The Growth and Welfare Analysis of Patent and Monetary Policies in a Schumpeterian Economy*

Chien-Yu Huang†
Southwestern University of Finance and Economics

Yibai Yang‡
University of Nottingham Ningbo China

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Abstract

This note compares the growth and welfare implications of patent policy and monetary policy in a Schumpeterian growth model where the market power of firms is subject to patent breadth whereas consumption and R&D investment are subject to cash-in-advance (CIA) constraints, respectively. The main findings are as follows. First, implementing monetary policy is more effective than implementing patent policy and implementing the mix of these policies in terms of stimulating economic growth if initial patent protection in the economy is strong. Second, the welfare difference between patent policy and monetary policy is ambiguous, depending on the levels of predetermined instruments in these policies. However, these policy regimes are (weakly) dominated by their combination in terms of raising social welfare.

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†School of Economics, Southwestern University of Finance and Economics, Chengdu, Sichuan, 611130, China. *Email* address: hqy@swufe.edu.cn.

‡GEP China, IAPS and School of Economics, University of Nottingham Ningbo China, Ningbo, Zhejiang, 315100, China. *Email* address: yibai.yang@hotmail.com.

1 Introduction

What is the effect of monetary policy on economic growth and social welfare given a degree of Intellectual Property Right (IPR) protection and vice versa? How does the interaction of two policy authorities (monetary and IPR protection) affect mutual optimal policy targets? Does the coordination of these two authorities possibly achieve a higher level of welfare than the non-coordination case? In this study, we build up an endogenous Schumpetarian growth model to stress the above interesting questions. Specifically, in our model, we introduce IPR protection by imposing patent breadth on the markup that determines the market power of firms and simultaneously incorporate the money demand by imposing cash-in-advance (CIA) constraints on households' consumption and R&D investment.

Our study is motivated by two series of well-known policy events. Firstly, most of developed countries have strengthened their IPR protection as a result of policy reform according to World Trade Organizations Agreement on Trade-Related Aspects of Intellectual Property Rights (TRIPS).[1] For example, the empirical survey of Park (2008) shows that 107 out of 122 countries have experienced an increase in the strength of patent rights during 1995 and 2005.[2] In particular, he finds that developing countries experienced a higher average increase in the strength of patent rights than developed and under-developed countries, because developing countries have larger market size and innovative capacity allowing them to implement a stronger patent system.[3]

In addition, a very low (close to zero) level of nominal interest rate target has recently been announced in several countries one after another. Since December 2008, the federal funds rate in the US has been targeted at around 0% to 0.25%. In October 2012, Federal Open Market Committee (FOMC) guaranteed a long-term low level of nominal interest rate target till mid 2015. In August 2015, the Fed reclaimed that it would postpone its low level of federal fund rate until the economy recovers. Similarly, the benchmark interest rate in Japan has been fluctuating between 0% and 0.1% since December 2008. Moreover, from 2014 to mid 2015, China also lowered its nominal interest rate several times for response to the collapse of its stock market. In this note, we find that a zero-nominal-interest-rate policy is optimal regardless of the degree of IPR protection, but the optimality of IPR protection does depend on the level of nominal interest rate target.

In a related study, Chu, Lai, and Liao (2012) establish a potential theoretical interaction between the effects of monetary policy and IPR protection policy on growth and welfare. This note thereby complements their study by extending their analysis of the interaction of monetary and patent policies to the comparison of optimal design for monetary and patent policies. For the analytical study, we firstly find that given two policy authorities that execute their policy independently, the single optimal monetary policy generates a higher (a lower) equilibrium growth rate than the single optimal patent policy if the initial scope of patent breath is larger (lower) than a critical level. This result in fact implies that the growth effect of monetary policy depends on the degree

[1]The WTOs TRIPS Agreement, which was initiated in the 1986-94 Uruguay Round, establishes a minimum level of intellectual property right protection that must be provided by all member countries by 2006.

[2]According to Park (2008), the measure of IPR protection, which is called Ginarte-Park index, includes 122 countries and sets a scale of 0 to 5. Of 122 countries, 107 countries experienced an increase of IPR protection, in which the average scale of Ginarte-Park index rises from 2.58 in 1995 to 3.34 in 2005, showing a high degree of IPR protection across countries.

[3]See Gillman and Kejak (2005) for a survey of this literature.

of IPR, which affects the structure of firm's market power.[4] In particular, on the one hand, a larger patent breath enhances firms' incentives for innovations, leading to a higher level of labor employment in R&D sector and a higher rate of economic growth. On the other hand, the rate of nominal interest also exhibits a monotone decreasing relationship with economic growth, given that a higher nominal interest rate implies larger costs for innovations in the presence of a CIA constraint on R&D. Then, the optimal monetary policy that happens at zero nominal interest rate (i.e., the Friedman rule) indicates the growth-maximizing level. Hence, a sufficiently large patent breath along with the Friedman Rule can create a larger growth effect than a single patent breath policy. This argument provides a policy recommendation such that the policy choice between money and patent for boosting economic growth depends on the strength of IPR. For this reason, those developing countries that experience a recent increase in the strength of IPR protection may find that it is more effective to enhance economic growth by choosing monetary policy rather than patent policy.

Second, we find that the coordinated optimal policy always yields a higher welfare level for the economy than the non-coordinated scenarios. First, optimal combined policy would generate a higher welfare level than optimal patent policy, because the nominal interest rate is lower under optimal combined policy than under optimal patent policy. Therefore, more labor is assigned to R&D under optimal combined policy yielding a larger growth effect on welfare, whereas less labor is allocated to manufacturing under this regime yielding a smaller consumption effect and leisure effect on welfare; the former effect dominates the latter two effects resulting in a higher level of welfare when policies are coordinated. In addition, optimal combined policy would generate a higher welfare level than optimal monetary policy. Intuitively, since the Friedman rule is optimal for both policy regimes, which does not alter the consumption-leisure decision of individuals, the amount of leisure is unaffected and thus is identical under these two regimes. The welfare difference between these regimes mainly stems from the different extent of patent breath. Interestingly, the extent of patent breath only plays a role on labor reallocation between R&D and manufacturing production in this case. If patent protection under optimal monetary policy is stronger (weaker) than the counterpart under optimal combined policy, the former policy devotes too much (too little) labor from production to R&D yielding a stronger (weaker) growth effect but a weaker (stronger) consumption effect. Nonetheless, the welfare level is lower under the former policy in either situation.

Finally, we calibrate our model to match the US data. Our quantitative analysis shows that optimal monetary policy is superior than optimal patent policy in most cases. This result suggests that for plausible parameter values, policymakers can help the economy achieve a better welfare outcome by adopting monetary policy than using patent policy in an environment where the two policies instruments could not be coordinated.

1.1 Literature Review

Our study is closely related to the literature on economic growth and monetary policy. One strand of this literature explores the growth and welfare effects of monetary policy in the framework with R&D-based endogenous growth. The pioneer work by Marquis and Reffett (1994) firstly introduce a cash-in-advance constraint into the Romer's model and investigate the growth effect of

[4] See Chu and Lai (2013) for a similar prediction.

monetary policy. Funk and Kromen (2006, 2010) incorporate nominal price rigidity – a short-run new Keynesian feature – into a long-run R&D growth model to analyze the effects of inflation on economic growth. Recently, Chu and Lai (2013) and Chu, Cozzi, Lai, and Liao (2015) consider CIA constraints on R&D and consumption and examine the relation among inflation, growth and welfare in a closed economy and in an open economy, respectively. Moreover, Huang, Chang, and Ji (2015) study the same issue by considering various CIA constraints in an R&D growth model with endogenous market structure.

Our study also relates to the literature on economic growth and patent policy. The pioneer study in this literature is Judd (1985), who analyzes the effects of patent length on economic growth in a dynamic-general-equilibrium (DGE) framework; Iwaisako and Futagami (2003) and Futagami and Iwaisako (2007) also explore the effects of patent length on economic growth in addition to social welfare. Instead of patent length, more recent studies in a similar DGE setting, such as Li (2001), Kwan and Lai (2003), Furukawa (2007), and Cysne and Turchick (2012), focus on an alternative IPR policy tool – patent breath – against imitations, given that it is a better instrument to describe firms' market power. Thus, our paper follows this strand of studies by considering patent breath to represent the strength of IPR protection.

