Evolutionary Game of Labor Division in a Perfectly Competitive Economy

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Abstract. This paper inspects the coordination of individuals' decisions on specialization and the determination of labor division in a large competitive economy from a perspective of evolutionary game theory. It is shown that the equilibrium structure of labor division defined by new classical economics is in fact an evolutionarily stable state. In an evolutionary economic environment where the law of "survival of the fittest" prevails, individuals' pursuits of self-interest can spontaneously lead to a steady state of labor division without any central planning or bargaining processes that coordinate individuals' decisions in an intended way. In this state, resource allocation is not only Pareto efficient, but also beneficent as if it was chosen by a welfare planner who holds a welfare criterion with aversion of inequality.

Key words: specialization, labor division, evolutionary game, evolutionarily stable strategy,

new classical economics

1. Introduction

Since the 70's of last century, a growing literature on specialization and labor division emerges, represented by Rosen (1978, 1983), Baumgardner (1988), Yang & Boland (1991) , Becker & Murphy (1992), and Yang & Ng 1993 (See Yang & Ng (1998) for a complete survey). Among these authors, Yang & Ng 1993 and their followers have developed a theoretical framework based on inframarginal analysis which regards all aspects of labor division as endogenous outcomes of the market (Yang & Ng, 1998). In the recent decade, this theoretical framework, named as new classical framework/economics by Yang & Ng (1993), has been broadly applied to various fields of economics, such as economic growth and development, trade theory, urbanization and industrialization, institutional economics, macroeconomics and economic policies, public economics and political economics (See Cheng & Yang, 2003 for a more complete survey). However, as a developing theory, new classical economics still has some unsolved problems in its foundation. This paper takes into account of one of them. That is the lack of a reasonable coordination mechanism compatible with the equilibrium concept it defined.

In new classical literature, the general equilibrium of a large economy is defined as an efficient allocation of population over different patterns of specialization that clears all markets, equalizes individuals' utility and reaches the maximal level of such equalized utility level (See Sun, Yang & Yao, 1999; Yang & Ng, 1993, 1995; Yang, 2001). According to the definition, in every particular model of new classical economics, the structure of labor division that makes markets clear and individuals obtain maximal equalized utility is picked out from the other possibilities as a simulation of the real state of economy and hence a basis of consequent applications. If it is true that the function of the market is “not only to efficiently allocate resources, but also to search the efficient levels of specialization and division of labor” (Yang & Ng, 1993, p.11), and to make people equally well-being, the concept above will result in a reasonable prediction of the economic state and all applications based on such a prediction is acceptable. However, the problem is “*why it is true*”.

To answer this question, new classical economics follows neoclassical framework (See Arrow & Debreu 1954 for a complete discussion) that regards the competitive market as a Walrasian regime and individuals’ decisions as independent responses to explicit price signal (See Yang & Ng, 1993; and Yang, 2001). However, the shortage of this approach is obvious. Firstly, as noticed by Yang & Ng (1993, p.71) themselves, Walrasian regime is open to criticism since it has not explicitly spelt out the operation process of the functioning of competitive markets (Kreps, 1990). Secondly, as I will show in Section 2, even ignoring the above problem, Walrasian regime is still not a sufficient simulation of the determination mechanism of labor division since it is ineligible in coordinating individuals’ decisions on specialization which are usually of mixed strategy.

In order to avoid the above problems, Nash’s bargaining model was introduced by Yang & Ng (1993, p.71-93) as another explanation of the determination mechanism of labor division. The bargaining process is a possible way of determination of labor division; however, such a process happens only when “no action taken by one of the individuals without the consent of the others can affect the well-being of the others” (Nash, 1950). This implies the bargaining model is applicable only when the population is small and well organized as a collective. In a large economy where millions of people trade thousands of goods among them as the modern society we can see, individuals' behaviors might influence one another; however, they are separate decision makers and do not act as a collective which allocates resources via common choice or bargaining process. Bargaining model is no doubt not a reasonable explanation of the determination of labor division at the social level.

Since the existing literature has not shown how the efficient state of labor division, defined as general equilibrium, can result from decentralized decisions of individuals as a "spontaneous order" in a competitive circumstance with large population, this paper attempts to afford a possible explanation based on the theory of evolutionary games. Using a simple economy for instance, it is shown that the equilibrium of labor division defined by new classical economics cannot be attained in a Walrasian regime (Section 2). However, if we take into account of the evolutionary aspects of the competitive economy, we will see the determination of labor division is in fact an evolutionary process, in which people interact at the population level (Section 3) and the equilibrium defined by new classical economics will be reached by the economic system as the evolutionarily stable state (Section 4). It is shown that in this stable state, resource allocation is efficient and beneficent (Section 5).

