta)]t} k_t^{\alpha} h_t^{\beta} \epsilon_t^P, \quad (2)$$

where the lower case variables, y, k, and h, denote per-working-hour output, physical capital and human capital, respectively. We can also detrend equation (2) so that all variables can be expressed in the form of per efficiency-unit working hour in production as

$$\hat{y}_t = A\hat{k}_t^{\alpha}\hat{h}_t^{\beta}\epsilon_t^P, \text{ where: } \hat{y}_t = \frac{Y_t}{(1+x)^t L_t}; \ \hat{k}_t = \frac{K_t}{(1+x)^t L_t}; \ \hat{h}_t = \frac{H_t}{(1+x)^t L_t}. \quad (3)$$

To simplify our mathematical derivation, we assume that both types of capital stock depreciate completely in every period, but their dynamic paths and the nature of steady-state values are not affected even when the depreciation rates are less than 100%.[1] Thus, the investment flow at time, t, becomes the capital stock at time, $t+1$. Output, $\hat{y}_t$, at each point in time is distributed to consumption, $\hat{c}_t$, and the investment of physical capital and human capital. An endogenous variable s_t denotes the fraction of investment to accumulate physical capital, so $(1-s_t)$ denotes the same to human capital. Assuming a log-utility function, the household-worker's lifetime utility maximization problem is to solve

[1] Capital accumulation with a 100% depreciation rate, though unrealistic, is generally accepted in previous studies, such as Glomm and Ravikumar (1994) for a solvable mathematical derivation.

$$V(\hat{k}_0, \hat{h}_0) = \max_{\{\hat{c}_t, s_t, \hat{k}_{t+1}, \hat{h}_{t+1}\}_{t=0}^{\infty}} \mathbb{E}_0(\textstyle\sum_{t=0}^{\infty} \delta^t \ln \hat{c}_t); \text{ discount factor } \delta \in (0,1), \tag{4}$$

subject to

$$\hat{k}_{t+1} = s_t \hat{y}_t - \hat{c}_t = s_t A \hat{k}_t^{\alpha} \hat{h}_t^{\beta} \epsilon_t^P - \hat{c}_t; \tag{5}$$

$$\hat{h}_{t+1} = (1 - s_t)\hat{y}_t = (1 - s_t) A \hat{k}_t^{\alpha} \hat{h}_t^{\beta} \epsilon_t^P; \tag{6}$$

$$\hat{k}_0,\ \hat{h}_0 \text{ are given; and} \tag{7}$$

$$\hat{c}_t,\ \hat{k}_{t+1},\ \hat{h}_{t+1} \geq 0,\ s_t \in [0,1], \tag{8}$$

for all $t \in \mathbb{N}$.

In order to solve household-worker's intertemporal utility maximization problem subject to constraints (5) to (8), we apply the Bellman (1957) principle of optimality and re-write equation (4) as the following limiting value function

$$V(\hat{k}_t, \hat{h}_t, \epsilon_t^P) = \max_{\{\hat{c}_t, s_t, \hat{k}_{t+1}, \hat{h}_{t+1}\}_{t=0}^{\infty}} \textstyle\sum_{t=0}^{\infty} \ln \hat{c}_t + \delta \mathbb{E}_0\left[V\left(\hat{k}_{t+1}, \hat{h}_{t+1}, \epsilon_{t+1}^P | \hat{k}_t, \hat{h}_t, \epsilon_t^P\right)\right], \tag{9}$$

subject to constraints (5) to (8). A guess of the above value function takes the form of

$$V(\hat{k}_t, \hat{h}_t, \epsilon_t^P) = B_0 + B_1 \ln \hat{k}_t + B_2 \ln \hat{h}_t + B_3 \ln \epsilon_t^P, \tag{10}$$

where B_0, B_1, B_2, and B_3 are combinations of the parameters in the system and are verified later. The optimality conditions for a maximum on the RHS of the combination of equations (9) and (10) can be obtained by the following two first-order derivatives with respect to $\hat{c}_t$ and s_t:

$$\frac{1}{\hat{c}_t} = \frac{\delta B_1}{\hat{k}_{t+1}}, \text{ and} \tag{11}$$

$$\frac{\hat{k}_{t+1}}{\hat{h}_{t+1}} = \frac{B_1}{B_2}. \tag{12}$$

Equation (11) shows household-worker's optimal intertemporal resource allocation on current consumption and future physical capital stock. Equation (12) shows that the ratio between physical capital and human capital should be constant and time-invariant for optimality. Substitute equations

(11) and (12) to equations (5) and (6) to obtain

$$\hat{c}_t = \left(\frac{1}{1+\delta B_1}\right) s_t A \hat{k}_t^{\alpha} \hat{h}_t^{\beta} \epsilon_t^P, \tag{13}$$

$$s_t = \frac{1+\delta B_1}{1+\delta B_1+\delta B_2}, \tag{14}$$

$$\hat{k}_{t+1} = \left(\frac{\delta B_1}{1+\delta B_1}\right) s_t A \hat{k}_t^{\alpha} \hat{h}_t^{\beta} \epsilon_t^P, \text{ and} \tag{15}$$

$$\hat{h}_{t+1} = \left(\frac{\delta B_2}{1+\delta B_1}\right) s_t A \hat{k}_t^{\alpha} \hat{h}_t^{\beta} \epsilon_t^P. \tag{16}$$

Equations (13) to (16) can be substituted back to the RHS of the combination of equations (9) and (10) to verify

$$B_1 = \frac{\alpha}{1-\delta(\alpha+\beta)} \text{ and } B_2 = \frac{\beta}{1-\delta(\alpha+\beta)}. \tag{17}$$