Motivated by the above two series of policy events, the literature that investigates the interactive effects of monetary and patent policies in a growth-theoretic framework is relatively rare. One notable exception is Chu, Lai, and Liao (2012), who firstly explore the growth and welfare effects of the interaction between monetary and patent policies in an endogenous growth model with expanding variety (i.e., Romer's model) and a CIA constraint on only consumption. Nevertheless, the present study follows Chu, Lai, and Liao (2012) to focus on the mutual interaction of these policies in a quality-ladder growth model (i.e., Grossman-Helpman model), in which CIA constrains on consumption and R&D are both taken into account as in Chu and Cozzi (2014). Furthermore, differing from their focus where the policy decisions are exogenously given, the policy variables in our model are determined by the related authorities for the purpose of welfare maximization. Consequently, the main contribution of this note is to provide a complete comparison of the optimal design of monetary and IPR protection policy regimes in order to analyze their impacts on growth and welfare.

2 The Model

To consider the optimal design of IPR policy and of monetary policy for comparing their growth and welfare effects in a Schumpeterian growth model, we adopt a version of the Grossman-Helpman quality-ladder model as in Chu and Cozzi (2014) where (a) patent breadth is introduced to determine the market power of monopolistic firms through markup, (b) cash-in-advance constraints on households' consumption and R&D investment are incorporated to model money demand, and (c) elastic labor supply is allowed.

2.1 Households

Suppose that at time t, each household has a population size of N_t, which grows at a rate of $n \geq 0$ such that $\dot{N}_t = nN_t$. There is a unit continuum of identical households, and each member's lifetime utility function is given by

$$U = \int_0^\infty e^{-\rho t} \left(\ln c_t - \theta l_t \right) dt, \tag{1}$$

where $\rho > 0$ represents the discount rate, c_t is the consumption of final goods per person, and l_t is the per capita supply of labor at time t. Following Chu, Lai, and Liao (2012), we use a form of felicity separable in consumption and labor supply with a unit intertemporal elasticity of substitution (IES) for consumption (which is 1) that is different from the IES of labor (which is infinity). The parameter $\theta > 0$ determines the intensity of leisure preference relative to consumption. The law of motion for assets of each household member is

$$\dot{a}_t + \dot{m}_t = (r_t - n)a_t + w_t l_t + \tau_t - c_t - (\pi_t + n)m_t + i_t b_t, \tag{2}$$

where a_t is the real asset value, r_t is the real interest rate, and w_t denotes the wage rate that each individual receives by supplying labor l_t. τ_t is the lump-sum transfer from the government (i.e., the monetary authority), π_t is the inflation rate that reflects the cost of holding money, and m_t is the real money balance that each household member holds in order to purchase consumption goods as well as to facilitate entrepreneur's loans b_t, which finance R&D investment on the return rate of i_t. Therefore, the CIA constraints are defined by $\xi c_t + b_t \leq m_t$, where $\xi \geq 0$ pins down the strength of the CIA constraint on consumption.

Maximizing (1) subject to the household member's asset accumulation yields the optimality condition for consumption such that

$$1/c_t = \gamma_t(1 + \xi i_t), \tag{3}$$

where γ_t is the Hamiltonian costate variable on (2). The optimality condition for labor supply is given by

$$w_t = \theta c_t(1 + \xi i_t), \tag{4}$$

and the intertemporal optimality condition is

$$-\dot{\gamma}_t/\gamma_t = r_t - \rho - n. \tag{5}$$

Combining (5) and the optimality condition for real money balance implies the familiar Fisher equation such that $i_t = r_t + \pi_t$.

2.2 Final Goods

Final goods y_t are produced competitively using a unit continuum of intermediate goods indexed by variety $s \in [0, 1]$ according to the standard Cobb-Douglas production function

$$\ln y_t = \int_0^1 \ln x_t(s) ds, \tag{6}$$

where $x_t(s)$ is the quantity of intermediate goods in variety s. Denote $p_t(s)$ as the price of $x_t(s)$ and assume that there is free entry into the final-goods sector. This assumption together with (6) yields the demand for variety s such that

$$x_t(s) = y_t/p_t(s). \tag{7}$$

2.3 Intermediate Goods

In each variety, there exists a monopolistic leader who holds a patent on the latest innovation to produce the intermediate goods. The leader's intermediate goods are replaced by the products of an entrant who has a new innovation due to the *Arrow replacement effect*. The current leader's production function for the intermediate goods is given by

$$x_t(s) = z^{q_t(s)} L_{x,t}(s), \tag{8}$$

where the parameter $z > 1$ measures the size of quality improvement, $q_t(s)$ is the number of innovations in variety s between time 0 and time t, and $L_{x,t}(s)$ is the employment level of production labor in this variety. Then, (8) implies that the marginal cost of producing intermediate goods for the leader is given by

$$mc_t(s) = w_t/z^{q_t(s)}. \tag{9}$$

In each variety, the current and previous leaders participate in Bertrand competition. Similar to previous studies such as Li (2001), Goh and Olivier (2002), and Iwaisako and Futagami (2013), patent breadth is introduced as a policy instrument, which is set by patent authority, represents the markup that the leader is able to charge over the marginal cost. Thus, the current leader's profit-maximizing price is

$$p_t(s) = \mu_t mc_t(s), \tag{10}$$

where $b_t \in (0, \infty]$ denotes the degree of patent breadth and we use $\mu_t = z^{b_t} > 1$ to represent patent breadth throughout for simplicity. The case with $b_t = 1$ corresponds to the canonical quality-ladder model with complete protection against imitation as in Grossman and Helpman (1991). Finally, the leader's profit in variety s is given by

$$\Pi_{x,t}(s) = (1 - 1/\mu_t)\, p_t(s) x_t(s) = (1 - 1/\mu_t)\, y_t = (\mu_t - 1) w_t L_{x,t}(s), \tag{11}$$

where substituting (6)-(10) into $\Pi_{x,t}(s)$ yields the second and third equalities.

2.4 Innovations and R&D

The value of owning a monopolistic firm in variety s is denoted as $v_t(s)$. Following the standard literature (see, for example, Cozzi, Giordani, and Zamparelli (2007)), we assume a symmetric equilibrium, yielding that $\Pi_{x,t}(s) = \Pi_{x,t}$ and $v_t(s) = v_t$ for $s \in [0, 1]$. Denote λ_t as the *aggregate-level* Poisson arrival rate of innovation. Then, the Hamilton-Jacobi-Bellman (HJB) equation for v_t is given by

$$r_t v_t = \Pi_{x,t} + \dot{v}_t - \lambda_t v_t, \tag{12}$$

which is the no-arbitrage condition for the asset value. In equilibrium, the return on this asset $r_t v_t$ equals the sum of the flow payoffs $\Pi_{x,t}$, the potential capital gain $\dot{v}_t$, and the capital loss $\lambda_t v_t$ when creative destruction occurs.

New innovations in each variety are invented by a unit continuum of R&D firms indexed by $j \in [0,1]$ and each of these firms employs a level of R&D labor $L_{r,t}(j)$ for producing inventions. We follow Chu and Cozzi (2014) and Huang, Chang, and Ji (2015) to incorporate a CIA constraint on R&D investment at time t, such that households lend the j-th entrepreneur an amount $B_t(j) = b_t(j)N_t$ of money, which finances the wage payment for R&D labor $w_t L_{r,t}$ on the return rate of i_t. Thus, the expected profit of the j-th R&D firm is

$$\Pi_{r,t}(j) = v_t \lambda_t(j) - (1 + \alpha i_t) w_t L_{r,t}(j), \tag{13}$$

where $\alpha \in [0,1]$ represents the strength of the CIA constraint on R&D. Moreover, we formulate the *firm-level* arrival rate of innovation $\lambda_t(j)$ as in Chu and Cozzi (2014) such that

$$\lambda_t(j) = \varphi L_{r,t}(j)/N_t, \tag{14}$$

which captures the dilution effect that removes scale effects as in Laincz and Peretto (2006). In equilibrium, the aggregate-level arrival rate of innovation equals the firm-level counterpart for each variety, namely, $\lambda_t = \lambda_t(j)$. In addition, free entry into the R&D sector implies the following zero-expected-profit condition for the R&D firms

$$v_t \lambda_t = (1 + \alpha i_t) w_t L_{r,t}. \tag{15}$$

This equation is a condition that pins down the allocation of labors between production and R&D.