2. Can Walrasian regime coordinate labor division?

In this section, I shall use a simple economy to review the equilibrium concept defined in new classical economics and explicate the difficulty of attaining such equilibrium in a Walrasian regime.

Throughout the paper, we will consider an simple economy $E = (I, u, G, L, k, a)$ with an infinite set of *ex ante* identical consumer-producers, I , a set of three consumer goods $G \equiv \{x, y, z\}$, and one type of resource L. moreover, each individual has the same utility function

$$u(x, x^d, y, y^d, z, z^d) = (x + kx^d)(y + ky^d + z + cz^d) \qquad (1)$$

where x, y and z are the self-provided quantities of the three consumer goods respectively, x^d , y^d and z^d are the quantities of them she purchases from others, $k, c \in [0,1]$ represent the transaction efficiencies of the three goods. It is assumed that x and y has the same transaction efficiency k , while z has a transaction efficiency c .

Each individual also has the same system of production functions and the same time constraint as following

$$x - x^s = l_x^a; \quad y - y^s = l_y^a \qquad z - z^s = l_z^a \qquad 2$$

$$l_x + l_y + l_z = 1 \qquad 3$$

where x^s, y^s and z^s are the negative quantities of the three goods she sells to others; l_x , l_y and l_z are the amounts of the resource (labor) input in producing the three goods respectively. The total available quantity of resource for each individual is L, which is simplified as one unit. $a > 1$ indexes the degree of the economy of specialization. While the economy is assumed to be a Walrasian regime, every individual is a price accepter. Let p_y, p_z denote the price of y and z measured by x respectively. Each individual has a budget constraint as below

$$x^s + p_y y^s + p_z z^s + x^d + p_y y^d + p_z z^d = 0 \qquad 4$$

To sum up, the individual's decision problem is to choose a production plan $(l_x, l_y, l_z) \in \mathbb{R}_+^3$ and a trade plan $(x^d, y^d, z^d, x^s, y^s, z^s) \in \mathbb{R}_+^3 \times \mathbb{R}_-^3$ to maximize her utility function (1), subjected to conditions (2)-(4). Let $\tilde{S} \equiv \{(l_x, l_y, l_z; x^d, y^d, z^d, x^s, y^s, z^s) \in \mathbb{R}_+^6 \times \mathbb{R}_-^3 :$ satisfying (2) and (3)}. The individual's decision problem (1)-(4) is then equivalent to maximization of function

$$v(s) = (l_x^a + x^s + kx^d)(l_y^a + y^s + ky^d + l_z^a + y^s + cz^d) \qquad (5)$$

by choosing $s = (l_x, l_y, l_z; x^d, y^d, z^d, x^s, y^s, z^s) \in S$, which is called a production-trade plan, subjected to constraint (4).

In new classical economics, the *general equilibrium* (Yang & Ng, 1993) or *competitive equilibrium* (Sun, Yang & Yao, 1999) of this economy is defined as a price vector (p_y^*, p_z^*) and a list of individuals' production-trade plans $\{s_i^*\}_{i \in I}$, satisfying the market clearing conditions and utility equalization conditions and maximizing the equalized utility. The general equilibrium can be obtained by a multiple-step approach as following (See Yang & Ng, 1993, p.52-70, or Sun, Yang & Yao, 1999 for a more complete discussion):

(i) Configuration[1]. According to Wen Theorem, in economy E, for any $(p_y, p_z) \in \mathbb{R}_+^2$, the optimal choice of the individual must be one of the five configurations (see Table 1.).

(ii) Corner equilibrium. Three market structures[2] are available in this economy. A corner equilibrium exists in every market structure, giving the distribution of population over different configurations, the market clearing price, and the optimal production-trade plan for individuals in one market structure (see Table 2.).

(iii) General equilibrium. Given parameters, the corner equilibrium that reaches the highest utility level is the general equilibrium of the economy, which gives a distribution of population in different configurations, called the equilibrium structure of labor division.

Configuration	***Utility***	***Production plan & Trade plan***

[1] In new classical literature, a profile of zero and positive values of decision variables in the individual's decision problem is called a configuration if it is consistent with Wen Theorem (Wen, 1998).

[2] Every possible combination of configurations that is compatible with the market clearing condition is called a market structure.

A Autarky	$u(A) = 4^{-a}$	$l_x = l_y = 1/2$ or $l_x = l_z = 1/2$; $x^s = x^d = y^s = y^d = z^s = z^d = 0$
X/Y sell x and buy y	$u(X/Y) = \dfrac{k}{4p_y}$	$l_x = 1$; $x^s = 1/2$, $y^d = x^s / p_y$
Y/X sell y and buy x	$u(Y/X) = \dfrac{kp_y}{4}$	$l_y = 1$; $y^s = 1/2$, $x^d = p_y y^s$
X/Z sell x and buy z	$u(X/Z) = \dfrac{k}{4p_z}$	$l_x = 1$; $x^s = 1/2$, $z^d = x^s / p_z$
Z/X sell z and buy x	$u(Z/X) = \dfrac{cp_z}{4}$	$l_z = 1$; $z^s = 1/2$, $x^d = p_z z^s$

Table 1.