B_1 and B_2 are two keys to solve the optimal sequence of $\{\hat{c}_t, s_t, \hat{k}_{t+1}, \hat{h}_{t+1}\}_{t=0}^{\infty}$ for the representative household-worker. With B_1, equation (13) shows the optimal consumption path as

$$\hat{c}_t = (1-\delta\alpha-\delta\beta) A \hat{k}_t^{\alpha} \hat{h}_t^{\beta} \epsilon_t^P, \tag{18}$$

in which the marginal propensity to consume is one minus the sum of one-period discounted shares of physical capital and human capital in the production process, $(\delta\alpha+\delta\beta)$. These discounted shares, $\delta\alpha$ and $\delta\beta$, also represent the optimal fractions of output invested in physical capital and human capital, respectively. This can be shown by substituting B_1 and B_2 to equations (15) and (16) as

$$\hat{k}_{t+1} = \delta\alpha A \hat{k}_t^{\alpha} \hat{h}_t^{\beta} \epsilon_t^P; \tag{19}$$

$$\hat{h}_{t+1} = \delta\beta A \hat{k}_t^{\alpha} \hat{h}_t^{\beta} \epsilon_t^P. \tag{20}$$

The optimal investment fraction, s_t, is implied by equations (6) and (20), or derived by substituting B_1 and B_2 to equation (14) as

$$s_t = 1-\delta\beta. \tag{21}$$

Thus, the optimal s_t is equal to one minus one-period discounted share of human capital in output. When β gets close to zero, s_t is close to one so that the household-worker has no incentive to develop human capital. In this case, the model is reduced to a conventional Ramsey planning economy where physical capital is the only accumulable factor in the system. Conversely, a higher share of human capital in output implies that human capital accumulation is a more productive strategy, compared with physical capital accumulation.

The above model covers four possible scenarios in an economy, depending on the combination of parameters, shown in Table 1.

[Table 1 is placed here]

Assuming that $\epsilon_t^P = 1$ and $0 < \alpha + \beta < 1$ for scenarios (a) and (b), from equations (19) and (20), the steady-state values of $\hat{k}^*$ and $\hat{h}^*$ are unique and stable as follows

$$\hat{k}^* = \phi^{\frac{1-\beta}{1-(\alpha+\beta)}}, \text{ and} \tag{22}$$

$$\hat{h}^* = (\delta\beta A)^{\frac{1}{1-\beta}} \phi^{\frac{\alpha}{1-(\alpha+\beta)}}, \text{ where } \phi = (\delta\alpha A)(\delta\beta A)^{\frac{\beta}{1-\beta}} > 0. \tag{23}$$

Scenarios (a) and (b) indicate that, if the reproducible factors, $\hat{k}_t$ and $\hat{h}_t$, exhibit diminishing returns to scale, per-working-hour output can grow in the long run only when the exogenous rate of technological progress, x, is positive. On the other hand, for scenarios (c) and (d), the imposition of constant returns to reproducible factors in output $(\alpha + \beta = 1)$ leads to the fact that the steady-state values of $\hat{k}^*$ and $\hat{h}^*$ will no longer exist. In such cases, the long-run growth rates of per-working-hour variables, along the balanced growth path, are identical, perpetual and non-explosive as

$$\gamma_y = \gamma_k = \gamma_h = \gamma_c = x + \delta A \alpha^{\alpha}(1-\alpha)^{1-\alpha} - 1,^{2} \text{ where } x \geq 0. \tag{24}$$

[2] γ_z is the growth rate of variable z.

2-2. Simulation and Calibration

Based on the previous theoretical results, it is possible to simulate short-run and long-run transitions of the variables in the system. Equations (18) to (20) characterize the solutions to the household-worker's intertemporal utility maximization problem. Along with equations (3) and (21), we can re-write the exact log-linear equilibrium conditions in the state-space form as

$$\begin{bmatrix} 1 \\ \ln \hat{k}_{t+1} \\ \ln \hat{h}_{t+1} \\ \ln \epsilon_t^P \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \ln(\delta\alpha A) & \alpha & \beta & 1 \\ \ln(\delta\beta A) & \alpha & \beta & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ \ln \hat{k}_t \\ \ln \hat{h}_t \\ \ln \epsilon_t^P \end{bmatrix}; \text{ and} \tag{25}$$

$$\begin{bmatrix} \ln \hat{c}_t \\ \ln \hat{y}_t \end{bmatrix} = \begin{bmatrix} \ln[(1-\delta\alpha-\delta\beta)A] & \alpha & \beta & 1 \\ \ln A & \alpha & \beta & 1 \end{bmatrix} \begin{bmatrix} 1 \\ \ln \hat{k}_t \\ \ln \hat{h}_t \\ \ln \epsilon_t^P \end{bmatrix}. \tag{26}$$

In order to calibrate the system described above, all parameters are assumed in reasonable ranges. First, for scenarios (a) and (b) in Table 1, we assume that $A = 2$, $\alpha = 0.3$, and $\beta = 0.2$. With household-worker's subjective discount rate $\delta = 0.95$, the optimal investment fraction is $s_t = (1 - \delta\beta) = 0.81$ for all time. This implies that, at every point of time, output should be distributed to consumption, physical capital accumulation, and human capital accumulation as 52.5%, 28.5%, and 19%, respectively. Concurrently, we also assume that $\epsilon_t^P = 1$ for a deterministic case and this shock is introduced later in our theoretical impulse-response analysis. The following figure 1 shows the optimal paths of $\hat{k}_t$, $\hat{h}_t$, $\hat{y}_t$, and $\hat{c}_t$, for the given initial values of $\hat{k}_0 = 0.1$ and $\hat{h}_0 = 0.05$.