2.5 Monetary Authority

Denote the nominal money supply as M_t and its growth rate as $\Phi_t \equiv \dot{M}_t/M_t$. Then, the real money balance is given by $m_t N_t = M_t/p_t$, where p_t is the price of final goods. Consider that the growth rate of money supply Φ_t is a policy instrument that can be exogenously set by monetary authority. Hence, the inflation rate is determined by $\pi_t = \Phi_t - \dot{m}_t/m_t - n$. Furthermore, combining this condition with the Fisher equation, namely, $i_t = r_t + \pi_t$, reveals the relationship between the nominal interest rate and the nominal money supply such that[5]

$$i_t = \Phi_t + \rho. \tag{16}$$

Finally, the monetary authority redistributes the increase in money supply as a lump-sum transfer to households, namely, $\tau_t N_t = \dot{M}_t/p_t = \Phi_t m_t N_t = [(\pi_t + n)m_t + \dot{m}_t]N_t$.

3 Decentralized Equilibrium

An equilibrium consists of a sequence of allocations $[c_t, m_t, y_t, x_t(s), l_t, L_{x,t}(s), L_{r,t}(j)]_{t=0;s,j\in[0,1]}^{\infty}$ and a sequence of prices $[p_t(s), r_t, p_t, w_t, v_t]_{t=0;s\in[0,1]}^{\infty}$. Moreover, in each instant of time,

- households choose $[c_t, l_t]$ to maximize their utility given $[r_t, i_t, w_t]$;
- final-goods producers choose $[y_t]$ to maximize profits given $[p_t(s)]$;

[5] On the balance growth path, which will be characterized in Section 3, c_t, m_t, and $1/\gamma_t$ all grow at the same rate of $r_t - \rho - n$ due to the Euler equation.

• monopolistic leaders for intermediate-goods produce $[x_t(s)]$ and choose $[p_t(s), L_{x,t}(s)]$ to maximize profits given $[w_t]$;

• R&D firms choose $[L_{r,t}(j)]$ to maximize profits given $[w_t, v_t]$;

• the goods market clears such that $c_t N_t = y_t$;

• the labor market clears such that $L_{x,t} + L_{r,t} = l_t N_t$;

• the innovations value adds up to households' assets value such that $v_t = a_t N_t$;

• the R&D entrepreneurs finance their wage payments through borrowing such that $\alpha w_t L_{r,t} = b_t N_t$; and

• the monetary authority balances its budget such that $\tau_t N_t = (i_t - \rho) m_t N_t$.

3.1 Balanced Growth Path

In this section, we characterize the decentralized equilibrium for this model and show that the economy grows in a uniquely stable balanced growth path (BGP). Before doing so, we use (6) and (8) to derive $y_t = Z_t L_{x,t}$, where Z_t is defined as the aggregate technology such that $\ln Z_t \equiv \ln z \int_0^1 q_t(s) ds = \ln z \int_0^t \lambda_\varsigma d\varsigma$, where the second equality is given by the law of large numbers. Differentiating this equation with respect to time yields the growth rate of technology, i.e., $g_t = \dot{Z}_t / Z_t = \lambda_t \ln z$, where λ_t follows (14).

For an arbitrary path of patent breadth and the growth rate of money supply $[\mu_t, \Phi_t]_{t=0}^{\infty}$, we obtain the following result.

Lemma 1. *Holding constant μ and Φ, the economy jumps to a unique and stable balanced growth path.*

Proof. See the Appendix. □

According to (16), a constant Φ means that the nominal interest rate is stationary. Throughout the rest of this study, we will use i_t to represent the monetary policy instrument for simplicity.

Given a constant i, it can be shown that the equilibrium labor allocation is stationary along the BGP. Define $l_{x,t} \equiv L_{x,t}/N_t$ as manufacturing labor per capita and $l_{r,t} \equiv L_{r,t}/N_t$ as R&D labor per capita, respectively. First, the production-labor income in (11) implies $w_t = y_t/(\mu L_x)$. Combining this equation with (3), (4), and the resource condition (i.e., $y_t = c_t N_t$) immediately yields the equilibrium manufacturing labor per person

$$l_x = \frac{1}{\mu\theta(1+\xi i)}. \tag{17}$$

In addition, (11) and (12) imply $\rho + \lambda_t = \Pi_t / v_t$ along the BGP because $\dot{v}_t / v_t = \dot{\Pi}_t / \Pi_t = \dot{y}_t / y_t = \dot{c}_t / c_t + n = r_t - \rho$ from (5). Substituting (11), (14), (15), and the resource condition into the above condition yields the equilibrium R&D labor per person

$$l_r = \frac{\mu - 1}{\mu\theta(1+\xi i)(1+\alpha i)} - \frac{\rho}{\varphi}. \tag{18}$$

Lastly, the labor-market-clearing condition simply implies that the equilibrium labor supply per person equals the sum of $l_{x,t}$ and $l_{r,t}$, which is

$$l = \frac{\mu + \alpha i}{\mu\theta(1+\xi i)(1+\alpha i)} - \frac{\rho}{\varphi}. \tag{19}$$

4 Welfare Effects of Patent Policy and Monetary Policy

In this section, given that patent authority and monetary authority in the government share a common objective for maximizing social welfare, we first consider the optimality of one policy instrument when the other tool is fixed at a constant level in order to design optimal single policies, which are optimal patent policy (PP) and optimal monetary policy (MP), respectively. In addition, we consider an experiment in which both policy levers are adjusted in unison to establish an optimal mix of policy instruments (CP). Then, we compare the growth and welfare effects of these optimal policy regimes. However, it is useful to note that assuming the separable utility function in (1) implies that the intertemporal elasticity of substitution of labor is infinity, so the first-best labor allocation does not exist in this model. In this case, the optimal policy instruments are obtained by maximizing households' lifetime utility through combining the equilibrium labor allocation (i.e., second-best outcomes).[6]

Consequently, imposing the balance growth on (1) yields

$$U = \frac{1}{\rho}\left(\ln c_0 + \frac{g}{\rho} - \theta l\right), \tag{20}$$

where $c_0 = Z_0 l_x$ and $g = (\varphi \ln z) l_r$. Dropping the exogenous term Z_0 and using (17)-(19), optimal patent policy and optimal monetary policy are obtained by maximizing the discounted lifetime utility of households $U = \frac{1}{\rho}\left(\ln l_x + (\varphi \ln z) l_r/\rho - \theta l\right)$ with respect to μ and i, respectively.

4.1 Optimal Patent Policy

Suppose that patent policy is implemented for maximizing welfare while policymakers are restricted to change monetary policy (i.e., the nominal interest rate), which is held constant. Thus, given $i \geq 0$, substituting (17)-(19) into (20) and maximizing it with respect to μ yields

$$\frac{\partial U}{\partial \mu} = 0 \Rightarrow \bar{\mu} = \frac{\varphi \ln z/(\theta\rho) + \alpha i}{(1+\xi i)(1+\alpha i)}, \tag{21}$$

[6] As will be shown, using the utility function as in Chu, Lai, and Liao (2012) leads to the result that for a predetermined level of patent breadth, optimal monetary policy is always given by zero nominal interest rate (namely, the Friedman rule) regardless of the presence of the CIA constraints on consumption and/or R&D. Chu and Cozzi (2014) instead use a separable utility function $u(c_t, l_t) = \ln c_t + \theta \ln(1 - l_t)$ with the IES for consumption equaling the IES of leisure (i.e., $1 - l_t$), which derives the socially optimal labor allocations. When the CIA constraint on R&D is present in their model, the (sub)optimality of the Friedman rule depends on whether R&D over-(under-) investment occurs in the zero-nominal-interest-rate equilibrium. However, in this setting there does not exist a closed-form solution for the optimal positive nominal interest rate (if it exists), and hence this complicates the subsequent welfare analysis as compared to optimal patent policy and the optimal mix of these policies.

which pins down the level of optimal patent breadth $\bar{\mu}$.[7] Additionally, to ensure that patent breadth in (21) is greater than unity, we focus on a range of values i that is bounded by an upper bound such that

Assumption 1. $i < \frac{-1+\sqrt{1+4\alpha(\varphi \ln z/(\theta\rho)-1)/\xi}}{2\alpha}$.

Note that Assumption 1 implies $\varphi > \theta\rho/\ln z$. In this case, the equilibrium labor allocation under this policy scheme $\{l_x(\bar{\mu}, i), l_r(\bar{\mu}, i), l(\bar{\mu}, i)\}$ follows (17)-(19) such that:

$$l_x(\bar{\mu}, i) = \frac{1}{\bar{\mu}\theta(1+\xi i)}. \tag{22}$$

$$l_r(\bar{\mu}, i) = \frac{\bar{\mu}-1}{\bar{\mu}\theta(1+\xi i)(1+\alpha i)} - \frac{\rho}{\varphi}. \tag{23}$$

$$l(\bar{\mu}, i) = \frac{\bar{\mu}+\alpha i}{\bar{\mu}\theta(1+\xi i)(1+\alpha i)} - \frac{\rho}{\varphi}, \tag{24}$$

where $\bar{\mu}$ is given by (21) and i depends on the presence of the Friedman rule.