Market structure	***utility***	***Corner equilibrium***
A	$u_A = 4^{-a}$	Each individual chooses autarky
X-Y	$u_{XY} = k/4$	1/2 population choose X/Y, 1/2 choose Y/X; $p_y = 1$
X-Z	$u_{XZ} = \sqrt{ck}/4$	$\sqrt{k}/(\sqrt{k}+\sqrt{c})$ population choose X/Z, the others choose Z/X; $p_z = \sqrt{k/c}$

Table 2.

In neoclassical economics, the competitive market is abstracted as a Walrasian regime or Walrasian auction mechanism, where the structure of labor division is given, individuals' decisions are coordinated by a Walrasian auctioneer, and the competitive equilibrium is reached when all markets are clear and each individual maximizes his objective function. However, can Walrasian regime coordinate individuals' decisions on specialization to endogenize the equilibrium structure of labor division defined above?

To see the answer, let's suppose $k = 9/16$, $c = 1/4$ and $a = 2$ in economy E. According to the above analysis, the general equilibrium is the corner equilibrium of market structure X-Y because $u_{XY} > u_{YZ}$ and $u_{XY} > u_A$. Assume the initial state of economy is at the corner equilibrium of X-Z, i.e. 60% of the population choose X/Z and 40% choose Z/X. If the Walrasian auctioneer gives the first price vector, say $(p_y, p_z) = (3, 2)$, all individuals will choose Y/X as their optimal choice after comparing the utilities of the five configurations under this price. Thus the economy deviates from the corner equilibrium of the market structure X-Z and shifts to a situation where every individual chooses Y/X, a configuration of market structure X-Y. If the auctioneer can coordinate individuals' decisions to reach the corner equilibrium of market structure X-Y, then we obtain the general equilibrium. However, individuals' decisions result in a positive excess supply of y, a positive excess demand of x, and a balance between the demand and supply of z in the market. Observing this, the auctioneer will decrease the price of y and remain the price of z.

In the whole process, the structure of labor division will not change until p_y decreases to 1. However, when p_y =1, there is no difference for an individual to choose X/Y or Y/X. Without any further information, she might choose these two options with any probability distribution. Because the auctioneer is not allowed to assign the configurations to individuals directly, to attain the corner equilibrium that requires half of the population choosing X/Y and the others choosing Y/X, the only way is to coordinate individuals' decisions on probabilities of taking the two different configurations (i.e. mixed strategies) to make the average frequency of choosing X/Y in the whole population equal to that of choosing Y/X. However, coordinating individuals' decisions on probability to obtain such a frequency distribution over different configurations is a job that beyond the auctioneer's capability.

It should be noticed that the above analysis does not reject the existence of a general equilibrium or competitive equilibrium (See Sun, Yang & Yao, 1999) in Walrasian regime because one can always find a frequency distribution over different configurations and a price vector to clear the markets, equalize the utility and maximize the equalized utility. For example, when $k = 9/16$, $c = 1/4$ and $a = 2$, any ($p_y, p_z, \{s_i\}_{i\in I}$) is a general equilibrium or competitive equilibrium if it satisfies the following conditions: 1 $p_y = 1, p_z < 9/4$; (2) there is a subset $I_0 \subset I$ such that $s_i = (1,0,0,1/2,1/2,0)$ for $\forall i \in I_0$, $s_i = (0,1,0,1/2,1/2,0)$ for $\forall i \notin I_0$, and I_0 has half population of the economy. Hence, the general equilibrium or competitive equilibrium does exist; the difficulty is that it is not reachable in the neoclassical Walrasian regime.

To sum up, new classical economics has given a definition of the equilibrium structure of labor division, but it fails in illuminating how such equilibrium spontaneously arises from decentralized decisions of individuals. Before we can show that such an equilibrium is attainable for the competitive economy without any fictitious coordination, it is arbitrary to apply such a concept to other issues.

3. An environment of evolutionary games

Neoclassical economics abstracts the function of competitive economies in coordinating individuals' decisions as a Walrasian regime. However, this fails to take into account of all the evolutionary features of real markets. As many studies (See Fudenberg & Levine 1998, Hofbauer & Sigmund 1988, Mailath 1998, Samuelson 1997, van Damme 1991, Vega-Redondo 1996, Weibull 1995, and Young 1998 for surveys) have shown, the competitive economy is a typical environment of evolution, where the strategies or behaviors (e.g. selling or buying) are *inherited* in a "asexual" way by individuals' memory and learning, the "*selection* mechanism" works in the way that relatively fitter strategies are more likely imitated and spread in the population, and *mutations* also happen occasionally while some individuals experiment their new strategies that are unrelated with the strategies' payoffs in the previous. Such a dynamic process makes the natural law of "survival of the fittest" prevail.