[Figure 1 is placed here]

Next, to demonstrate scenarios (c) and (d) in Table 1, we simply change the share of human capital in output from $\beta = 0.2$ to $\beta = 0.7$ so that $\alpha + \beta = 1$. In this case, the steady-state equilibrium demonstrated in Figure 1 does not exist, and the growth rate of the system, from equation (24), can be derived as

$$\begin{aligned}\gamma_y = \gamma_k = \gamma_h = \gamma_c = x + \delta A \alpha^{\alpha}(1-\alpha)^{1-\alpha} - 1 &= 3.15\% \quad \text{for } x = 0 \text{ (scenario (c))} \\ &= 5.15\% \quad \text{for } x = 0.02 \text{ (scenario (d))}\end{aligned}.$$

The model contains a stochastic component $\epsilon_t^P \sim^{iid} N(1, \sigma_P^2)$ that allows us to perform theoretical impulse-response analysis from short to long run. Scenario (b) is a good example for us to demonstrate this. Assuming that there is a one-percentage increase of TFP, the percentage responses of $\hat{k}_t$, $\hat{h}_t$, $\hat{y}_t$, and $\hat{c}_t$ are shown in Figure 2.

[Figure 2 is placed here]

In Figure 2, $\hat{y}_t$ and $\hat{c}_t$ respond contemporaneously and positively to the shock by one-percentage point, while the responses of $\hat{k}_t$ and $\hat{h}_t$ lag by one period. All responses of the four variables are positive and declining. The shock dies away after 6 to 7 time periods. This positive standard shock also causes the long-run rate of growth to fluctuate. In the case of Scenario (b), we assume that the exogenous rate of technological progress is $x = 0.02$, which is also the long-run rate of growth in per-working-hour output. After having a one-percentage increase in TFP at time T, Figure 3 shows that this growth rate increases from 2% to 3% contemporaneously, but at time $T+1$, it falls to 1.5%, which is consistent with the reduction of the positive response from $\hat{y}_t$ shown in figure 2. The long-run growth rate of 2% is then restored after 6 to 7 time periods.

[Figure 3 is placed here]

2-3. Theory-implied Cointegration Equations

In the previous subsection, while our simulation results are reasonable, the calibrated technological parameters, α, β, and x, cannot be arbitrarily assumed rather should be properly estimated. In our theoretical model, we provide theory-consistent cointegrating spaces for the per-working-hour variables, y_t, k_t, and h_t. By log-linearizing equations (19), (20), and (3), and applying a lag operator, L, we have

$$[1-(\alpha+\beta)L](\ln k_t - xt) = \lambda_1 + L \ln \epsilon_t^P; \qquad (27)$$

$$[1-(\alpha+\beta)L](\ln h_t - xt) = \lambda_2 + L\ln\epsilon_t^P; \quad (28)$$

$$[1-(\alpha+\beta)L](\ln y_t - xt) = \lambda_3 + \ln\epsilon_t^P, \quad (29)$$

where λ_1, λ_2, and λ_3 are some constant combination of parameters, and $\ln\epsilon_t^P$ is an *i.i.d.* shock with a zero mean and a constant variance.

From a time-series perspective, $\ln y_t$, $\ln k_t$, and $\ln h_t$ are possibly I(1) variables, so there can be at maximum two linearly independent cointegrating vectors. Assuming that $(\alpha+\beta)=1$ for non-explosive and perpetual growth, equations (27) to (29) yield the cointegrating space as

$$\ln y_t - \ln k_t = \kappa_1 + \ln\epsilon_t^P; \text{ and} \quad (30)$$

$$\ln y_t - \ln h_t = \kappa_2 + \ln\epsilon_t^P, \text{ where } \kappa_1 \text{ and } \kappa_2 \text{ are constant.} \quad (31)$$

If the existence of the cointegrating space with two cointegrating vectors is rejected, then at most one cointegrating equation can be found. By multiplying equation (30) on both sides by α and equation (31) on both sides by $(1-\alpha)$, and then summing them up, we have

$$\ln y_t - \alpha\ln k_t - (1-\alpha)\ln h_t = \kappa_3 + \ln\epsilon_t^P, \text{ where } \kappa_3 \text{ is constant.} \quad (32)$$

If we release the restriction that $(\alpha+\beta)=1$, the unrestricted cointegrating equation can be derived from equation (2) yielding

$$\ln y_t - \alpha\ln k_t - \beta\ln h_t - [1-(\alpha+\beta)]xt - \ln A = \ln\epsilon_t^P. \quad (33)$$

It can be noted that equation (32) is a restricted case of (33) under the hypothesis of $(\alpha+\beta)=1$. This hypothesis, together with the effectiveness of the exogenous rate of technological progress, x, will be tested in the empirical part of this paper.

3. Empirical Estimation

3-1. Data

The data source of all variables in the theoretical model is directly available from Penn World Table (PWT) version 8.1, covering sufficient time period from 1950 to 2011 for the United States and most of advanced countries. It is so far the most updated and complete worldwide macroeconomic dataset constructed by Feenstra, *et al.* (2015). In this dataset, we have real GDP (with PPP adjustment) for the variable Y_t, total working hours for the variable L_t, real aggregate physical capital stock for the variable K_t, and the index of aggregate human capital stock for the variable H_t.[3] We deliberately exclude net export from real GDP because the theoretical model introduced in this paper is a closed-economy system. Including net export may introduce more complicated cointegrating space in our time-series analysis. All real variables are at constant 2005 national prices (in million 2005 US$). By dividing Y_t, K_t, and H_t by total working hours, L_t, we can overview the growing trend of the logarithmic lower case variables, $\ln y_t$, $\ln k_t$, and $\ln h_t$, in Figure 4.

[Figure 4 is placed here]

In our sample period, physical capital stock per working hour (k_t) and GDP per working hour (y_t) grew steadily by average annual growth rates of 2.04% and 1.89%, respectively. On the other hand, the growth of human capital stock per working hour, h_t, showed a clear slowdown from 1980 onwards. The average annual growth rates of h_t were 1.15% and 0.28% in the periods of 1950–1980 and 1981–2011, respectively. This fact echoes the observation that "*the United States has been under-producing college-going workers since 1980*" by Carnevale and Rose (2011).