4.2 Optimal Monetary Policy

Suppose that monetary policy is set to maximize social welfare while patent policy (i.e., patent breadth) is difficult to be altered (due to antitrust laws, e.g., Chu, Cozzi, and Galli (2012)), which is fixed at some predetermined level. Therefore, given μ, substituting (17)-(19) into (20) and taking the first-order condition with respect to i yields

$$\frac{\partial U}{\partial i} = \begin{bmatrix} -\alpha^2\xi i^2\theta(\mu-1+\xi i\mu) - \xi((\varphi\ln z/\rho)(\mu-1)+\xi i\mu\theta) \\ +\alpha(-(1+2\xi i)(\varphi\ln z/\rho)(\mu-1)+\theta(\mu-1-2\xi^2 i^2\mu)) \end{bmatrix} /[\mu\theta(1+\xi i)^2(1+\alpha)^2]. \tag{25}$$

This equation determines the optimal nominal interest rate $\hat{i}$. Then we have the following results.

Lemma 2. *When the nominal interest rate is chosen for maximizing welfare, then the Friedman rule is always optimal regardless of whether a CIA constraint on consumption and/or R&D is present.*

Proof. First, it can be seen that when one of the CIA constraints is absent (i.e., either $\alpha = 0$ or $\xi = 0$), we obtain that $\partial U/\partial i|_{\alpha=0} = -\xi i - (1-1/\mu)\varphi\ln z/(\theta\rho) < 0$ or $\partial U/\partial i|_{\xi=0} = (1-1/\mu)(1-\varphi\ln z/(\theta\rho))/(1+\alpha i) < 0$. This result implies that the Friedman rule is optimal in these cases (i.e., $\hat{i} = 0$).

Nevertheless, the optimality of the Friedman rule also applies when both CIA constraints on consumption and R&D are present. Denote the numerator of (25) as a function of i, that is $\Omega(i)$. For any given $\mu > 0$, $\Omega(i)$ is a cubic function of i. For $i \geq 0$, we have $\Omega''(i) = -2\alpha\xi\theta(2\xi\mu + \alpha(\mu - 1 + 3\xi i\mu)) < 0$. Furthermore, we know that $\Omega(i)|_{i=0} = -((\alpha+\xi)(\varphi\ln z/\rho) - \alpha\theta)(\mu-1) < 0$ and $\Omega'(i)|_{i=0} = -\xi(2\alpha(\varphi\ln z/\rho)(\mu-1) + \xi\mu\theta) < 0$. Also, the turning points of $\Omega(i)$ are represented by

[7] Note that $\partial^2 U/\partial\mu^2 = -\frac{2(\varphi\ln z/(\theta\rho)+\alpha i)}{\mu^3(1+\xi i)(1+\alpha i)} + \frac{1}{\mu^2}$, which (locally) satisfies the second-order condition at $\bar{\mu}$.

the roots of $\Omega'(i) = 0$, which are

$$i^{\pm}|_{\Omega'(i)=0} = \frac{-\theta(\alpha(\mu-1) + 2\xi\mu) \pm \sqrt{\theta M(\varphi)}}{3\alpha\xi\theta\mu}, \tag{26}$$

where $M(\varphi) \equiv \alpha^2\theta(\mu-1)^2 - 2\alpha\xi(3(\varphi \ln z/\rho) - 2\theta)(\mu-1)\mu + \xi^2\theta\mu^2$. Denote $\hat{\varphi} \equiv \frac{\theta\rho\left((\alpha(\mu-1)+\xi\mu)^2 + 2\alpha\xi\mu(\mu-1)\right)}{6 \ln z \alpha\xi\mu(\mu-1)}$, which is greater than the lower bound of φ implied by Assumption 1. Specifically, when $\varphi < \hat{\varphi}$, $M(\varphi) > 0$ implying that there exist two negative turning points in $\Omega(i)$; when $\varphi = \hat{\varphi}$, $M(\varphi) = 0$ implying that there exists a negative inflection point in $\Omega(i)$; and when $\varphi > \hat{\varphi}$, $M(\varphi) < 0$ implying that (26) does not exist so $\Omega(i)$ is monotonically decreasing in i. In any of the above three situations, $\Omega(i)|_{i \geq 0}$ is decreasingly concave in i. So $\partial U/\partial i$ in (25) is always negative implying that U achieves its maximum when $\hat{i} = 0$; the Friedman rule is optimal regardless of the level of μ. Figure 1 shows the case with two turning points in $\Omega(i)$. □

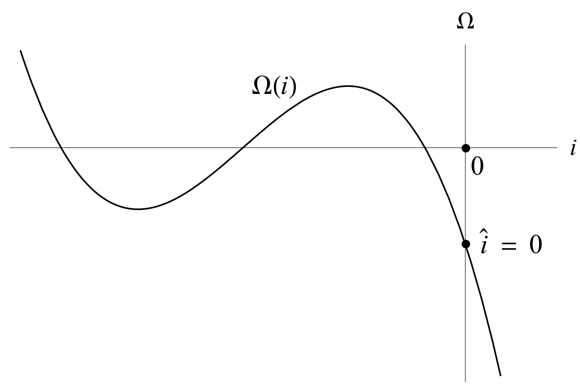


Figure 1: Diagram of $\Omega(i)$ with two turning points

Consequently, substituting $\hat{i} = 0$ into (17)-(19) yields the equilibrium labor allocation under this policy regime $\left\{l_x(\mu, \hat{i}), l_r(\mu, \hat{i}), l(\mu, \hat{i})\right\}$ such that:

$$l_x(\mu, \hat{i}) = \frac{1}{\mu\theta}, \tag{27}$$

$$l_r(\mu, \hat{i}) = \frac{\mu - 1}{\mu\theta} - \frac{\rho}{\varphi}, \tag{28}$$

$$l(\mu, \hat{i}) = \frac{1}{\theta} - \frac{\rho}{\varphi}. \tag{29}$$

4.3 Optimal Combination of Policy Instruments

Suppose that patent authority and monetary authority coordinate their behaviors; both patent and monetary policies can be jointly adjusted so that policymakers set an optimal mix of policy instruments $\{\mu^*, i^*\}_{t=0}^{\infty}$ simultaneously in order to maximize social welfare.[8] It is straightforward

[8] In reality, we do not admit that such a cooperation in policy design exists between patent authority and monetary authority. Nonetheless, patent policy affects production and research labor through monopolistic markup, whereas

to see that the optimal monetary policy instrument i^* is given by the Friedman rule because for a fixed level of patent breadth, the welfare level is always decreasing in i (see the derivation in Section 4.2). Also, welfare maximization with respect to μ implies optimal patent breadth is given by (21) with $i^* = 0$ such that

$$\mu^* = \frac{\varphi \ln z}{\theta \rho}, \tag{30}$$

which is greater than $\bar{\mu}$ for a given i. Notice that these optimal policy choices are unaffected by the absence of CIA constraints on consumption and/or R&D since it does not change the optimality of the Friedman rule. In this case, substituting μ^* in (30) and $i^* = 0$ into (17)-(19) yields the equilibrium labor allocations under this policy scheme $\{l_x(\mu^*, i^*), l_r(\mu^*, i^*), l(\mu^*, i^*)\}$ such that

$$l_x(\mu^*, i^*) = \frac{1}{\mu^* \theta}, \tag{31}$$

$$l_r(\mu^*, i^*) = \frac{\mu^* - 1}{\mu^* \theta} - \frac{\rho}{\varphi}, \tag{32}$$

$$l(\mu^*, i^*) = \frac{1}{\theta} - \frac{\rho}{\varphi}. \tag{33}$$

4.4 Optimal Patent Policy versus Optimal Monetary Policy

This subsection compares the growth rate and the welfare level under optimal patent policy to those under optimal monetary policy. Recall that the nominal interest rate i under PP and patent breadth μ under MP are fixed at a constant level (namely, they are considered parameters). Substituting $\bar{\mu}$ in (21) into $l_r(\bar{\mu}, i)$ in (23) and comparing it to $l_r(\mu, \hat{i})$ in (28) yields

$$l_r(\bar{\mu}, i) - l_r(\mu, \hat{i}) = \frac{\mu \left[\frac{\varphi \ln z/(\theta\rho) + \alpha i}{(1+\xi i)(1+\alpha i)} - 1\right] - (\mu - 1)\left[\varphi \ln z/(\theta\rho) + \alpha i\right]}{\bar{\mu}\mu\theta(1+\xi i)(1+\alpha i)}, \tag{34}$$