The evolutionary aspects of human economies have been well understood and do not require further discussion here. Since the evolution of economic behavior may be a "wing-form" type that has nothing to do with the interactions among individuals but only related with the exogenous conditions, the main concern of this section is to show that the decision of specialization and hence the determination of the structure of labor division at the social level is a game-form or "dispersal-behavior" type evolution, rather than independent optimizations (See Maynard Smith 1982 Chapter 1 for the discussion of evolution of wing form and of dispersal behavior in soaring birds).

To start our analysis, let's introduce some notations and definitions. Let h be the function from real numbers to real numbers such that $h(r)=\begin{cases}0, r=0\\1, r\neq 0\end{cases}$. For $\forall s \in \mathcal{S}$, define $h(s)=(h(l_x),h(l_y),h(l_z);h(x^d),h(y^d),h(z^d),h(x^s),h(y^s),h(z^s))$. We have an equivalent relation $\approx$ on $\mathcal{S}$ defined by $\forall s \approx s' \Leftrightarrow h(s)=h(s')$, $\forall s, s' \in \mathcal{S}$. The relation $\approx$ divides $\mathcal{S}$ into several equivalent classes. Each equivalent class is formally called a *pure strategy*, which is a subset of $\mathcal{S}$, containing a group of production-trade plans that are equivalent under $\approx$. A pure strategy defines a pattern of specialization or economic behaviors of the individual: what to produce, what to buy and what to sell. It is obvious that any configuration is a pure strategy. Naturally, each probability distribution on the set of pure strategies, is called a *mixed strategy*.

Because for any given price in the market, an individual's optimal production-trade plan must belongs to one of the five configurations (i.e. A, X/Y, Y/X, X/Z, and Z/X), there are only five pure strategies relevant with our analysis. Let $S=\{S_1, S_2, S_3, S_4, S_5\}$ be the set of these five pure strategies, where S_1, S_2, S_3, S_4, and S_5 represent A, X/Y, Y/X, X/Z, and Z/X respectively. The set of mixed strategies is $\Delta=\{(\delta_1,\delta_2,\delta_3,\delta_4,\delta_5)\in \mathbb{R}_{+}^{5}:\sum_{j=1}^{5}\delta_j=1\}$, where $\delta_1, \delta_2, \delta_3, \delta_4$, and δ_5 are probabilities of choosing S_1, S_2, S_3, S_4, and S_5 respectively. While δ_j is explained as the fraction of the population who choose strategy S_j, any $\delta \in \Delta$ also represents a possible distribution of population over different configurations and is called a population state.

In the economy, an individual needs to do three things in one period: firstly, choosing a mixed strategy; secondly, using the mixed strategy to determine a pure strategy randomly; thirdly, (if necessary) joining the market to trade goods with other people. The market is perfectly competitive such that every individual regards the price as given, and maximizes his utility at the market clearing price by choosing a production-trade plan compatible with his pure strategy. To see this economy is an environment of evolutionary games, we need to confirm that each individual's fitness (the utility or real income after behavior) depends on both his own strategy and the population state.

As shown in Table 2, there are three market structures in the economy. The first market, denoted as AA, is a degenerated one which consists of individuals choosing autarky and contains no exchange at all. The second market, denoted as X-Y, is a place where people trade x and y. The third market, denoted as X-Z, is a place where people trade x and z. Table 1 has listed the utility values and optimal production-trade plans of different pure strategies. If the population state is $\eta=(\eta_1,\eta_2,\eta_3,\eta_4,\eta_5)\in\Delta$, the total supply of x in market X-Y is $\eta_2/2$, and the total demand of x in X-Y is $\eta_3 p_y/2$. Market clearing implies that $p_y=\eta_2/\eta_3$. Similarly, we can obtain the market clearing price in X-Z as $p_z = \eta_4 / \eta_5$. Thus, p is a function of population state η, which can be denoted by $p(\eta)$. According to results of Section 1, utility of an individual choosing pure strategy S_j and facing with the price $p = (p_y, p_z) \in \mathbb{R}_+^2$ can be defined by $V(S_j, p) \equiv \max\{v(s) : s \in S_j$ and subjected to (4)}. These conclusions make the fitness, denoted by π, of an individual choosing strategy S_j a function of η such that $\pi(S_j, \eta) = V(S_j, p(\eta))$. In particular, for any $\eta = (\eta_1, \eta_2, \eta_3, \eta_4, \eta_5) \in \Delta$, we have $\pi(S_1, \eta) = 4^{-a}$, $\pi(S_2, \eta) = \frac{k\eta_3}{4\eta_2}$, $\pi(S_3, \eta) = \frac{k\eta_2}{4\eta_3}$, $\pi(S_4, \eta) = \frac{k\eta_5}{4\eta_4}$, and $\pi(S_5, \eta) = \frac{k\eta_4}{4\eta_5}$. Finally, an individual who chooses mixed strategy δ and faces with a population state η will have a fitness $\pi(\delta, \eta) = \sum_{j=1}^{5} \delta_j \pi(S_j, \eta)$.