3-2. Stationarity of variables

In general, most macroeconomic variables, after logarithmic transformation, contain stochastic trends in the long run. To test the stationarity of $\ln y_t$, $\ln k_t$, and $\ln h_t$, Figure 4 suggests that both intercepts and time trends should be considered when performing conventional unit root tests for these variables in levels, and only intercepts in first differences. Table 2 summarizes the results of the Augmented Dickey–Fuller (ADF) unit root tests (Said and Dickey (1984)).

[Table 2 is placed here]

Given the conclusion that $\ln y_t$, $\ln k_t$, and $\ln h_t$ are integrated of order one (or I(1)) in Table 2,

[3] The aggregate human capital stock is calculated, based on the documentation of PWT 8.1, by multiplying the number of workers in an economy (emp) by their average human capital (hc). hc is an index of human capital that considers both years of schooling (Barro and Lee (2013)) and returns to education (Psacharopoulos (1994)).

their long-run cointegrating relationships are testable in balanced regressions. It may be stated that involving these integrated variables in an ad hoc single aggregate production function could cause endogeneity bias, but the cointegration properties of variables based on the theory-consistent model avoid spurious regressions and simultaneity bias. We show this by performing Johansen cointegration test (Johansen (1991)) based on a vector autoregressive (VAR) model.

3-3. Long-run cointegration relationships

With common stochastic trends, non-stationary variables demonstrate cointegration relationships in some stationary and linear combinations. Based on proper lag selection criteria,[4] a VAR model with lag two, of VAR(2) model, is estimated, and the consequent Johansen cointegration test results are summarized in Table 3.

[Table 3 is placed here]

Table 3 indicates that, at usual significance level of 5%, both trace and maximum eigenvalue test statistics conclude that there are two cointegration vectors (CVs) among $\ln y_t$, $\ln k_t$, and $\ln h_t$, with a time trend. Only one cointegration vector is concluded at a significance level of 1% from trace test statistic. The possibility of one or two CVs is consistent with the prediction of the theoretical model. By transforming the VAR(2) model to a vector error correction model with order one, or VEC(1) model, the following two cointegration equations are estimated for $\ln y_t$, $\ln k_t$, and $\ln h_t$, with a time trend.

$$\ln y_t = 5.0182 + \underset{(0.2079)}{0.3358^{**}} \ln k_t + \underset{(0.0012)}{0.0189^{***}}\ t + \ln \epsilon_t^P; \tag{34}$$

$$\ln k_t = 3.8176 + \underset{(0.2638)}{0.0449} \ln h_t + \underset{(0.0021)}{0.0170^{***}}\ t + \ln \epsilon_t^P, \tag{35}$$

where standard errors are in parentheses. Significance levels of 5% and 1% are denoted by ** and ***, respectively. The first CV represents a conventional production function in the Solow–Swan model,

[4] These lag selection criteria, at maximum five lags, are sequential modified LR test statistic (5% level), Schwarz information criterion, and Hannan–Quinn information criterion.

in which labor and physical capital are the only factor inputs under constant returns to scale. The share of physical capital is significantly estimated as 0.3358, and the economy is observed to grow exogenously by an annual rate of technology progress of 1.89%.

If the significance level used in the Johansen cointegration tests is reduced to 1%, based on trace test statistic in Table 3, there exists only one CV as

$$\ln y_t = 3.4195 + \underset{(0.2079)}{0.4188^{**}} \ln k_t + \underset{(0.2588)}{0.3546^{*}} \ln h_t + \underset{(0.0026)}{0.0118^{***}}\ t + \ln \epsilon_t^P . \quad (36)$$

where standard errors are in parentheses. Significance levels of 10%, 5% and 1% are denoted by *, **, and ***, respectively. The estimates in equation (36) are significant and in reasonable ranges. The shares of physical and human capital in the production process, α and β, are estimated as 0.4188 and 0.3546, respectively. This evidently shows that the input of human capital stock was as important as the input of physical capital stock in the United States over the past 60 years. In addition, the coefficient of time trend, 0.0118, also shows a stable and exogenous growing tendency of technology in the United States. Last but not least, the optimal share of investment on human capital was derived as $\delta\beta$ in the previous theoretical section; so, for an assumed reasonable size of the discount rate $\delta = 0.95$ and the estimated α and β, we can calculate that 33.69% and 39.79% of net GDP[5] are suggested to invest in developing human capital and accumulating physical capital, respectively, to achieve the long-run optimality in the United States.

In our theoretical model, we categorize four scenarios in Table 1, depending on the estimates of the coefficient. Based on formal log-likelihood ratio (LR) tests for the VEC(1) model, Table 4 summarizes the test results. The first test considers the null hypothesis of constant returns to per-working-hour variables in equation (33). Under reasonable level of significance, $\alpha + \beta = 1$ cannot be rejected, which leads to the case of perpetual endogenous growth in the United States. The second test is for the null hypothesis of zero Harrod-neutral rate of technological progress ($x = 0$),

[5] As mentioned before, the output series of the United States used in our empirical estimation is GDP net of net export for being consistent with the theoretical model.

which cannot be rejected, either. These two results are jointly tested and supported in the third test, leading to the fact that, over the past 60 years, the United States had experienced strict endogenous growth. With the contribution of human capital stock to final-good sector, growth in the United States was not driven by exogenous components but by physical and human capital stocks, although it is important to note that the dominance of physical capital is relatively emphasized in the economic development of the United States.

[Table 4 is placed here]

3-4. Granger Causality Tests and Impulse Response Analysis

As our VEC(1) model satisfies the stability condition because $\ln y_t$, $\ln k_t$, and $\ln h_t$ are stationary after taking first difference, it allows us to conduct Granger causality tests to understand the predictability of one variable in terms of another. The pair-wise Granger causality tests are summarized in Table 5.