which is positive (negative) if $\mu < (>) \frac{(\varphi \ln z/(\theta\rho) + \alpha i)(1+\alpha i)(1+\xi i)}{1 + i(\alpha + \xi + \alpha\xi i)(1 + \varphi \ln z/(\theta\rho) + \alpha i + \xi i)} \equiv \tilde{\mu}_1$, and $\tilde{\mu}_1 > 1$ according to Assumption 1. Hence, for a given level of i, (14) and the growth equation that $g_t = \lambda_t \ln z$ imply that in terms of allocating R&D labor for promoting innovations and economic growth, the effect of a sufficiently large patent breadth along with the Friedman rule under MP can outweigh the effect of optimal patent breadth under PP.[9] Moreover, comparing $l(\bar{\mu}, i)$ in (24) and $l(\mu, \hat{i})$ in (29) yields

$$\frac{l(\bar{\mu}, i) + \rho/\varphi}{l(\mu, \hat{i}) + \rho/\varphi} = \frac{\bar{\mu} + \alpha i}{\bar{\mu}(1+\alpha i)(1+\xi i)} \leq 1, \tag{35}$$

monetary policy affects them through CIA constraints on consumption and R&D. Thus, it is interesting to analyze how taking both policy tools into consideration would improve social welfare by affecting labors through different channels.

[9] Of course, this condition that determines the growth difference between PP and MP can be rewritten as a comparison between the level of i under PP and a threshold value $\tilde{i}$ for a given μ. Nevertheless, using the nominal interest rate for comparison significantly complicates the analysis of the growth difference between these optimal policy regimes and does not change the results in welfare differences.

meaning that MP always allocates no less labor (i.e., no more leisure) than PP. Then, comparing $l_x(\bar{\mu}, i)$ in (22) and $l_x(\mu, \hat{i})$ in (27) yields

$$\frac{l_x(\bar{\mu}, i)}{l_x(\mu, \hat{i})} = \frac{\mu}{\bar{\mu}(1+\xi i)}, \tag{36}$$

which is greater (smaller) than 1 if $\mu > (<) \frac{\varphi \ln z/(\theta\rho)+\alpha i}{1+\alpha i} \equiv \tilde{\mu}_2$, and it can be shown that $\tilde{\mu}_2 \geq \tilde{\mu}_1$ where the equality holds when $i = 0$.[10]

As for the welfare difference, denote $\bar{U}$ and $\hat{U}$ as the lifetime utility along the BGP under PP and under MP, respectively. Using (20), a direct comparison of the discounted utility (i.e., ρU) yields

$$\rho\bar{U} - \rho\hat{U} = \underbrace{\ln\frac{l_x(\bar{\mu}, i)}{l_x(\mu, \hat{i})}}_{\substack{\text{consumption effect} \\ (+) \text{ or } (-)}} + \underbrace{\frac{\varphi \ln z}{\rho}\left(l_r(\bar{\mu}, i) - l_r(\mu, \hat{i})\right)}_{\substack{\text{growth effect} \\ (+) \text{ or } (-)}} + \underbrace{\theta\left(l(\mu, \hat{i}) - l(\bar{\mu}, i)\right)}_{\substack{\text{leisure effect} \\ (+)}}. \tag{37}$$

(37) shows that the welfare difference between these optimal policy regimes features three effects resulting from the labor-allocation comparison: (a) the consumption effect, depending on the production-labor levels; (b) the growth effect, depending on the R&D-labor levels, and (c) the leisure effect, which is no weaker under PP than under MP. Moreover, substituting $\bar{\mu}$ and $\hat{i}$ into (37) reveals that

$$\begin{aligned} \rho\bar{U} - \rho\hat{U} &= \ln\frac{\mu}{\bar{\mu}(1+\xi i)} + \frac{\varphi \ln z}{\theta\rho}\left[\frac{\mu(\bar{\mu}-1) - \bar{\mu}(\mu-1)(1+\alpha i)(1+\xi i)}{\mu\bar{\mu}(1+\alpha i)(1+\xi i)}\right] + \left[1 - \frac{\bar{\mu}+\alpha i}{\bar{\mu}(1+\alpha i)(1+\xi i)}\right] \\ &\geq -\frac{i(\varphi \ln z/(\theta\rho) - 1)(\alpha(\mu-1)(1+\xi i) + \xi\mu)}{\mu(1+\xi i)(1+\alpha i)}, \end{aligned} \tag{38}$$

where in the inequality we use the property of natural logarithm such that $\ln\frac{\mu}{\bar{\mu}(1+\xi i)} \geq 1 - \frac{\bar{\mu}(1+\xi i)}{\mu}$ and the term in the second line is negative. Thus, (38) indicates that the welfare difference between PP and MP could be either positive or negative.

As shown in (37)-(38), the labor allocations between these optimal policy schemes are different, which are affected by the predetermined level of the monetary instrument (i) and that of the patent instrument (μ), respectively. On the one hand, the effect of i on all of the labors is negative through ξ; an increase in the nominal interest rate causes households to decrease consumption and increase leisure, which leads to a reduction in labor supply as shown in (19). Thus, both manufacturing labor and R&D labor decrease. Furthermore, the effect of i on R&D investment is also negative through α in (18); an increase in i makes R&D more costly leading to an additional reduction in the R&D labor, but it does not change the production labor as in (17). One the other hand, the effect of μ through monopolistic markup on R&D investment is positive such that a larger patent breadth increases the profits in the intermediate-goods sector, which attracts more innovations by shifting labor from production to R&D. In addition, the effect of μ on consumption production is negative such that a larger patent breadth enhances the market power of the monopoly firms, which reduces

[10] Suppose that $\mu = \tilde{\mu}_1$. Then the level of R&D labor is identical under both optimal policies (i.e., $l_r(\bar{\mu}, i) = l_r(\mu, \hat{i})$). In this case, it must be true that $\tilde{\mu}_1 = \mu \leq \tilde{\mu}_2$ in order to satisfy (35).

the demands for intermediate goods and consumption. Totally, an increase in μ decreases labor supply. Hence, the impacts of these two instruments on labor allocations are not identical. The difference in labor allocations that is affected by the interactions of these predetermined instruments will accordingly change the signs and magnitudes of the effects as decomposed in (37), which induces the ambiguity of the welfare comparison in (38).

Although the analytical comparison for a more welfare-improving optimal policy regime is ambiguous, we can explore how the welfare difference is altered by the level of a predetermined policy lever. On the one hand, for a given i, taking the derivative of $\rho\bar{U} - \rho\hat{U}$ with respect to μ yields

$$\frac{\partial \rho(\bar{U} - \hat{U})}{\partial \mu} = \frac{\mu - \varphi \ln z/(\theta\rho)}{\mu^2}. \tag{39}$$

Therefore, $\rho(\bar{U} - \hat{U})$ is decreasing (increasing) in μ when $\mu < (>) \varphi \ln z/(\theta\rho) = \mu^*$ and reaches its minimum at $\mu = \mu^*$, implying that the welfare difference is a U-shaped function in patent breadth under MP. On the other hand, for a given μ, taking the derivative of $\rho\bar{U} - \rho\hat{U}$ with respect to i yields

$$\begin{aligned}\frac{\partial \rho(\bar{U} - \hat{U})}{\partial i} &= \frac{(\varphi \ln z/(\theta\rho) - 1)[\alpha(1 + \xi i(1 + \xi i(1 + \alpha i))) - (\alpha + \xi + 2\alpha\xi i)(\varphi \ln z/(\theta\rho))]}{(1 + \alpha i)^2(1 + \xi i)^2(\varphi \ln z/(\theta\rho) + \alpha i)} \\ &< \frac{(\varphi \ln z/(\theta\rho) - 1)[\alpha(1 + \xi i(1 + \xi i(1 + \alpha i))) - (\alpha + \xi + 2\alpha\xi i)]}{(1 + \alpha i)^2(1 + \xi i)^2(\varphi \ln z/(\theta\rho) + \alpha i)} \\ &= -\frac{(\varphi \ln z/(\theta\rho) - 1)[\xi(1 + \alpha i)(1 - \alpha\xi i^2)]}{(1 + \alpha i)^2(1 + \xi i)^2(\varphi \ln z/(\theta\rho) + \alpha i)} < 0,\end{aligned} \tag{40}$$

where the second line applies $\varphi > (\theta\rho)/\ln z$ implied by Assumption 1 and the third uses the fact that α, ξ, and i are no greater than 1. In other words, the welfare difference $\rho(\bar{U} - \hat{U})$ is decreasing in the nominal interest rate i under PP. The above result implies that fixing the nominal interest rate, there exists a threshold level of patent breadth that maximizes the welfare level of optimal monetary policy; in contrast, fixing patent breadth, the zero nominal interest rate maximizes the welfare level of optimal patent policy.