It is clear now that in the economy each individual's fitness depends not only on his own probabilities of choosing different pure strategies, but also the frequencies of the different pure strategies prevailing in the whole population. This fact makes the economic environment discussed a typical case of evolutionary games. Evolutionary games are first formulated by biologists to study the biologic evolution at the phenotypic level, especially when stable population states are observed (See Maynard Smith 1982 for a more complete discussion of the origin of the theory). In my paper, each configuration can be regarded as a phenotype that the individual can choose, and the real income or utility can be explained as fitness. Moreover, because fitness of particular phenotypes here depends on their frequencies in the population, i.e., the success of an individual depends on what others are doing, game theory becomes relevant (Maynard Smith, 1982, p.1).

4. Evolutionarily stable structure of labor division

By now, a typical environment of evolution has been presented: An infinite set of *ex ante* individuals compete in the market repeatedly; each individual's fitness (utility or real income) depends on his mixed strategy and the population state and determines the number of individuals who inherit his strategy. In this environment, we naturally hope to know if some certain population state is able to exist stably in the long run under the "selection". And if yes, will it be the equilibrium population state (structure of labor division) defined by new classical economics? This section will answer these questions.

A concept called *evolutionarily stable strategy* or *state* (ESS) was first introduced by Maynard Smith & Price (1973) and Maynard Smith (1974), and has been broadly used by their follower to explain the existence of a stable population state in an evolutionary system. This concept is intended to capture the following idea: A population state $\eta \in \Delta$ that is stable must be unbeatable (Hamilton, 1967) or uninvadable (Friedman, 1991). I.e., once the population is in the state of η then any small fraction of the population employing any deviant behavior (an invasion of mutants) will eventually disappear under selection. When the natural law of "survival of the fittest" applies, this leads to the following definition:

Definition 1: A vector $\eta \in \Delta$ is said to be an evolutionarily stable strategy or state, if for any $\eta' \in \Delta$, $\eta' \neq \eta$, $\exists \bar{\varepsilon} > 0$ such that the inequality

$$\pi(\eta', \varepsilon\eta' + (1-\varepsilon)\eta) < \pi(\eta, \varepsilon\eta' + (1-\varepsilon)\eta) \quad (8)$$ holds

for all $0 < \varepsilon \leq \bar{\varepsilon}$.

The implication of this definition is clear. Suppose the stable state is $\eta \in \Delta$. At any time, an arbitrarily small but positive proportion of the population may choose a deviant mixed strategy (mutation) $\eta' \neq \eta$. If they survive in the competition, then the original distribution will be changed. The stability of η requires that the mutation strategy disappears under natural selection given that η is taken by almost every individual. When the natural law of “survival of the fittest” applies, this will happen if individuals choosing the mutation strategy obtain a lower fitness than those sticking to the original one in the state where the original strategy coexists with the mutation.

The concept of ESS has been successfully applied to biologic issues such as the emergence of the stable population state with a certain “species” (see Holbauer & Sigmund, 1988 for a survay). While each of the five configurations listed in Table 1 is regarded as a “phenotype” that an individual can choose, we have the following result:

Theorem 1. In economy E, when $k < k_0 \equiv 4^{1-a}$ and $c < k_0$, $\eta^A \equiv (1,0,0,0,0)$ is the only ESS; when $k > k_0$ and $k > c$, $\eta^{XY} \equiv (0, \frac{1}{2}, \frac{1}{2}, 0, 0)$ is the only ESS; when $\sqrt{ck} > k_0$ and $c > k$, $\eta^{XZ} \equiv (0,0,0,\frac{\sqrt{k}}{\sqrt{k}+\sqrt{c}}, \frac{\sqrt{c}}{\sqrt{k}+\sqrt{c}})$ is the only ESS.