[Table 5 is placed here]

In a VAR or VEC system, a variable is relatively endogenous if it can be Granger caused by other variables but there exists no reverse predictability. The test results reported in Table 5 suggest that the lagging effects of $\Delta \ln h_{\mathrm{t}}$ on $\Delta \ln y_t$ and $\Delta \ln k_t$ are not significant, but both $\Delta \ln y_t$ and $\Delta \ln k_t$ significantly Granger cause $\Delta \ln h_t$. We can also see that $\Delta \ln y_t$ and $\Delta \ln k_t$ have mutual Granger causality to each other, but the response of $\Delta \ln y_t$ is stronger to the lagging effects of $\Delta \ln k_t$ than the response of $\Delta \ln k_t$ to the lagging effects of $\Delta \ln y_t$. Based on these pair-wise Granger causality tests, our data show that physical capital is a relatively exogenous variable compared with output, and human capital is the most endogenous variable. These lagging effects explain the transmission channel among the variables. At any stage of economic development of any country, building physical capital stock is a prior means to provide output growth. Along the process of increasing output, more resources will and should be allocated to develop human capital stock to improve the quality of workers and further boost labor productivity. These results do not overthrow our previous growth conjectures that GDP grows endogenously with the inputs of physical capital and human capital because what was estimated is the long-run cointegration relationship among $\ln y_t$, $\ln k_t$, and $\ln h_t$, and not their short-run causality to each other.

These short-run Granger causal relationships enable us to perform generalized impulse response analyses[6] based on the test results of an order of $\{\ln k_t, \ln y_t, \ln h_t\}$ in our VEC(1) system. Three variables in the impulse response analysis create nine interactions between variables, though, we only report some meaningful and intuitive figures below. First, the responses of the two factor inputs, $\ln k_t$ and $\ln h_t$, to each other's shocks are shown in Figure 5.

[Figure 5 is placed here]

In the first two periods, both $\ln k_t$ and $\ln h_t$ respond to the shocks from each other positively, suggesting that these two factor inputs are mutually complement in the short run. From the third period onwards, the response of $\ln k_t$ to $\ln h_t$ stays positive but slightly declines, while their inverse impulse and response becomes negative and leans towards zero in the long run. In other words, upgrading human intelligence helps building more robots, but increasing number of robots seems to reduce the incentive to train more smart people.

Next, a one-off increase in output shown in Figure 6 leads to a two-period slight reduction in physical capital and human capital. After the third period, the responses of both types of capital stock to this shock keep increasing, creating more factor inputs in the long run.

[Figure 6 is placed here]

Lastly, temporarily positive shocks of the two types of capital stock are, by intuition, beneficial to output increase, but it is so only after the second period as shown in Figure 7.

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[6] Because the conventional impulse response may suffer from autocorrelated errors and causal-ordering problems, the generalized impulse response functions of Koop, *et al.* (1996) and Pesaran and Shin (1998) are used to avoid them.

[Figure 7 is placed here]

4. Conclusion

By considering optimal resource allocation on consumption, and accumulation of physical capital and human capital, this paper extends the theoretical framework of Mankiw, *et al.* (1992) to solve the optimality problem. The theoretical results motivate the empirical findings using the data of the United States. Measured by per-working-hour unit, long-run cointegration relationships can be found among final output, physical capital, and human capital, from 1950 to 2011 in the United States. These variables also bilaterally influence each other in the short run. This information on how variables respond to each other when shocks are introduced to certain variables could be useful. In the long-run perspective, the estimates of the shares of physical capital and human capital in the production process not only emphasize their relative importance of contributions to the final output, but they also inform the optimal fraction of capital investment for policy maker's consideration of resource allocation.

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Table 1: Growth Scenario and Corresponding Parameters in the System

Parametric combination	Growth Scenario
(a) $\alpha+\beta<1$ and $x=0$	No perpetual growth, steady state exists
(b) $\alpha+\beta<1$ and $x>0$	Pure exogenous growth
(c) $\alpha+\beta=1$ and $x=0$	Strict endogenous growth
(d) $\alpha+\beta=1$ and $x>0$	Co-existence of endogenous and exogenous growth

Table 2: ADF unit root tests for logarithmic variables in levels and first differences

	Variables in Levels		
	$\ln y_t$	$\ln k_t$	$\ln h_t$
Sample period	1950–2011	1950–2011	1950–2011
Specification	Constant and trend	Constant and trend	Constant and trend
Leg length	1	1	1
ADF t-statistic	–2.2037	–2.5100	–0.5786
5% Critical value	–3.4865	–3.4865	–3.4865
	Variables in First Differences		
	$\Delta \ln y_t$	$\Delta \ln k_t$	$\Delta \ln h_t$
Sample period	1950–2011	1950–2011	1950–2011
Specification	Constant	Constant	Constant
Leg length	0	0	0
ADF t-statistic	–6.2027	–6.0638	–4.5950
5% Critical value	–2.9109	–2.9109	–2.9109

* Lag lengths were optimally chosen by Schwarz Information Criterion (SIC) with a maximum lag length of 10. At usual 5% level of significance, $\ln y_t$, $\ln k_t$, and $\ln h_t$ are all I(1) variables.

Table 3: Johansen cointegration tests for $\ln y_t$, $\ln k_t$, and $\ln h_t$, a linear time trend is included

Null hypothesis	Trace test statistic	Maximum eigenvalue test statistic	Conclusion
No CV	55.0670***	28.0992**	Reject the null
At most one CV	26.9679**	21.6602**	Reject the null
At most two CVs	5.3077	5.3077	Do not reject the null

* Significance levels of 5% and 1% are denoted by ** and ***, respectively.