Proposition 1. *Suppose that Assumption 1 holds. Then optimal monetary policy generates a higher equilibrium growth rate than optimal patent policy if patent protection in the economy is initially strong (i.e., $\mu > \tilde{\mu}_1$). Moreover, the welfare difference between these regimes is a U shape in patent breadth under optimal monetary policy and is decreasing in the nominal interest rate under optimal patent policy.*

Proof. Proven in the text. □

Notice that the above results remain in the absence of a CIA constraint on consumption and/or R&D. To gain a better understanding of the ambiguity of the welfare difference between optimal patent policy and optimal monetary policy, in Section 5 we calibrate this model for the US economy and conduct a numerical exercise to quantify this welfare difference. We also reveal the shapes for the effects of predetermined instruments on welfare differences, which are consistent with the model's predictions.

4.5 Optimal Single Policy versus Optimal Combined Policy

In this subsection, we consider the growth and welfare differences of the optimal coordination of policy instruments as compared to optimal single policies. In fact, optimal combined policy corresponds to a special case with $i = 0$ under optimal patent policy or with $\mu = \mu^*$ under optimal monetary policy.

First, we compare optimal combined policy (CP) and optimal patent policy (PP). For $i > 0$ under PP, comparing (23) and (32) shows that CP allocates more R&D labor than PP, such that

$$\frac{l_r(\mu^*, i^*) + \rho/\varphi}{l_r(\bar{\mu}, i) + \rho/\varphi} = \frac{\varphi \ln z/(\theta\rho) - 1}{\frac{\varphi \ln z/(\theta\rho) + \alpha i}{(1+\xi i)(1+\alpha i)} - 1} \left(1 + \frac{\alpha i}{\varphi \ln z/(\theta\rho)}\right) > 1. \tag{41}$$

Intuitively, optimal patent breadth under CP is broader than the counterpart under PP because $\frac{\mu^*}{\bar{\mu}} = \frac{(1+\xi i)(1+\alpha i)}{1+\alpha i/[\varphi \ln z/(\theta\rho)]} > 1$. Along with the zero nominal interest rate used in CP, the effects of the policy tools on R&D investment under CP are stronger than under PP, which develop more innovations and stimulate economic growth. Also, given that CP is invariant of i, an increase in i reduces the growth rate under PP due to a decrease in R&D investment, which enlarges the growth difference.

Moreover, denote U^* as the lifetime utility along the BGP under CP. Similarly, the discounted welfare difference between PP and CP (namely, $\rho\bar{U} - \rho U^*$) can be decomposed as in (37) in terms of labor allocation comparisons, which feature (a) a positive consumption effect (i.e., $l_x(\bar{\mu}, i) > l_x(\mu^*, i^*)$), (b) a negative growth effect, as discussed in (41) (i.e., $l_r(\bar{\mu}, i) < l_r(\mu^*, i^*)$), and a positive leisure effect (i.e., $l(\bar{\mu}, i) < l(\mu^*, i^*)$). Substituting μ^*, i^*, and $\bar{\mu}$ into this welfare difference and simplifying it as in (38) reveals that the negative effect dominates the two positive effects, such that

$$\rho\bar{U} - \rho U^* < -\frac{i(\varphi \ln z/(\theta\rho) - 1)[\alpha((1+\xi i)\varphi \ln z/(\theta\rho) - 1) + \xi\varphi \ln z/(\theta\rho)]}{(1+\xi i)(1+\alpha i)[\alpha i + \varphi \ln z/(\theta\rho)]} < 0. \tag{42}$$

This result reflects that optimal combined policy is more welfare-enhancing than optimal patent policy, and the growth effect is the most important determinant for this welfare difference. Interestingly, the effect of i on $\rho(\bar{U} - U^*)$ is equivalent to that in (40), implying that the welfare difference is monotonically decreasing in i.

Next, we compare optimal combined policy (CP) and optimal monetary policy (MP). Suppose that μ under MP does not coincide with μ^*, which is given by (30). Then using (28) and (32) yields

$$\frac{l_r(\mu^*, i^*) + \rho/\varphi}{l_r(\mu, \hat{i}) + \rho/\varphi} = \left(\frac{\mu^* - 1}{\mu^*}\right) \frac{\mu}{\mu - 1}, \tag{43}$$

which is smaller (greater) than 1 when $\mu > (<) \mu^*$. In other words, given that the optimality of the Friedman rule holds under both MP and CP, MP can generate a higher equilibrium growth rate than CP if initial patent protection under MP is sufficiently strong, which induces a higher level of R&D labor as compared to the case under CP. Furthermore, denoting $g(\mu, i)$ as the growth rate of technology under a regime and taking into consideration the threshold $\tilde{\mu}_1$, it is known that the ranking of the growth rates across these optimal policy regimes depends on μ under MP. Specifically, if $\mu > \mu^*$, then $g(\mu, \hat{i}) > g(\mu^*, i^*) > g(\bar{\mu}, i)$; if $\tilde{\mu}_1 < \mu < \mu^*$, then $g(\mu^*, i^*) > g(\mu, \hat{i}) > g(\bar{\mu}, i)$; and if $\mu < \tilde{\mu}_1$, then $g(\mu^*, i^*) > g(\bar{\mu}, i) > g(\mu, \hat{i})$. Moreover, the difference in (43) is decreasing in

μ, meaning that the positive (negative) gap between $g(\mu, \hat{i})$ and $g(\mu^*, i^*)$ enlarges (shrinks) as μ increases.

As for the welfare difference between these optimal policy regimes (namely, $\rho\hat{U} - \rho U^*$), which is decomposed analogously as in (37), comparing (29) and (33) implies that both MP and CP allocate the same amount of labor forces yielding a zero leisure effect (i.e., $l(\mu, \hat{i}) = l(\mu^*, i^*)$). In addition, from (43), it is obvious that the consumption effect and the growth effect have opposite signs depending on whether the magnitude of patent breadth under optimal monetary policy exceeds the threshold value μ^*. Nevertheless, substituting μ^*, i^*, and $\hat{i}$ into this welfare comparison and simplifying it shows that the negative effect overwhelms the positive one, such that

$$\rho\hat{U} - \rho U^* = \ln\frac{\mu^*}{\mu} + \mu^* \left(\frac{\mu - 1}{\mu} - \frac{\mu^* - 1}{\mu^*} \right) < 0, \tag{44}$$

where in the inequality we use the property of natural logarithm again such that $\ln\frac{\mu^*}{\mu} < \frac{\mu^*}{\mu} - 1$. Accordingly, optimal combined policy is also more effective than optimal monetary policy in terms of increasing welfare. In addition, $\partial\rho(\hat{U} - U^*)$ in (44) is increasing (decreasing) in μ when $\mu < (>) \mu^*$ and reaches its maximum at $\mu = \mu^*$, implying that the welfare difference between MP and CP is an inverted U-shaped function in μ. This result simply corresponds to the welfare comparison in (39) when the special case of $i = 0$ under PP arises, which is equivalent to CP.

To summarize, optimal monetary policy adopting the Friedman rule should be implemented only if the target of policymakers is to promote economic growth and a high degree of patent protection is given; otherwise, an optimal mix of policy instruments generally yields a higher growth rate and more social welfare than the other two optimal regimes adjusting a single policy lever. It is worthwhile noting that when patent breadth under optimal monetary policy is predetermined at the welfare-maximizing level under optimal combined policy (i.e., $\mu = \mu^*$), the policy-tool selection between these two regimes becomes identical implying that the growth and welfare differences are no longer present. This argument similarly applies to the situation in which the nominal interest rate under optimal patent policy is predetermined by the Friedman rule (i.e., $i = i^* = 0$).