Proof. Suppose $k < k_0 \equiv 4^{1-a}$ and $c < k_0$. For any $\lambda = (\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5) \in \Delta$, $\lambda \neq \eta^A$, and any $\varepsilon \in (0,1)$, let $\tilde{\eta} \equiv \varepsilon\lambda + (1-\varepsilon)\eta^A = (\varepsilon\lambda_1 + (1-\varepsilon), \varepsilon\lambda_2, \varepsilon\lambda_3, \varepsilon\lambda_4, \varepsilon\lambda_5)$. We have $\pi(\eta^A, \tilde{\eta}) = u(A) = 4^{-a}$, and

$$\pi(\lambda, \tilde{\eta}) = [\varepsilon\lambda_1 + (1-\varepsilon)]u(A) + \varepsilon\sum_{j=2}^{5}\lambda_j \pi(S_j, \tilde{\eta}) = [\varepsilon\lambda_1 + (1-\varepsilon)]u(A) + \varepsilon\frac{k}{4}(1-\lambda_1) .$$

Since $k < k_0 \equiv 4^{1-a}$ and $c < k_0$ implies $u(A) = 4^{-a} > k/4$, we have $\pi(\lambda, \tilde{\eta}) < \pi(\eta^A, \tilde{\eta})$. This proves the first conclusion of the theorem.

Suppose $k > k_0$ and $k > c$. For any $\lambda = (\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5) \in \Delta$, $\lambda \neq \eta^{XY}$, and any $\varepsilon \in (0,1)$, let $\tilde{\eta} = \varepsilon\lambda + (1-\varepsilon)\eta^{XY} = (\varepsilon\lambda_1, \varepsilon\lambda_2 + (1-\varepsilon)/2, \varepsilon\lambda_3 + (1-\varepsilon)/2, \varepsilon\lambda_4, \varepsilon\lambda_5)$. We have $\pi(\eta^{XY}, \tilde{\eta}) = \frac{k}{8}[\frac{\varepsilon\lambda_2 + (1-\varepsilon)/2}{\varepsilon\lambda_3 + (1-\varepsilon)/2} + \frac{\varepsilon\lambda_3 + (1-\varepsilon)/2}{\varepsilon\lambda_2 + (1-\varepsilon)/2}]$, and

$$\pi(\lambda, \tilde{\eta}) = \sum_{j=1}^{5} \lambda_j \pi(S_j, \tilde{\eta}) = \lambda_1 u(A) + \frac{k}{4}[\lambda_2 \frac{\varepsilon\lambda_3 + (1-\varepsilon)/2}{\varepsilon\lambda_2 + (1-\varepsilon)/2} + \lambda_3 \frac{\varepsilon\lambda_2 + (1-\varepsilon)/2}{\varepsilon\lambda_3 + (1-\varepsilon)/2} + \lambda_4 + \lambda_5)$$

$$< \frac{k}{4}(\lambda_1 + \lambda_4 + \lambda_5) + \frac{k}{4}[\lambda_2 \frac{\varepsilon\lambda_3 + (1-\varepsilon)/2}{\varepsilon\lambda_2 + (1-\varepsilon)/2} + \lambda_3 \frac{\varepsilon\lambda_2 + (1-\varepsilon)/2}{\varepsilon\lambda_3 + (1-\varepsilon)/2}].$$

There are five cases to discuss: (i) $\lambda_2 = \lambda_3 < 1/2$; (ii) $\lambda_2 < \lambda_3 \leq 1/2$; (iii) $\lambda_3 < \lambda_2 \leq 1/2$; (iv) $\lambda_2 < 1/2 < \lambda_3$; (v) $\lambda_3 < 1/2 < \lambda_2$. For Case (i), it is easy to see $\pi(\eta^{XY}, \tilde{\eta}) > \pi(\lambda, \tilde{\eta})$ holds for any $\varepsilon \in (0,1)$. For Case (ii), let $\frac{\varepsilon\lambda_3 + (1-\varepsilon)/2}{\varepsilon\lambda_2 + (1-\varepsilon)/2} = t$. Because for any $\varepsilon \in (0,1)$, there must be $t > 1 > \frac{1-2\lambda_2}{1-\lambda_2}$, we have $(1/2 - \lambda_2)t + (1/2 - \lambda_3)\frac{1}{t} \geq (1 - \lambda_2 - \lambda_3)$. This also implies $\pi(\eta^{XY}, \tilde{\eta}) > \pi(\lambda, \tilde{\eta})$ holds for any $\varepsilon \in (0,1)$. Similar results are also available in the other cases. The second conclusion is proved.

Suppose $\sqrt{ck} > k_0$ and $c > k$. For any $\lambda = (\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5) \in \Delta$, $\lambda \neq \eta^{XY}$, and any $\varepsilon \in (0,1)$, let $\tilde{\eta} = \varepsilon\lambda + (1-\varepsilon)\eta^{XZ} = (\varepsilon\lambda_1, \varepsilon\lambda_2, \varepsilon\lambda_3, \varepsilon\lambda_4 + (1-\varepsilon)\delta, \varepsilon\lambda_5 + (1-\varepsilon)(1-\delta))$, where $\delta = \frac{\sqrt{k}}{\sqrt{k} + \sqrt{c}}$. We have $\pi(\eta^{XZ}, \tilde{\eta}) = \frac{k}{4}\delta \frac{\varepsilon\lambda_5 + (1-\varepsilon)(1-\delta)}{\varepsilon\lambda_4 + (1-\varepsilon)\delta} + \frac{c}{4}(1-\delta)\frac{\varepsilon\lambda_4 + (1-\varepsilon)\delta}{\varepsilon\lambda_5 + (1-\varepsilon)(1-\delta)}$, and