Table 4: Hypothesis tests for endogenous and/or exogenous growth scenarios

Cointegrating equation: $\ln y_t = \beta_0 + \beta_1 \ln k_t + \beta_2 \ln h_t + \beta_3 t + \ln \epsilon_t^P$ (equation (33))				
Null hypothesis	Model implication	LR test statistic	Critical value	Conclusion
(1) $\beta_1 + \beta_2 = 1$	$\alpha + \beta = 1$	0.2057	$\chi^2_{0.05}(1) = 3.84$	Do not reject the null
(2) $\beta_3 = 0$	$x = 0$	1.2563	$\chi^2_{0.05}(1) = 3.84$	Do not reject the null
(3) $\beta_1 + \beta_2 = 1$ and $\beta_3 = 0$	$\alpha + \beta = 1$ and $x = 0$	4.9729	$\chi^2_{0.05}(2) = 5.99$	Do not reject the null

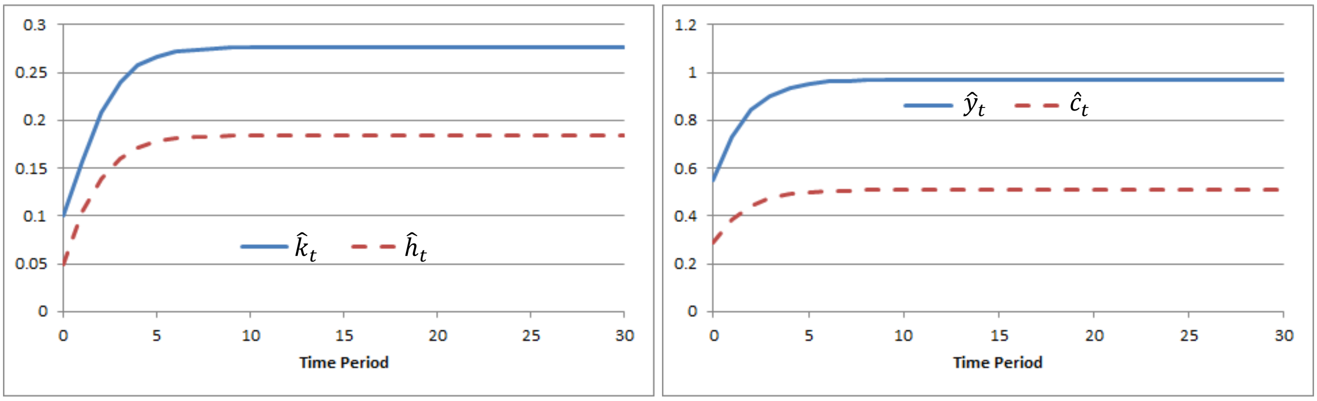
* The calculation of LR test statistics are based on the determinants of residual covariance of restricted and unrestricted VEC(1) models.

Table 5: Pair-wise Granger Causality Tests for first-differenced $\ln y_t$, $\ln k_t$, and $\ln h_t$

Null Hypothesis	χ^2 Statistic	Critical Value	Conclusion
$\Delta \ln k_t$ does not Granger cause $\Delta \ln y_t$	10.9182	$\chi^2_{0.05}(1) = 3.84$	Reject the null
$\Delta \ln h_t$ does not Granger cause $\Delta \ln y_t$	1.2263	$\chi^2_{0.05}(1) = 3.84$	Do not reject the null
$\Delta \ln k_t$ and $\Delta \ln h_t$ do not Granger cause $\Delta \ln y_t$	14.1792	$\chi^2_{0.05}(2) = 5.99$	Reject the null
$\Delta \ln y_t$ does not Granger cause $\Delta \ln k_t$	4.9231	$\chi^2_{0.05}(1) = 3.84$	Reject the null
$\Delta \ln h_t$ does not Granger cause $\Delta \ln k_t$	0.0735	$\chi^2_{0.05}(1) = 3.84$	Do not reject the null
$\Delta \ln y_t$ and $\Delta \ln h_t$ do not Granger cause $\Delta \ln k_t$	6.4060	$\chi^2_{0.05}(2) = 5.99$	Reject the null
$\Delta \ln y_t$ does not Granger cause $\Delta \ln h_t$	4.4186	$\chi^2_{0.05}(1) = 3.84$	Reject the null
$\Delta \ln k_t$ does not Granger cause $\Delta \ln h_t$	17.4919	$\chi^2_{0.05}(1) = 3.84$	Reject the null
$\Delta \ln y_t$ and $\Delta \ln k_t$ do not Granger cause $\Delta \ln h_t$	25.4493	$\chi^2_{0.05}(2) = 5.99$	Reject the null

* The above Granger causality tests are performed at 5% level of significance.

Figure 1: Optimal Transitions of $\hat{k}_t$, $\hat{h}_t$, $\hat{y}_t$, and $\hat{c}_t$, in the case of scenarios (a) and (b)



[*] Note: Based on our calibrated parameters, the steady-state values of $\hat{k}_t$, $\hat{h}_t$, $\hat{y}_t$, and $\hat{c}_t$ are unique and equal to 0.28, 0.18, 0.97, and 0.51, respectively.

Figure 2: The percentage responses of $\hat{k}_t$, $\hat{h}_t$, $\hat{y}_t$, and $\hat{c}_t$ to 1% increase in TFP, in the case of scenario (b)