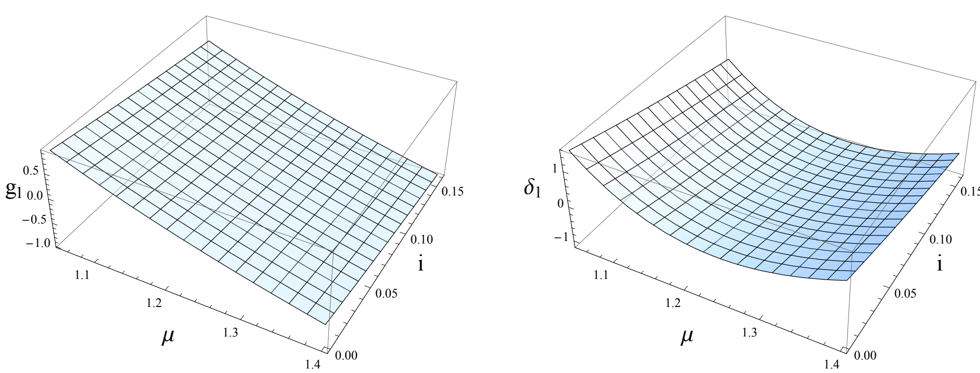
Proposition 2. *Suppose that Assumption 1 holds. Then optimal combined policy generates a higher equilibrium growth rate than optimal single policies if initial patent breadth in the economy is narrower than μ^*. However, optimal combined policy always (weakly) dominates optimal single policies in terms of increasing welfare.*

Proof. Proven in the text. □

5 Quantitative Analysis

In this section, we calibrate the model for the US economy to numerically evaluate the differences of growth rates and social welfare among the optimal policy regimes. To undertake this numerical exercise, steady-state values are assigned to the following structural parameters $\{\rho, z, \varphi, \phi, \alpha, \xi, i, \mu\}$ in the model. We follow Acemoglu and Akcigit (2012) to set the discount rate ρ to 0.05 and the step size of innovation to 1.05. We set the strength of the CIA constraint on consumption ξ to 0.16 for matching the ratio of M1 to consumption in the US and choose the counterpart on R&D investment of $\alpha = 1$ in the benchmark case. To calibrate the productivity parameter φ and the leisure intensity θ, we use the empirical long-run growth rate of GDP per capita in the US, which is 2%. However, following Comin (2004) and Chu and Cozzi (2014), we consider that the contribution of R&D investment drives only a fraction of long-run economic growth in the US, and we set this fraction to 0.4 suggested by the estimation in Chu (2010). As for the nominal interest rate i, which is predetermined under optimal patent policy, we focus on a range of values $[0, 0.16]$ and set its market/standard level to the long-run average value of 8% for matching the empirical moments.[11] As for the predetermined level of patent breadth μ under optimal monetary policy, we take into account a range of values $[1.05, 1.4]$ according to the empirical estimate of markup reported in Jones and Williams (2000), and its market/standard level is set to the average value of 1.225. Finally, using the standard level of i and of μ with (17) to compute the equilibrium growth rate $g = \varphi \ln z L_r$ and setting a usual moment of l to 1/3 with (19) yields the calibrated value of $\varphi = 3.57$ and of $\phi = 2.80$, respectively.

First, in Figure 2 we quantify the growth and welfare differences between optimal patent policy and optimal monetary policy (i.e., g_1 and δ_1).[12] It shows that MP tends to generate a higher (lower) rate of economic growth than PP as μ becomes larger (smaller), which is consistent with the result predicted by our model. The largest (absolute) growth difference between MP and PP can reach 0.98% in which $\mu = 1.4$ and $i = 0.16$. Over the interested ranges of μ and i, MP is generally more growth-enhancing than PP, since the average growth rate $g(\mu, \hat{i})$ under MP is roughly 0.11% higher than the counterpart $g(\bar{\mu}, i)$ under PP. Additionally, the shape for the welfare differences is in line with the implication of Proposition 1; δ_1 is U-shaped in μ for a given i but is decreasing in i for a given μ, and MP could yield a lower or higher level of social welfare than PP because δ_1 can be both positive and negative. The largest welfare difference between PP and MP is around 1.54% in which $\mu = 1.05$ and $i = 0$. Nevertheless, our calibration implies that PP is less welfare-improving than MP in most cases given that the average value of δ_1 is approximately -0.24%.[13]



(a) Growth Differences (b) Welfare Differences

Figure 2: Growth and welfare differences between patent policy and monetary policy.

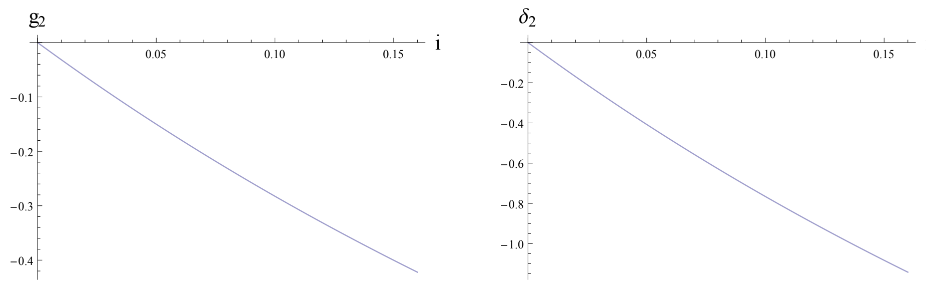
Second, as compared to optimal combined policy, Figures 3 and 4 show the discrepancies in growth and welfare of optimal patent policy (i.e., g_2 and δ_2) and of optimal monetary policy (i.e.,

[11] Data source: World Development Indicators.

[12] We express welfare differences as the usual equivalent variation in consumption flow denoted by $\delta \equiv \exp(\rho \Delta U) - 1$.

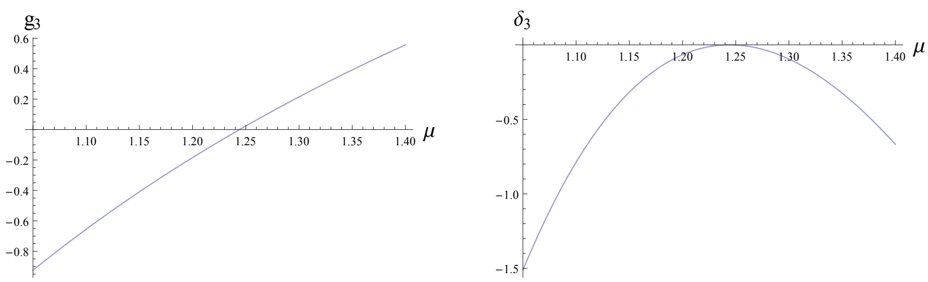
[13] Notice that the mean of welfare differences in this note is calculated by the average value of the function δ over the ranges of μ and i, that is $\frac{\int \int_{\mathcal{D}} \delta d\mu di}{\int \int_{\mathcal{D}} d\mu di}$ where $\mathcal{D} = [1.05, 1.4] \times [0, 0.16]$.

g_3 and δ_3), respectively. As predicted by our model, the growth rate under CP is at least as high as under PP since $\bar{\mu} \leq \mu^* = 1.24$. A similar pattern also applies to the welfare difference δ_2 unless i under PP equals 0. As i deviates from $i^* = 0$, the growth and welfare differences become larger (up to -0.42% and -1.14%, respectively). In contrast, the growth rate $g(\mu, \hat{i})$ under MP is higher (lower) than the counterpart $g(\mu^*, i^*)$ under CP when μ deviates further above (below) μ^*. However, CP yields a growth rate of 0.98%. It is about 0.12% higher than the average growth rate under MP over the range of μ and can be even 0.92% higher than the growth rate under MP when $\mu = 1.05$. Moreover, the inverted U shape of δ_3 with respect to μ is consistent with our previous finding, in which the maximum (i.e., no welfare loss) occurs at $\mu = \mu^*$, because the policy tools under MP coincide with those under CP in this case. The largest difference of δ_3 reaches roughly 1.51% and it occurs when μ lies at its lower bound 1.05. Table 1 presents the average growth rates and welfare differences of these optimal policy regimes over our calibrated ranges of μ and i.



(a) Growth Differences (b) Welfare Differences

Figure 3: Growth and welfare differences between combined policy and patent policy.



(a) Growth Differences (b) Welfare Differences

Figure 4: Growth and welfare differences between combined policy and monetary policy.

5.1 CIA Parameters on R&D and Consumption

Finally, we examine the sensitivity of this quantitative analysis by varying the strength of the CIA constraint on R&D α and on consumption ξ, respectively. A wide range of values for the strength of the CIA constraint on R&D investment (consumption) is considered, namely $\alpha \in [0, 1]$

($\xi \in [0, 1]$), where each value of α (ξ) corresponds to a specific value of φ and of θ.[14] Tables 1 and 2 display the main changes in the alternative sets of structural parameters and the corresponding results for each robustness check.