$$\pi(\lambda, \tilde{\eta}) = \lambda_1 u(A) + \frac{k}{4}(\lambda_2 + \lambda_3) + \frac{k}{4}\lambda_4 \frac{\varepsilon\lambda_5 + (1-\varepsilon)(1-\delta)}{\varepsilon\lambda_4 + (1-\varepsilon)\delta} + \frac{c}{4}\lambda_5 \frac{\varepsilon\lambda_4 + (1-\varepsilon)\delta}{\varepsilon\lambda_5 + (1-\varepsilon)(1-\delta)}$$

$$< \frac{\sqrt{ck}}{4}(\lambda_1 + \lambda_2 + \lambda_3) + \frac{k}{4}\lambda_4 \frac{\varepsilon\lambda_5 + (1-\varepsilon)(1-\delta)}{\varepsilon\lambda_4 + (1-\varepsilon)\delta} + \frac{c}{4}\lambda_5 \frac{\varepsilon\lambda_4 + (1-\varepsilon)\delta}{\varepsilon\lambda_5 + (1-\varepsilon)(1-\delta)}.$$

Because $1 - \delta > \frac{1}{2} > \delta$, there are seven cases to discuss: (i) $\lambda_4 = \delta$ or $\lambda_5 = (1-\delta)$; (ii) $\lambda_4 \leq \lambda_5 < \delta$; (iii) $\lambda_5 \leq \lambda_4 < \delta$; (iv) $\lambda_4 < \delta < \lambda_5$; (v) $\lambda_5 < \delta < \lambda_4$; (vi) $\delta < \lambda_4 \leq \lambda_5 < 1 - \delta$; (vii) $\delta < \lambda_5 \leq \lambda_4 < 1 - \delta$. For Case (i), it is easy to see $\pi(\eta^{XZ}, \tilde{\eta}) > \pi(\lambda, \tilde{\eta})$ holds for any

$\varepsilon \in (0,1)$. For Case (ii), let $\dfrac{\varepsilon\lambda_3 + (1-\varepsilon)/2}{\varepsilon\lambda_2 + (1-\varepsilon)/2} = t$. Because for any $\varepsilon \in (0,1)$, there must be $t > 1 > \dfrac{1-2\lambda_2}{1-\lambda_2}$, we have $(1/2-\lambda_2)t + (1/2-\lambda_3)\dfrac{1}{t} \geq (1-\lambda_2-\lambda_3)$. This also implies $\pi(\eta^{XY},\tilde{\eta}) > \pi(\lambda,\tilde{\eta})$ holds for any $\varepsilon \in (0,1)$. Similar results are also available for the other cases. The third conclusion is then proved.

Comparing Theorem 1 with Table 2, we can see that the equilibrium structure of labor division predicted by new classical economics is exactly as the same as the ESS we have shown above. Such a structure of labor division cannot be attained in the Walrasian regime; however, the evolutionary forces in the economy will spontaneously coordinate individuals' decisions to reach the structure of labor division which generates maximal equalized fitness among the population under the law of "survival of the fittest". This gives a possible explanation to the central idea of new classical economics that "the function of the market is not only to allocate resources for a given network structure of division of labor, but also to coordinate all individuals' decisions in choosing their patterns of specialization to utilize positive network effects of division of labor net of transaction costs" (Sun, Yang & Yao, 1999).

5. Further discussion

We have shown the existence of the evolutionarily stable structure of labor division in Section 3. There are two problems worthy of further discussion.

First, Theorem 1 only points out ESS for situations where maximal equalized fitness is obtained on a unique mixed strategy. What will happen to the other situations? It is clear, once any two of the three values (k_0, c, k) are equal and greater than the other, there will not be any uninvidable strategy. For example, suppose $k = k_0 > c$. In such a situation, the utility obtained from the market X-Y where every one choosing the mixed strategy η^{XY} is equal to that from autarky. When all individuals are choosing η^{XY} (or η^{A}) it is still possible to survival by doing autarky η^{A} (or η^{XY}), since both of the two mixed strategies generates the same fitness and will not be ruled out by "natural selection". Thus, The maximal equalized utility will be attained in any population state that is a convex combination of η^{A} and η^{XY} . The coexistence of autarky and labor division will be observed, which leads to an uncertain boundary of the market. Since any mixed strategy is invadable, there is not a ESS; however, the economy will only randomly wonder on a set of certain population states, which are said to be nutrally stable (Weibull, 1995, p.46).