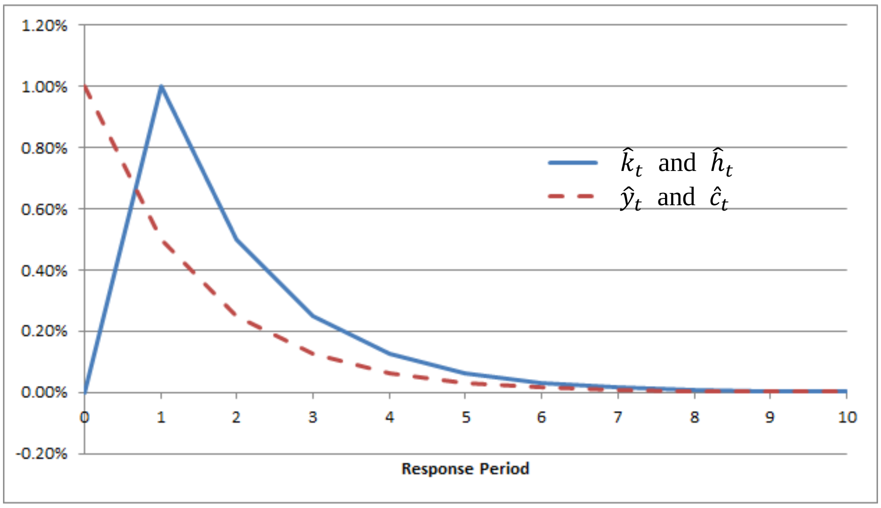


Figure 3: The response of the long-run growth rate in the case of scenario (b)

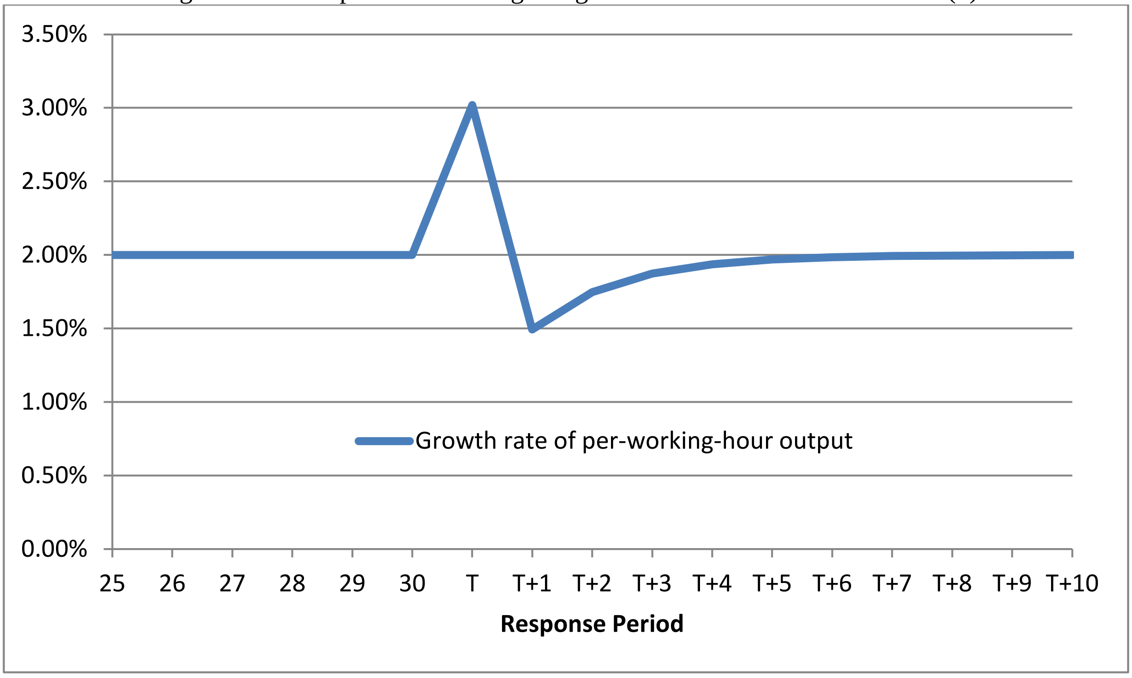


Figure 4: Historical trends of $\ln y_t$, $\ln k_t$, and $\ln h_t$ from 1950 to 2011