In particular, Table 1 shows that as α declines, the average value of $\tilde{\mu}_1$ decreases, which is lower than the average value of μ over the calibrated range. Therefore, the average growth rate under MP is always higher than under PP. The average value of μ^* is also increasing in α due to a lower φ/θ, and the average growth rate under MP is higher than under CP only when α is no greater than 0.6, for which μ^* is lower than the average value of μ. In addition, it is shown that as α declines, both δ_1 and δ_2 increase but δ_3 decreases. Intuitively, over the range of i, the average value of $\bar{\mu}$ gets closer to μ^* as α declines. Hence, the welfare differences between PP and CP that are caused by a different choice in optimal patent breadth are reduced, which yields an increase in δ_2. Furthermore, for a given μ, (44) implies that $\frac{\partial \rho(\hat{U}-U^*)}{\partial \mu^*} = \frac{1}{\mu^*} - \frac{1}{\mu} > 0$ when $\mu > \mu^*$. Then, a decline in α, which reduces μ^*, makes more choices of μ satisfy the above condition. As a result, there is a decrease in δ_3 due to a lower level of μ^*. Finally, combining the two comparisons of δ_2 and δ_3 leads to an increase in δ_1.

Table 1: The growth rates and welfare differences among optimal policy regimes under $\alpha \in [0, 1]$.

α	0	0.2	0.4	0.6	0.8	1
φ	3.34	3.39	3.44	3.48	3.53	3.57
θ	2.83	2.83	2.82	2.82	2.81	2.80
$\tilde{\mu}_1$	1.14	1.15	1.16	1.17	1.18	1.19
$\bar{\mu}$	1.14	1.15	1.17	1.18	1.20	1.21
μ^*	1.15	1.17	1.19	1.20	1.23	1.24
$g(\bar{\mu}, i)$	0.44%	0.50%	0.58%	0.62%	0.70%	0.75%
$g(\mu, \hat{i})$	0.78%	0.80%	0.82%	0.83%	0.85%	0.86%
$g(\mu^*, i^*)$	0.51%	0.60%	0.71%	0.78%	0.89%	0.98%
δ_1	0.30%	0.18%	0.05%	-0.04%	-0.15%	-0.24%
δ_2	-0.19%	-0.25%	-0.32%	-0.39%	-0.50%	-0.61%
δ_3	-0.49%	-0.42%	-0.37%	-0.35%	-0.35%	-0.37%

Notes: The benchmark parameter set is $\rho = 0.05$, $z = 1.05$, $\varphi = 3.57$, $\phi = 2.8$, $\alpha = 1$, $\xi = 0.16$, $\mu \in [1.05, 1.4]$, and $i \in [0, 0.16]$. $\tilde{\mu}_1$, $\bar{\mu}$, $g(\bar{\mu}, i)$, $g(\mu, \hat{i})$, δ_1, δ_2, and δ_3 display the respective average value over the ranges of μ and i for a specific value of α.

In Table 2, we find that as ξ declines from 1 towards the benchmark case, the average value of $\tilde{\mu}_1$ increases and μ^* decreases. However, the average value of μ is greater than $\tilde{\mu}_1$ and is smaller than μ^*, so that the average growth rate under MP is higher than under PP but is lower than under CP. Moreover, the magnitudes of the welfare differences in this robustness check are generally larger than in the benchmark case (except $\xi = 0$); a decline in ξ decreases μ^* and leads $\bar{\mu}$ to become closer to μ^*, which is similar to the effects of a decrease in α. Hence, the qualitative pattern of the welfare differences under the variation of ξ is the same as under the variation of α.

[14] Chu, Cozzi, Lai, and Liao (2015) estimate α to 0.33 for the US and 0.56 for the Euro Area in a two-country monetary Schumpeterian model with CIA constraints on consumption and R&D investment.

Table 2: The growth rates and welfare differences among optimal policy regimes under $\xi \in [0, 1]$.

ξ	0	0.16	0.4	0.6	0.8	1
θ	2.84	2.8	2.75	2.71	2.67	2.63
$\tilde{\mu}_1$	1.19	1.19	1.18	1.18	1.17	1.17
$\bar{\mu}$	1.21	1.21	1.21	1.21	1.21	1.21
μ^*	1.23	1.24	1.26	1.29	1.30	1.32
$g(\bar{\mu}, i)$	0.75%	0.75%	0.75%	0.74%	0.74%	0.75%
$g(\mu, \hat{i})$	0.85%	0.86%	0.88%	0.90%	0.92%	0.94%
$g(\mu^*, i^*)$	0.89%	0.98%	1.09%	1.18%	1.28%	1.38%
δ_1	0.06%	-0.24%	-0.70%	-1.10%	-1.51%	-1.93%
δ_2	-0.29%	-0.61%	-1.12%	-1.59%	-2.10%	-2.64%
δ_1	-0.35%	-0.37%	-0.42%	-0.50%	-0.59%	-0.71%

Notes: The benchmark parameter set is $\rho = 0.05$, $z = 1.05$, $\varphi = 3.57$, $\phi = 2.8$, $\alpha = 1$, $\xi = 0.16$, $\mu \in [1.05, 1.4]$, and $i \in [0, 0.16]$. $\tilde{\mu}_1$, $\bar{\mu}$, $g(\bar{\mu}, i)$, $g(\mu, \hat{i})$, δ_1, δ_2, and δ_3 display the respective average value over the ranges of μ and i for a specific value of ξ.

6 Conclusion

In this note, we compare the different implications of patent policy and monetary policy on economic growth and social welfare in a scale-invariant Schumpeterian growth model. Patent policy is incorporated by patent breadth determining the markup of monopolistic firms in the intermediate-goods sector, whereas monetary policy is introduced by the nominal interest rate determining the cash-in-advance constraints for consumption and R&D investment. It is found that implementing optimal monetary policy is more effective than implementing optimal patent policy in terms of promoting economic growth if the initial degree of patent protection is sufficiently strong; the optimality of the monetary tool (i.e., the Friedman rule) along with a high level of patent breadth helps allocate more R&D labor to developing more innovations than using the optimal patent tool. Nevertheless, the welfare difference between optimal patent policy and optimal monetary policy is ambiguous, depending on the levels of the predetermined instruments in these policy regimes. Thus, this result provides a rationale for either policy to be implemented in terms of raising social welfare. Finally, we conduct a policy experiment for an optimal coordination of patent policy and monetary policy. The growth rate under this optimal combined policy is always no lower than under optimal patent policy, but it may be higher or lower than under optimal monetary policy. In addition, optimal combined policy always yields a welfare level that is at least as high as optimal single policies. This outcome suggests that an interdepartmental authority (i.e., a superagency) choosing an optimal mix of policies would improve welfare rather than a single authority.

Appendix

Proof for Lemma 1

This proof follows the counterpart in Chu, Lai, and Liao (2012), but there is an extra CIA constraint on R&D. Given that Φ_t is constant, substituting (5) into the Fisher equation implies

$$i_t = r_t + \pi_t = \rho - \dot{\gamma}_t/\gamma_t + \Phi - \dot{m}_t/m_t, \tag{45}$$

where we use π_t in the real money balance. Differentiating the log of (3) with respect to time yields

$$\xi \dot{i}_t / (1 + \xi i_t) = -\dot{\gamma}_t / \gamma_t - \dot{c}_t / c_t. \tag{46}$$

Combining the resource constraint $y_t = c_t N_t$ and the final-goods production function $y_t = Z_t L_{r,t}$, we know that c_t grows at the rate of $g_t + g_{L_r,t} - n$ where g_t and $g_{L_r,t}$ are the growth rate of technology Z_t and of R&D labor $L_{r,t}$, respectively. Using the production-labor income $w_t L_{x,t} = y_t / \mu$ in (11) and the R&D entrepreneurs' balanced budget $\alpha w_t L_{r,t} = b_t N_t$ implies that $(b_t / c_t)(L_{x,t} / L_{r,t}) = \alpha / \mu$, so that c_t and b_t grow at the same rate provided that $L_{x,t}$ and $L_{r,t}$ grow at the same rate as $l_t N_t$ to satisfy the labor-market-clearing condition. Consequently, for the per capita CIA constraints to bind, we have $\xi c_t + b_t = m_t$, implying that m_t must grow at the rate as c_t does and that m_t / c_t is constant. Using the above facts, substituting (45) into (46) yields

$$\dot{i}_t = (i_t + 1/\xi)(i_t - \rho - \Phi). \tag{47}$$

This dynamic system of i_t is characterized by the saddle-point stability given that i_t is a control variable. Hence, i_t must jump to its steady-state value $i_t = \rho + \Phi$, which is consistent with (16).

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