Secondly, it has been shown that the above equilibrium structure of labor division is Pareto efficient (Yang, 2003, p336). But we can go further. Our main concern is the fact that ESS extremely rejects inequality among individuals. As the outcome of selection, in the stable state only the fittest survive. Any difference of fitness between two phenotypes implies that the phenotype (configuration or pure strategie) with less fitness will eventually disappear in the evolutionary process. Thus any genotype (mixed strategy) that generates differences of fitness among phenotypes will not prevail in the stable state. This property of ESS implies the reasonability of the aversion of inequality as a welfare criterion in the sense that it always chooses a population state that coincides with the outcome of natural selection. This point is summarized in the following theorem:

Theorem 2. In the evolutionarily stable state, resource allocation is not only Pareto optimal but also beneficent in the sense that it maximizes an inequality-averse social welfare.

To see the conclusion of this theorem is true, we can assume a fictitious social welfare planner who maximizes an anonymous[3] social welfare function $w[(u^i)_{i\in I}]$ by choosing a common mixed strategy for all individuals[4], where u^i is the utility of individual i. Suppose $k > k_0$ and $k > c$. The planner will allocate all population into the market X-Y. Because while $\eta_2 \to 0$, we have $\pi(S_2,\eta) \to \infty$ and $\pi(S_3,\eta) \to 0$; while $\eta_2 \to 1$, we have $\pi(S_2,\eta) \to 0$ and $\pi(S_3,\eta) \to \infty$, the social welfare planner can makes some individuals obtain very high utility by decreasing the size of the population who enjoy such utility. If the planner does not averse inequality, $w[(u^i)_{i\in I}]$ is at most of Utilitarian form[5]. Given the property of anonymity, it is an unweighted sum of all individuals' utilities. Hence, the planner's decision problem is to choose a $\eta \in \Delta$ to maximize $\eta_2\pi(S_2,\eta) + \eta_3\pi(S_3,\eta) = \frac{k}{4}[\eta_2 + \eta_3]$. Any $\eta \in \Delta$ such that $\eta_2 + \eta_3 = 1$ and $\eta_2\eta_3 > 0$ might be the optimal choice of the social welfare planner. The possibility of deviating from ESS makes Utilitarian a less favorable criterion of public choice comparing with those, more or less, taking into account of the importance of equality.

6. Concluding remarks

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3 A social welfare function $w[(u^i)_{i\in I}]$ is anonymous if and only if it does not favor any particular individuals, i.e. for any bijection $\varphi: I \to I$, there is $w[(u^i)_{i\in I}] = w[(u^{\varphi(i)})_{i\in I}]$. See Sen (1970) for a more complete discussion.

4 This is equivalent to assuming the planner chooses a common mixed strategy for all individuals.

5 It could also be an inequality-preference type that makes the social welfare contours concave to the origin.

The existing literature of new classical economics abstracts the competitive market as a Walrasian regime. This approach neglects the very important features of real economy as a "field" of evolutionary games. As we have shown, Walrasian regime can not sufficiently explain the automatic determination of labor structure that exists in our real society (Section 2). However, some certain network of labor division among individuals can be attained in the environment of evolutionary games as the evolutionarily stable state (Section 3 and 4). It seems to be necessary to introduce the evolutionary aspects of the economy into new classical framework while the theoretical analysis has been extended to endougenize labor division.

Since Adam Smith (1776), the thought of "invisible hand" has been at the heart of the economic analysis. Vaughn (1989) summarized the idea in three separate claims: (1) individuals' behaviors can lead to some social order; (2) the social order is not intended consequences of decentralized decisions (no planning, no bargaining); (3) the social order is beneficent. The first two claims are also named as "spontaneous order" by Hayek (1973). Our analysis has covered all of these three claims. As shown in the previous sections, in the evolutionary environment of economy, individuals' pursuits of self-interest can result in a spontaneous order of labor division without any central planning or bargaining process that artificially coordinate the separate decision makings; and the consequences are proved to be not only Pareto efficient, but also beneficent as if it was chosen by a welfare planner who holds a welfare criterion with aversion of inequality.

Because our main concern is the determination and emergence of stable structure of labor division, we consider neither the technology advance nor the economic development driven by that. However, our model shows that the market can spontaneously "find" the most efficient structure of labor division. Thus, when the transaction efficiencies increase, the dynamics of evolution will lead the population state from autarky to a new state where people “cooperate” in the network of labor division and enjoy a life with higher utilities. This is another type of development.

As an elementary attempt, this paper does not take into account of any dynamic mechanisms of selection or mutation. The concept of ESS, as with Nash equilibrium, does not explain how a population arrives at such a state; however, as noticed by many economists, this concept provides a robustness criterion for human behaviors in a wide range of situations, which gives a reasonable interpretation of *convention* (Weibull, 1995, p33).

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