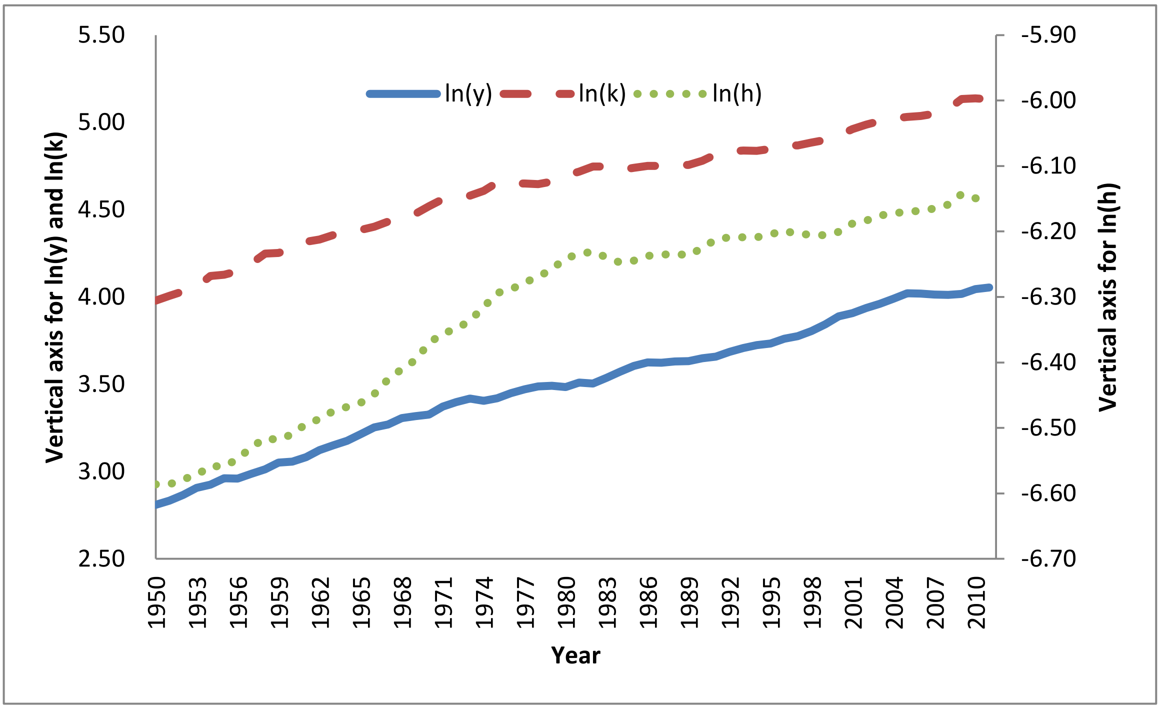


Figure 5: Mutual impulse responses between $\ln k_t$ and $\ln h_t$

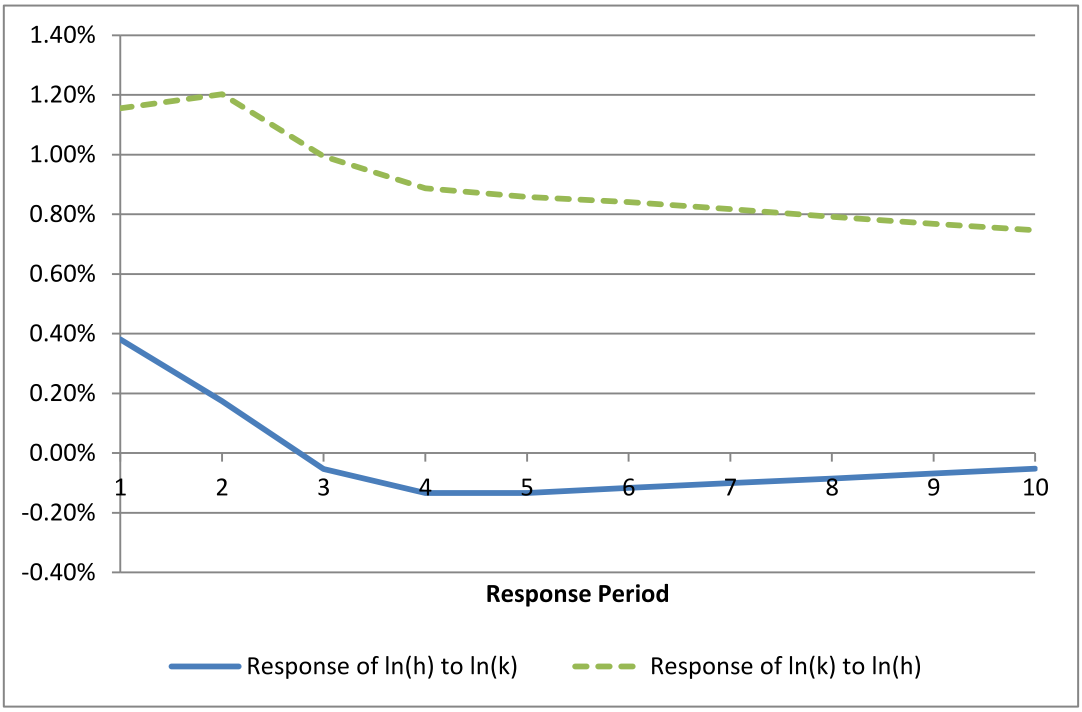


Figure 6: Responses of $\ln k_t$ to $\ln y_t$ and $\ln h_t$ to $\ln y_t$

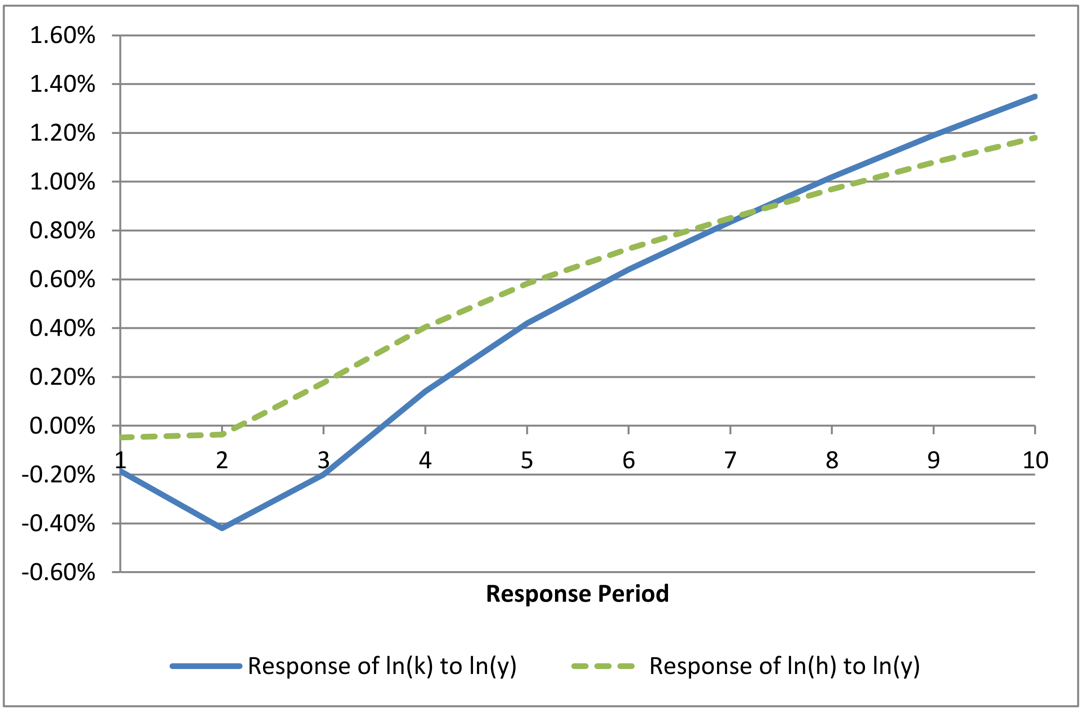


Figure 7: Responses $\ln y_t$ to $\ln k_t$ and to $\ln h_t$